UNIVERSITÄT
TRIER

Mutual Witness Proximity Drawings of Isomorphic Trees

Carolina Haase Philipp Kindermann
William J. Lenhart Giuseppe Liotta

Mutual Witness Proximity Drawings

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Gabriel Drawings:

[Gabriel, Sokal ’69]

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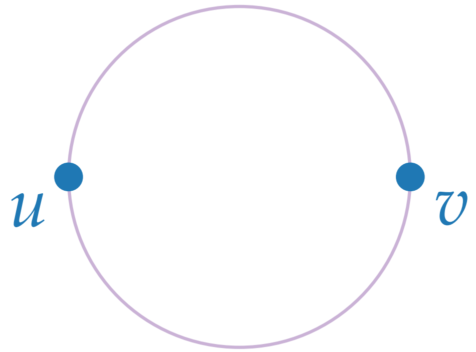

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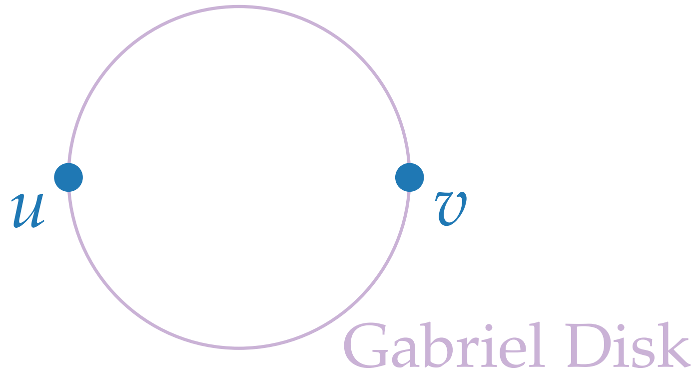

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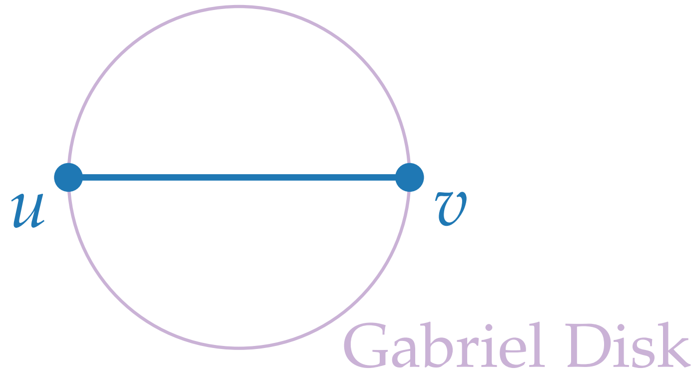

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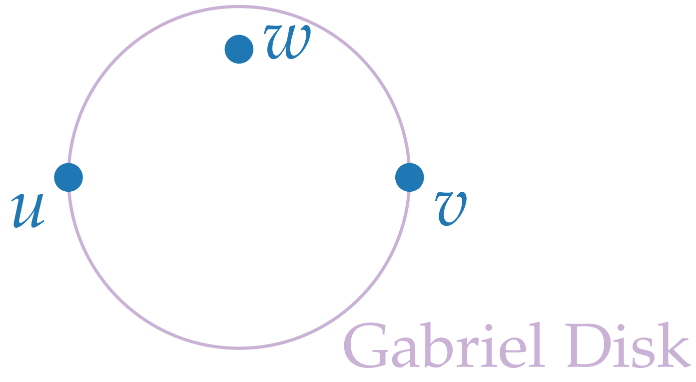

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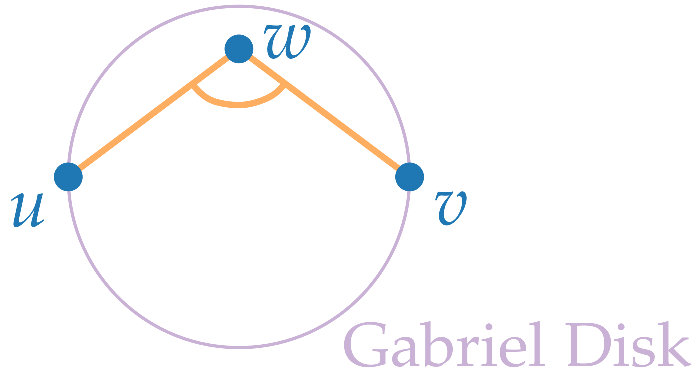

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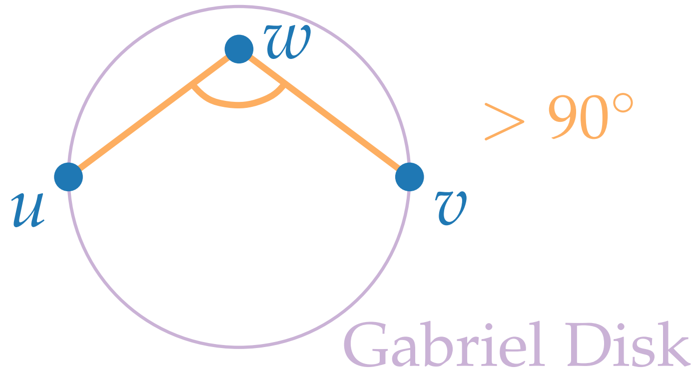

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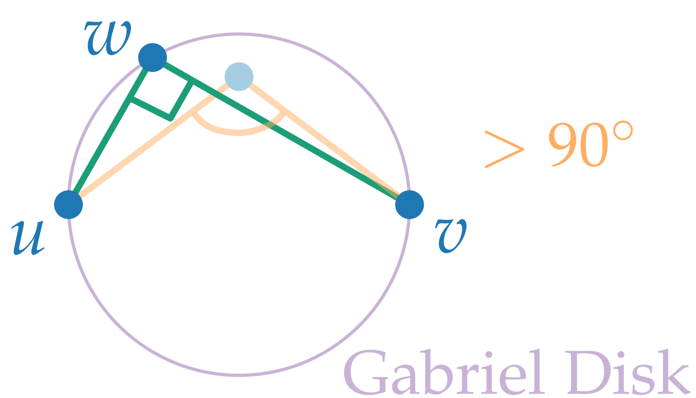

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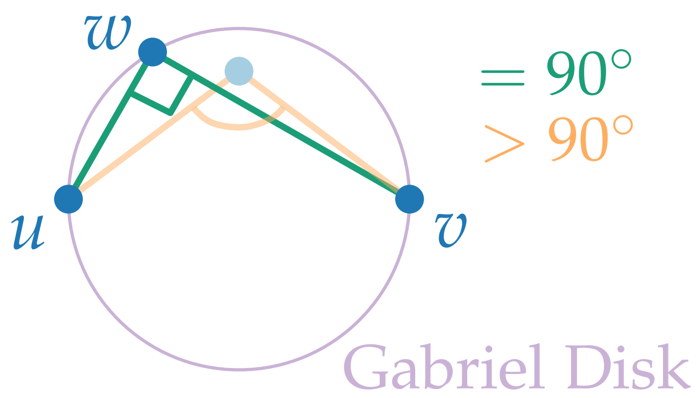

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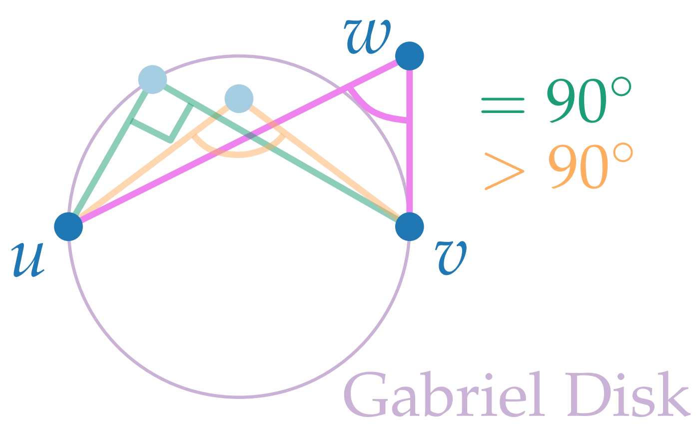

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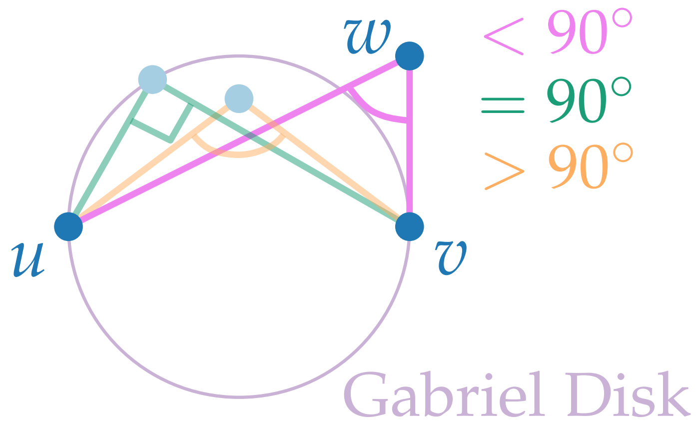

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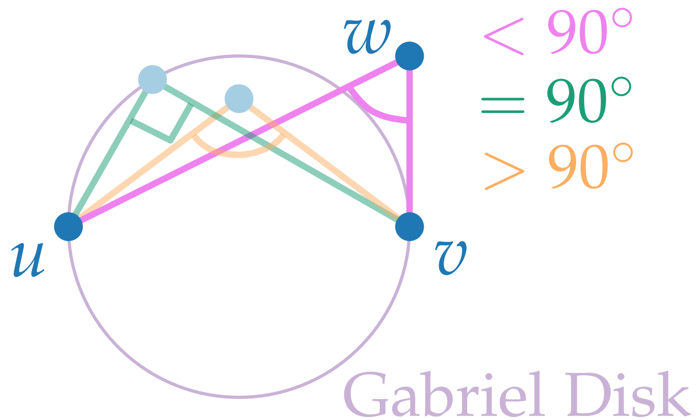

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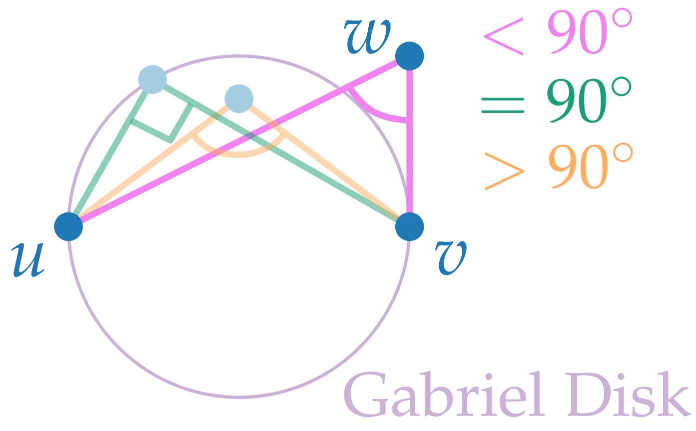

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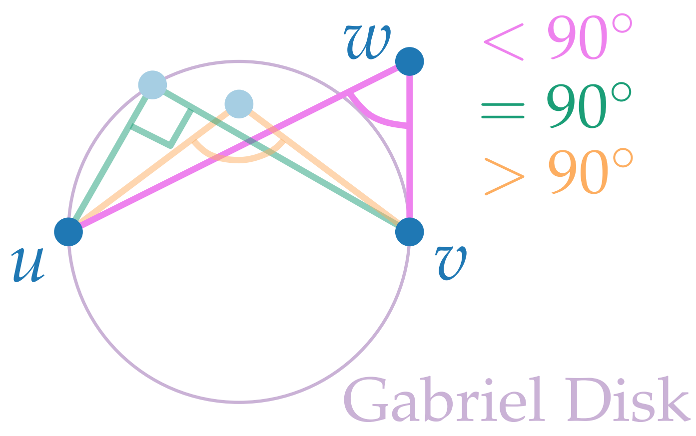

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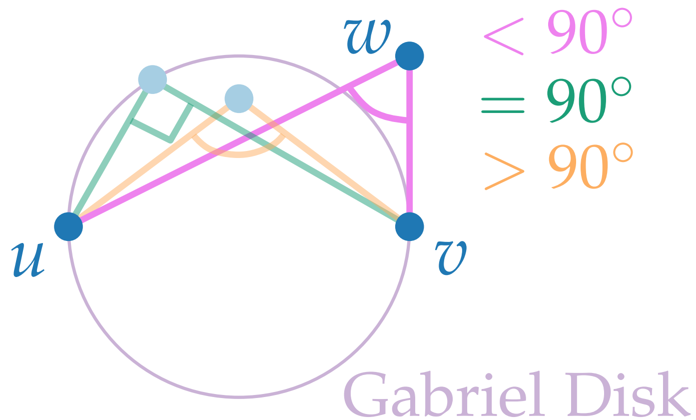

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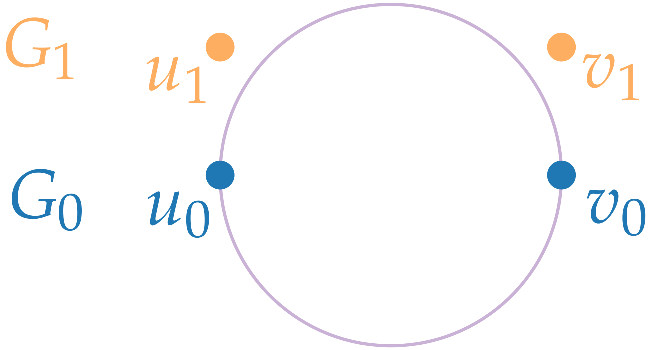

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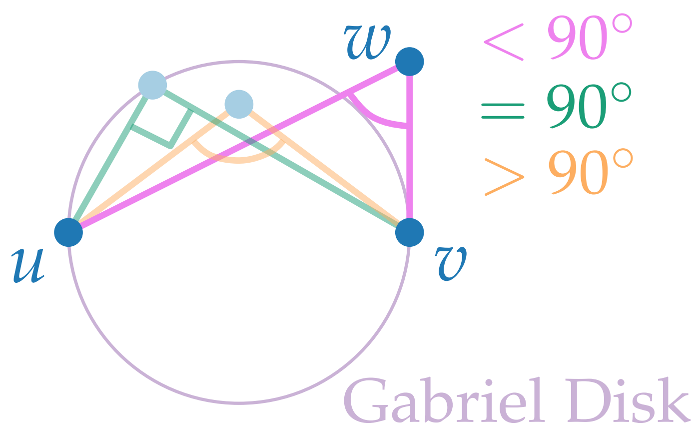

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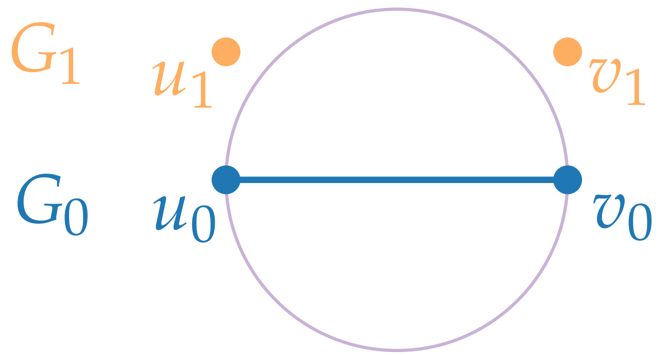

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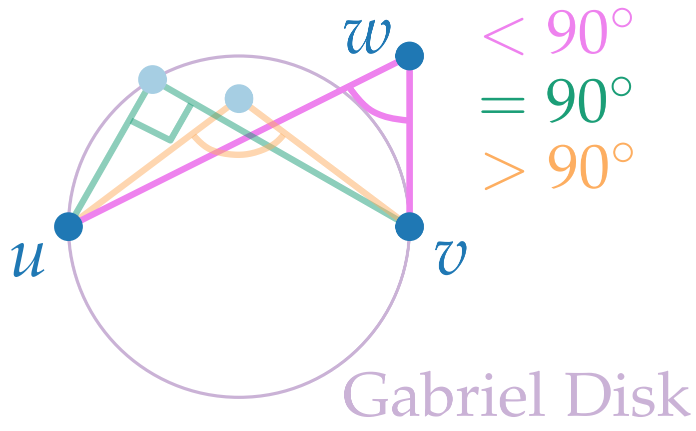

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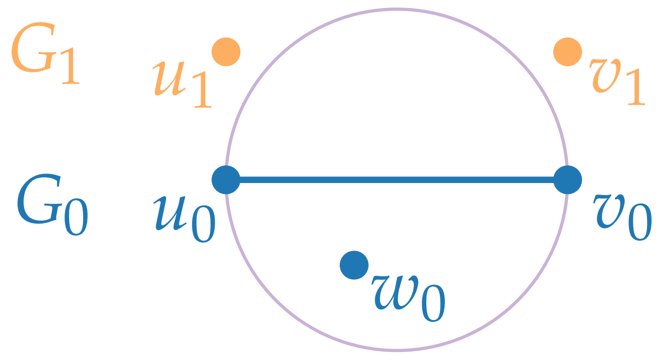

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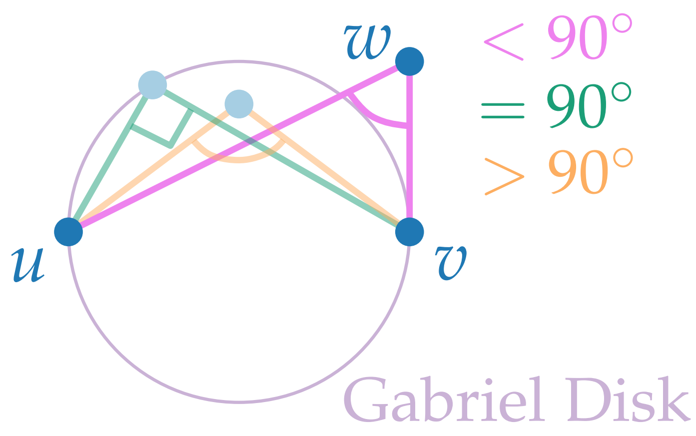

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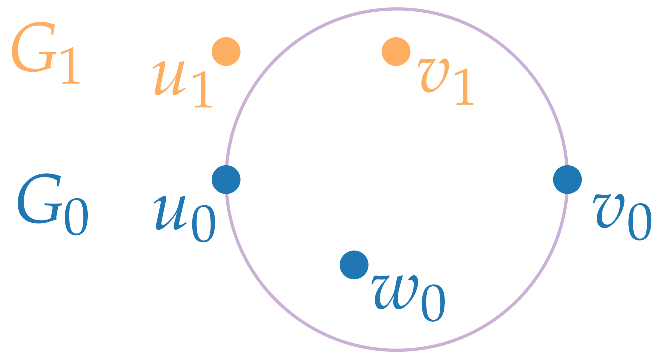

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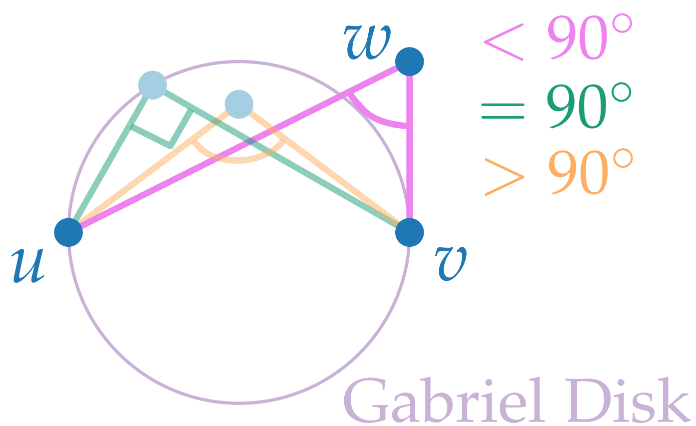

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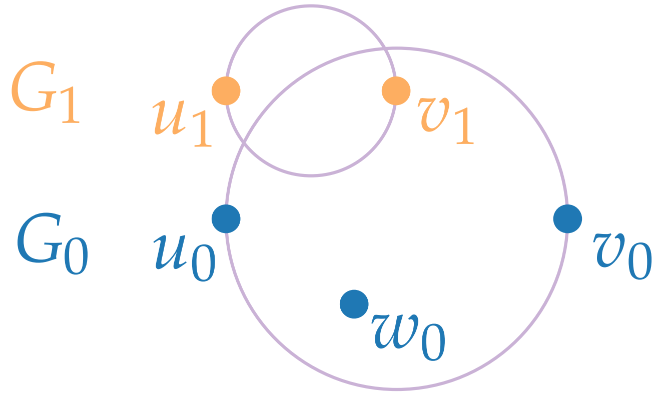

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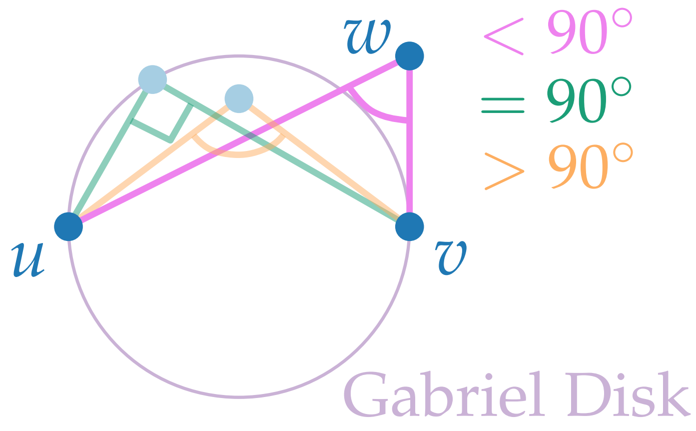

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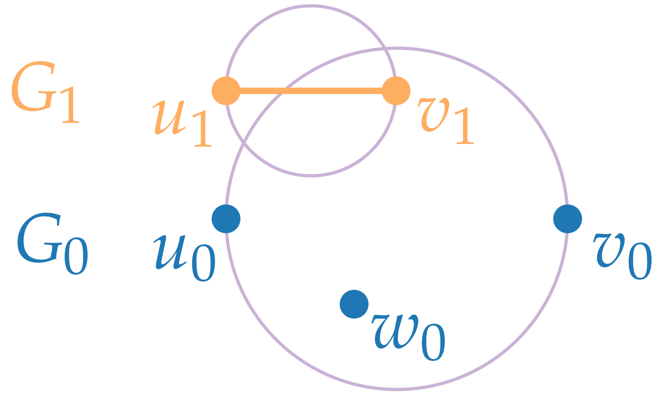

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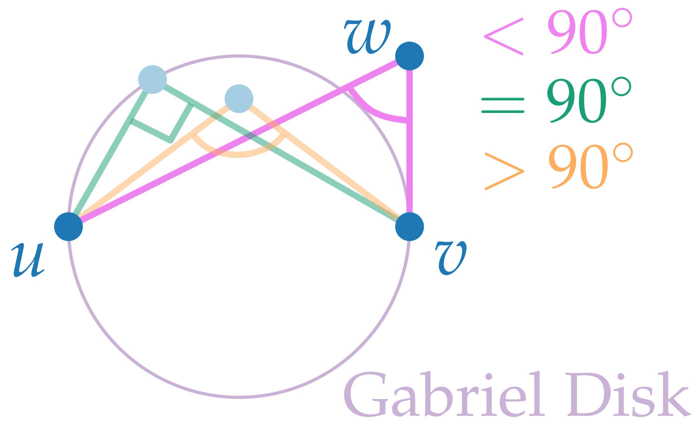

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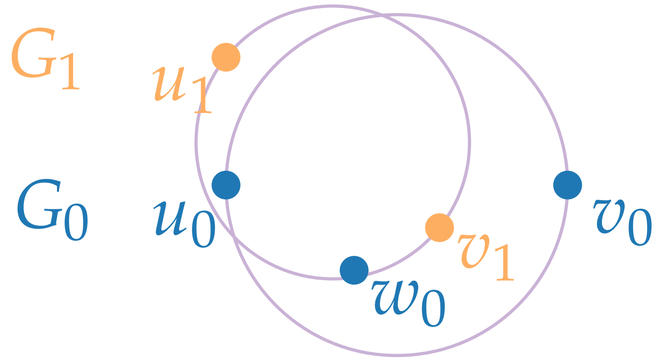

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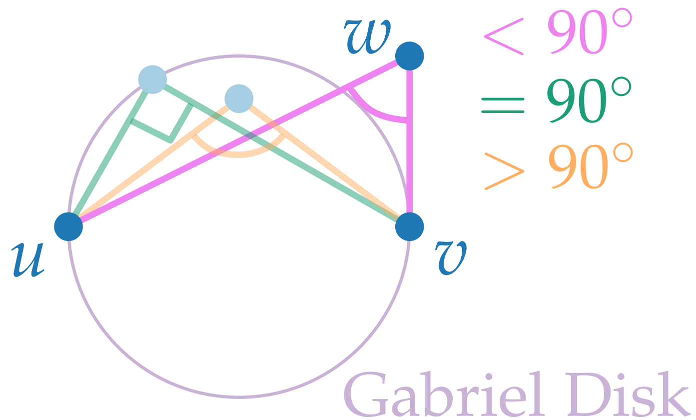

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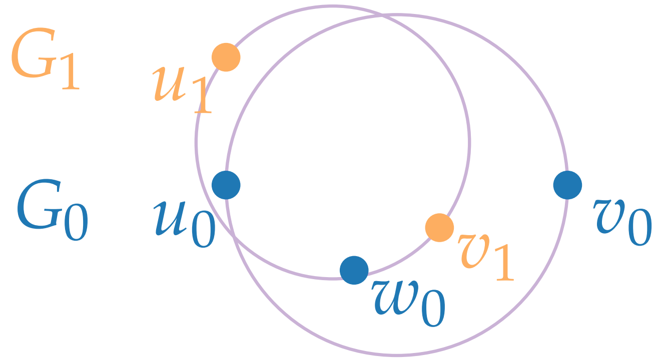

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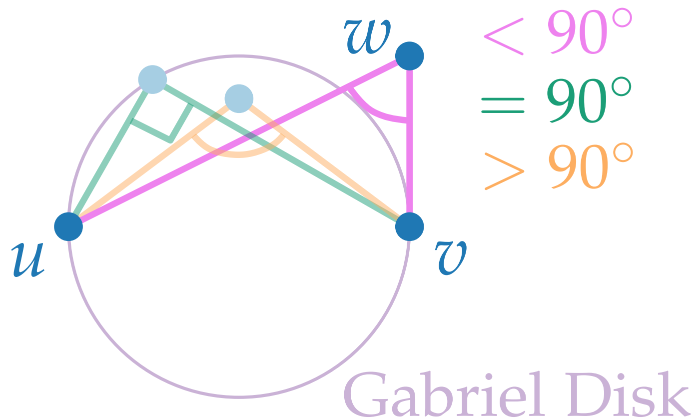

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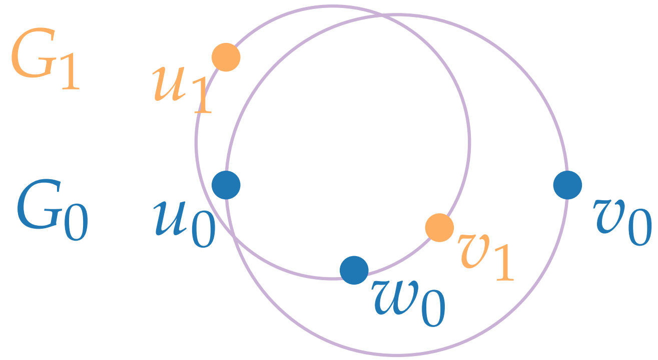

β-Proximity Drawings:

Mutual Witness Proximity Drawings

Gabriel Drawings:

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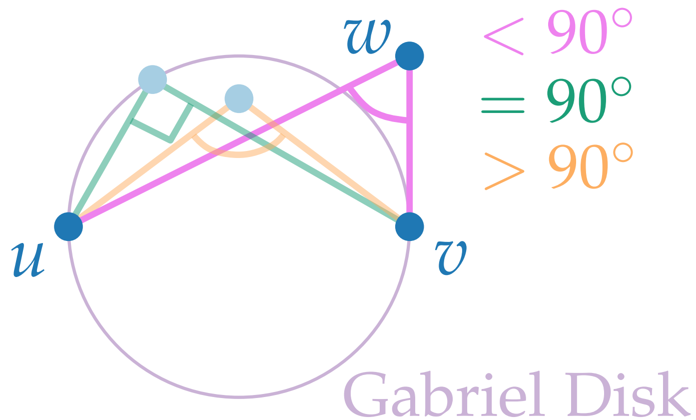

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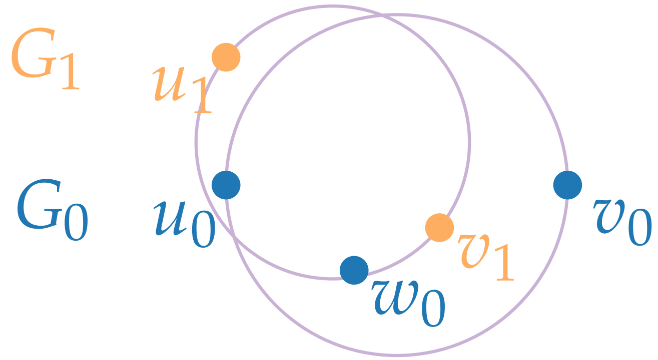

β-Proximity Drawings:

two disks:

Mutual Witness Proximity Drawings

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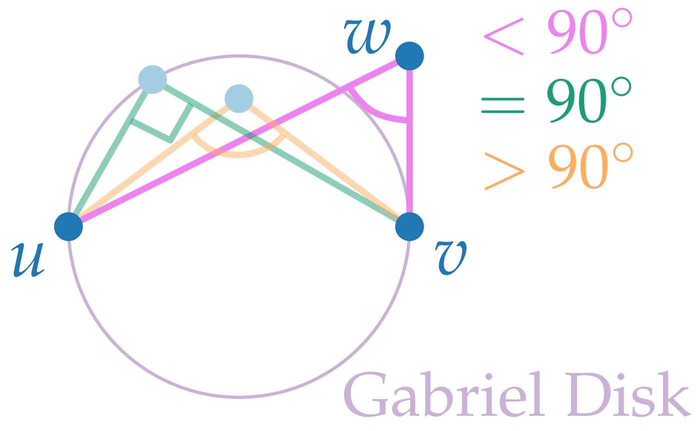

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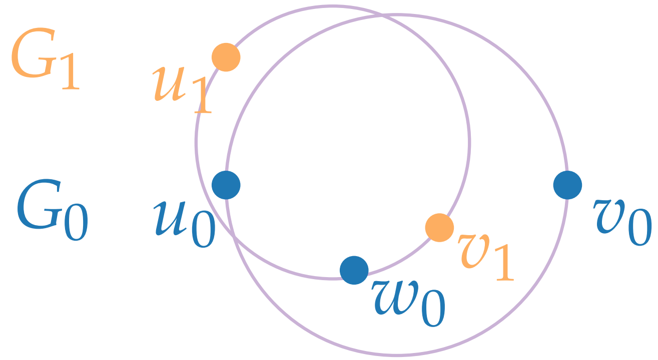

β-Proximity Drawings:

two disks:
radius $:= \frac{\beta d(u,v)}{2}$

Mutual Witness Proximity Drawings

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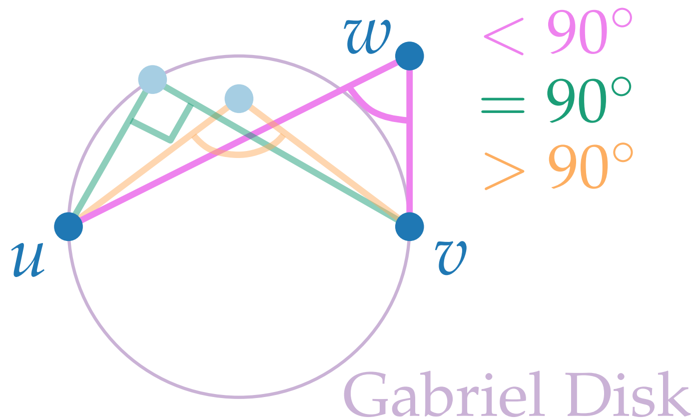

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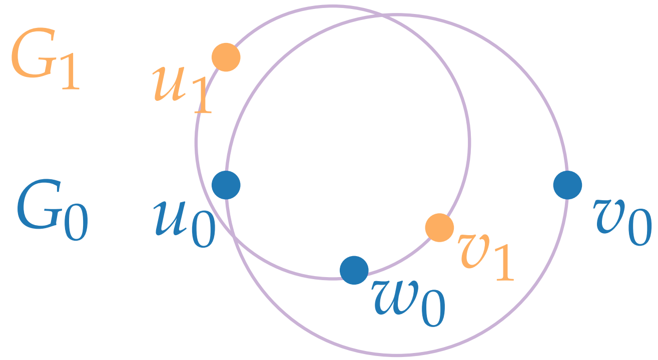
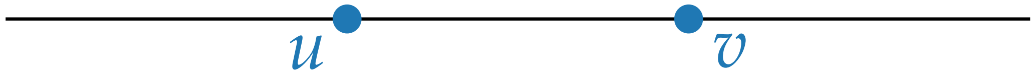


β-Proximity Drawings:

two disks:
radius := $\frac{\beta d(u,v)}{2}$

center on the line through u and v



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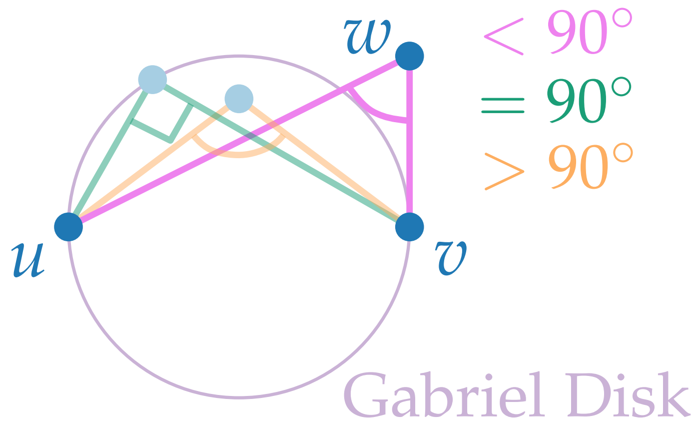

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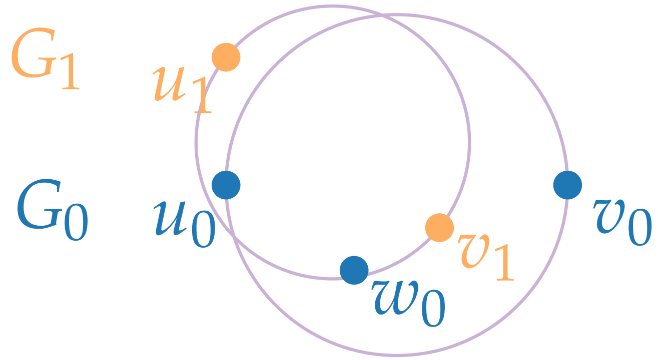

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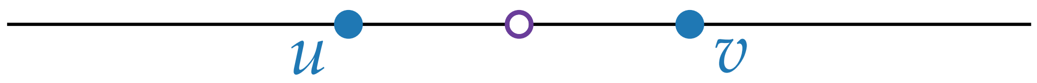

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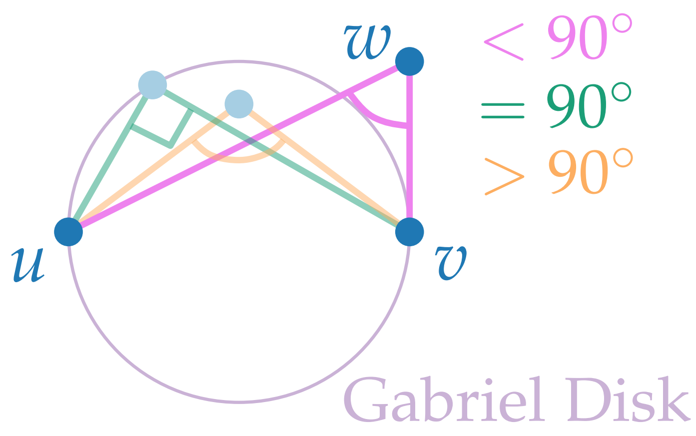

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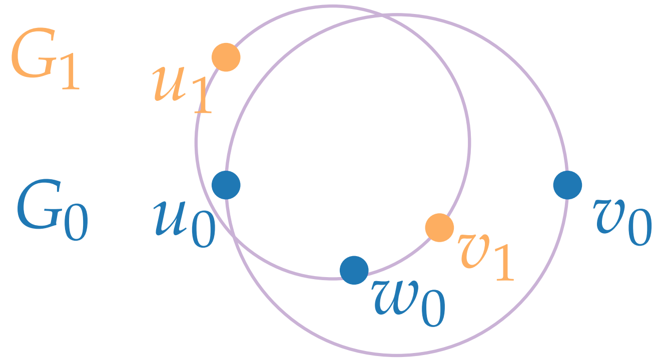

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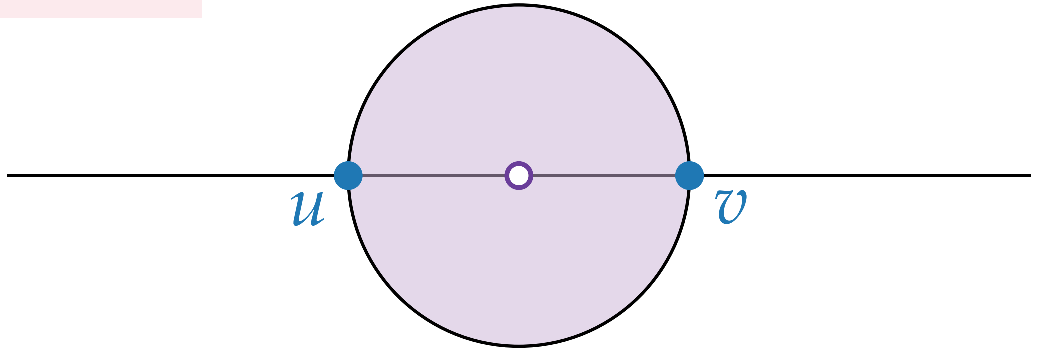

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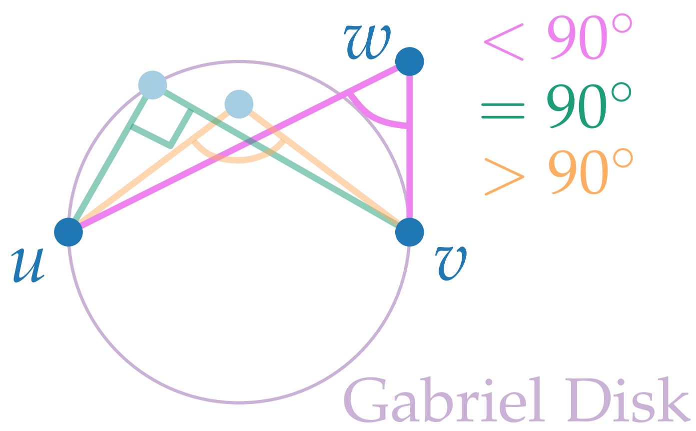

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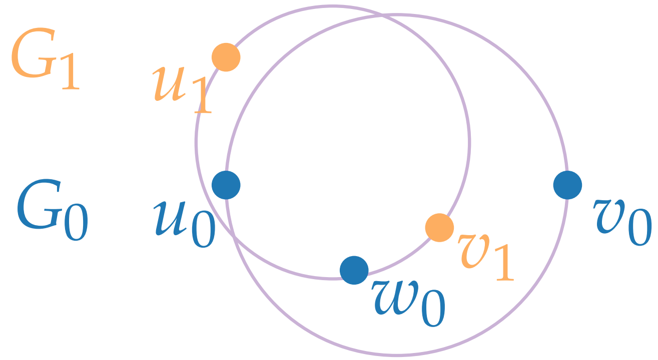

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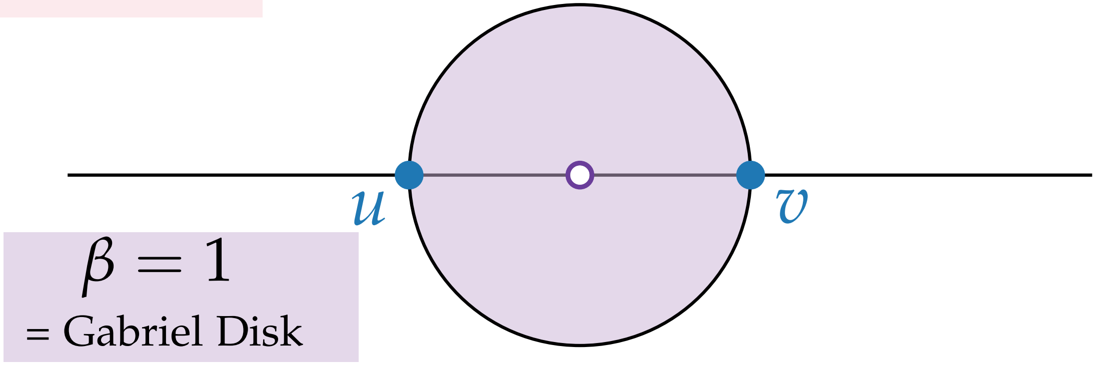

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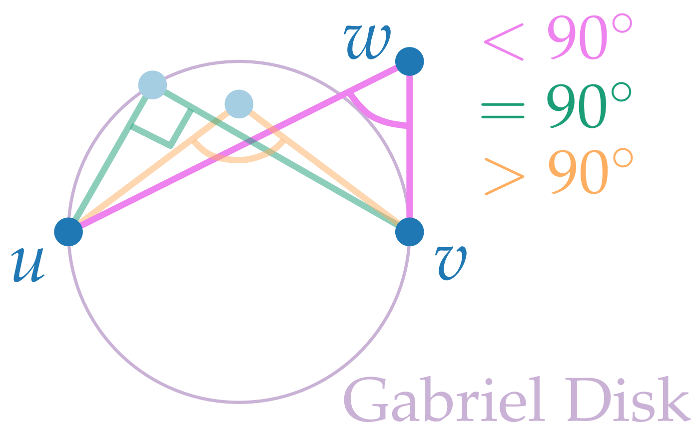

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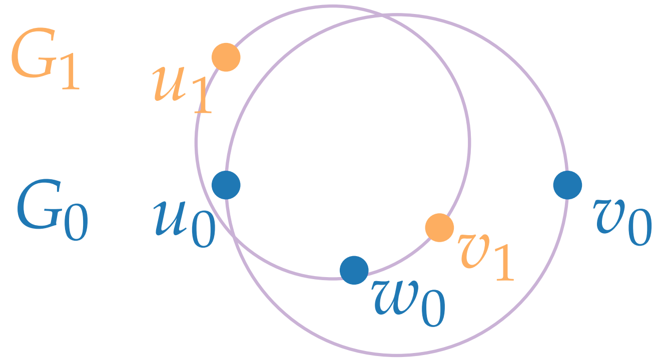

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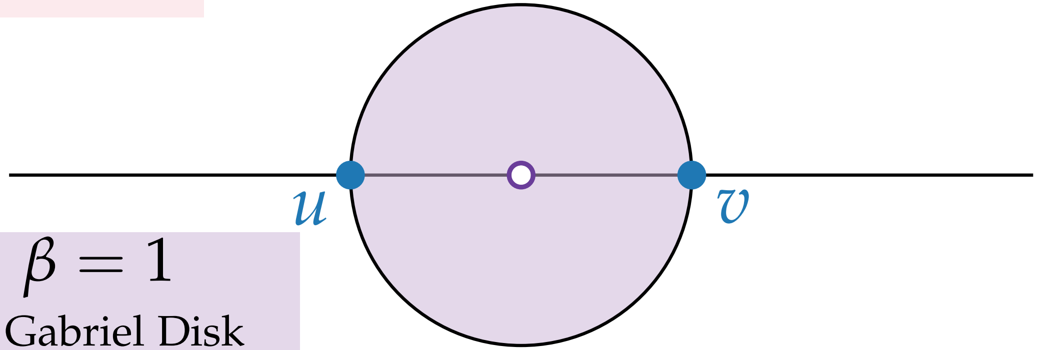

$\beta = 1$

= Gabriel Disk

$\beta = 2$

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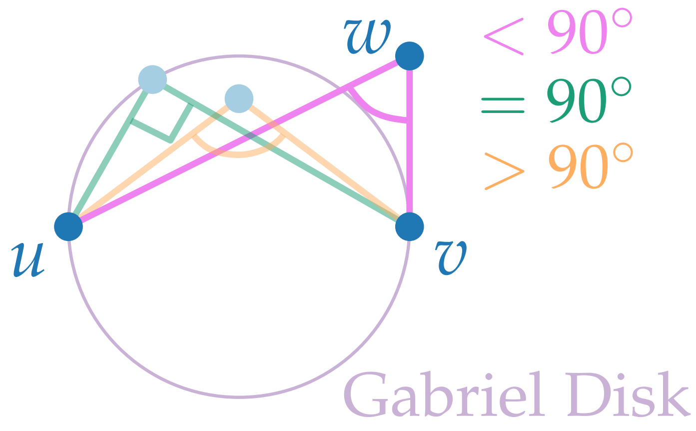

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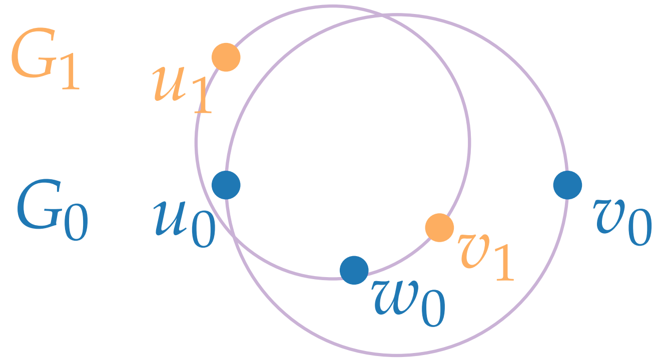

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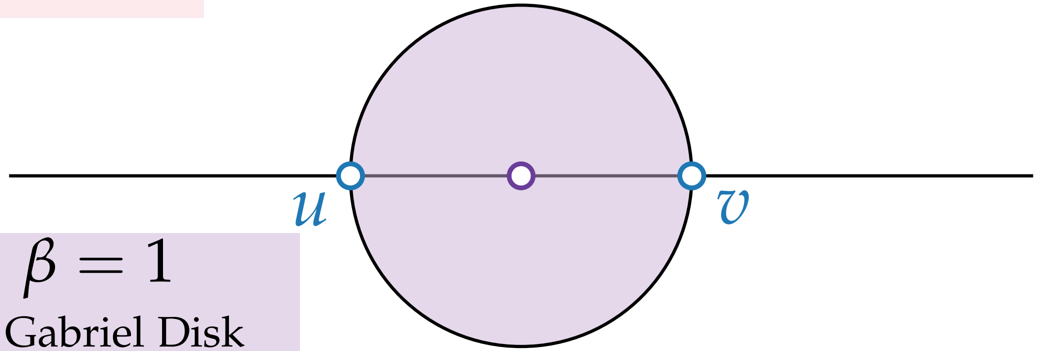

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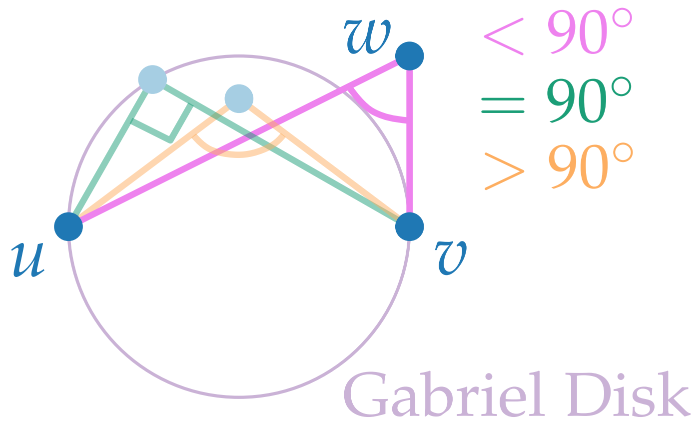

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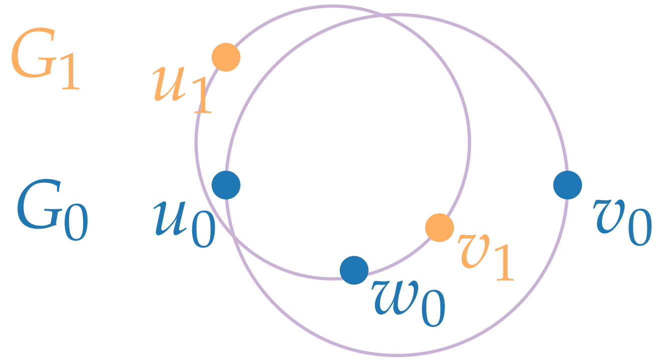

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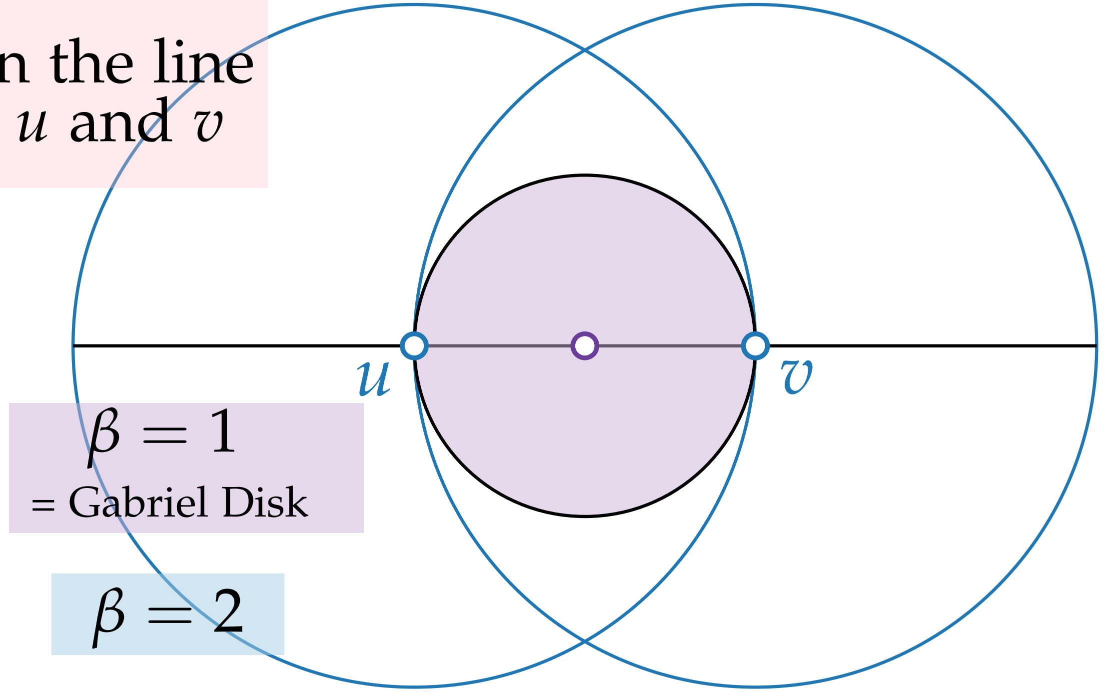

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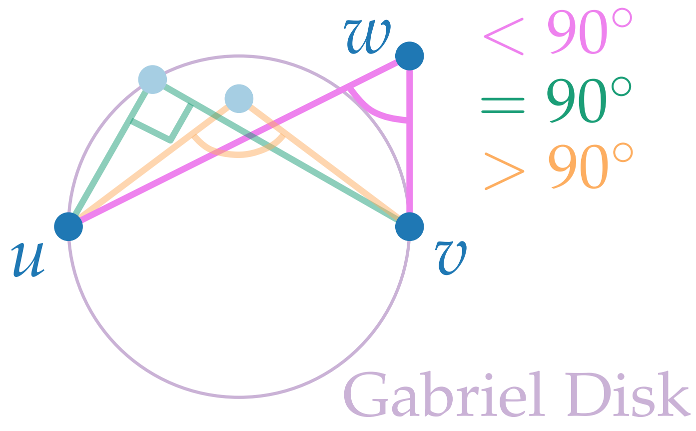

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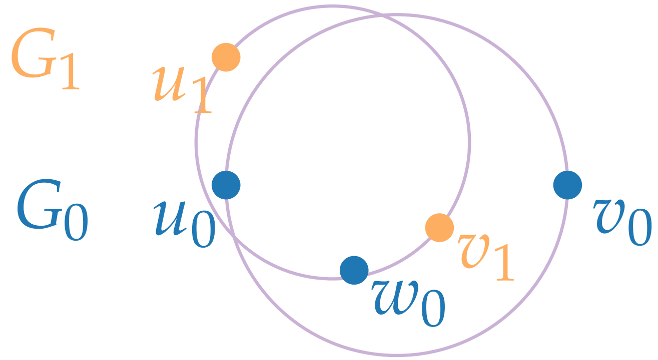

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radius := $\frac{\beta d(u,v)}{2}$

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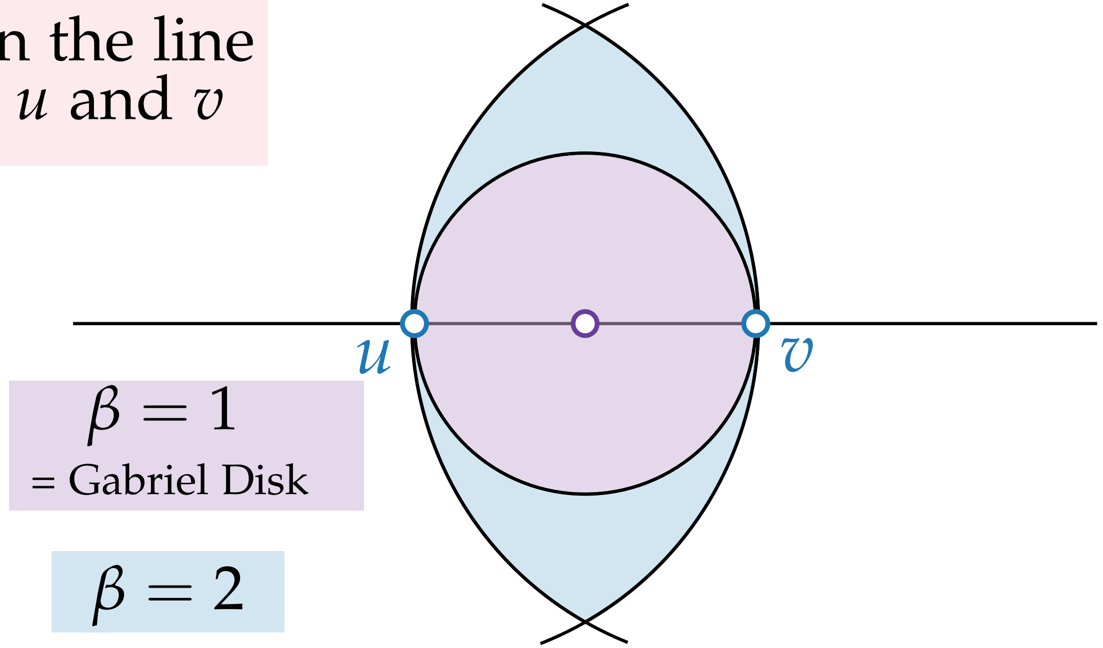

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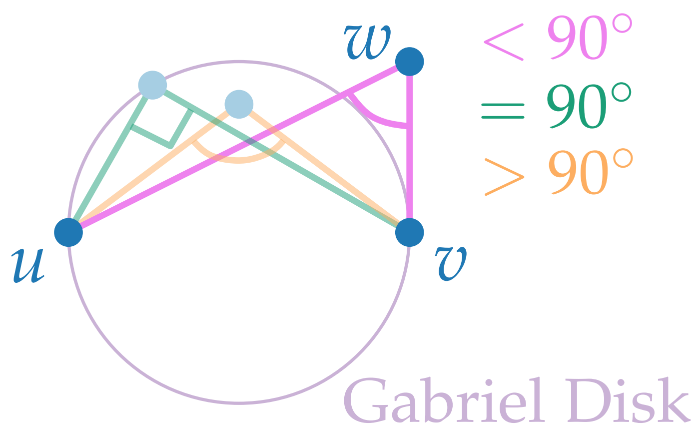

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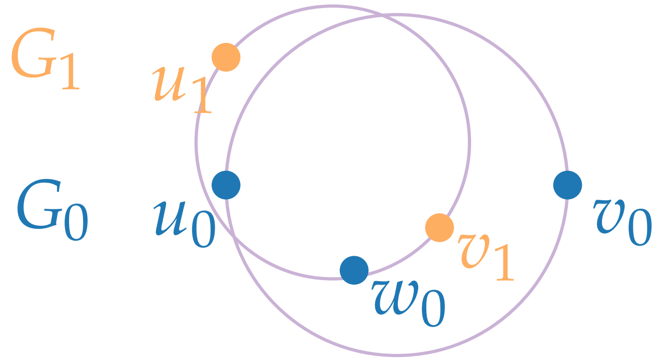

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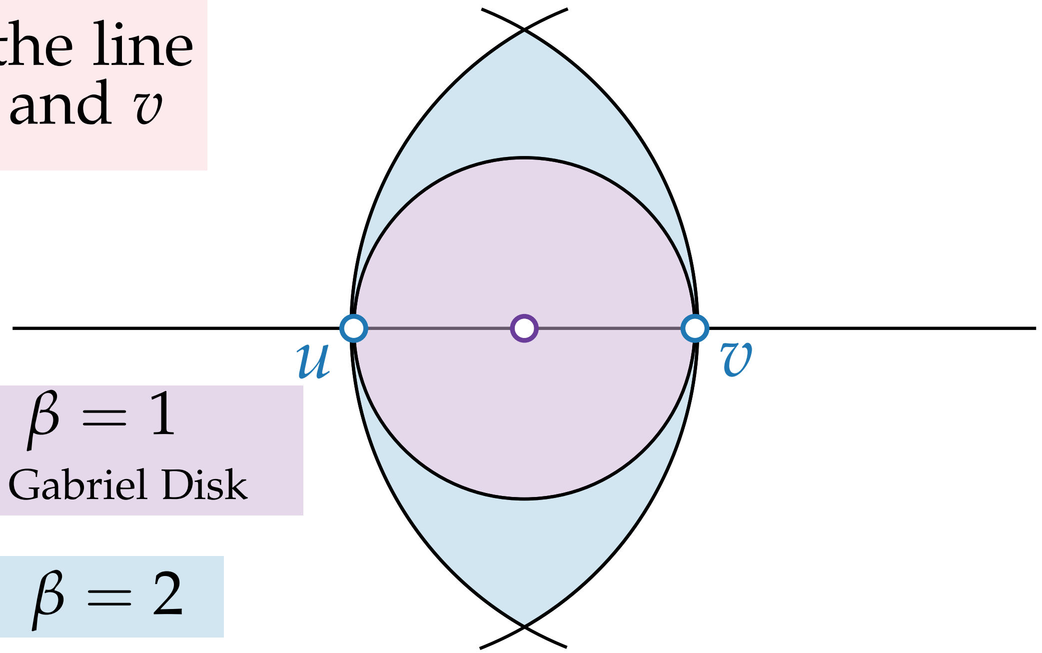


$\beta = 1$
= Gabriel Disk

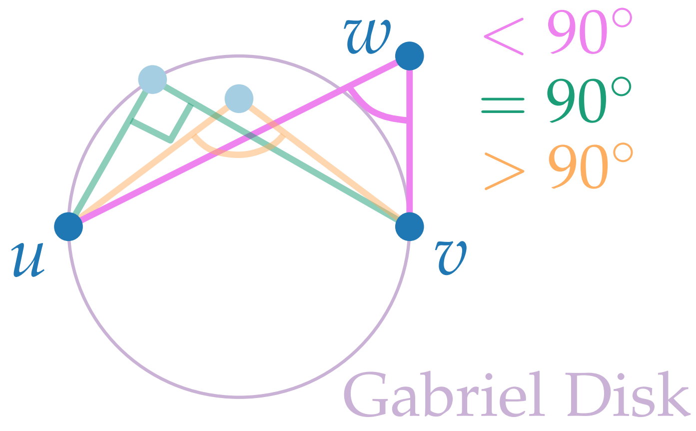
$\beta = 2$

$\beta = 4$

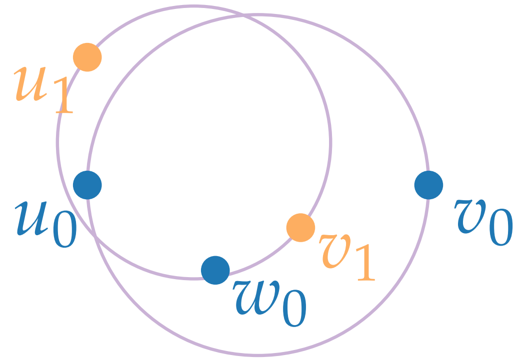
Mutual Witness Proximity Drawings

Gabriel Drawings:

[Gabriel, Sokal '69]



Mutual Witness Gabriel Drawings:

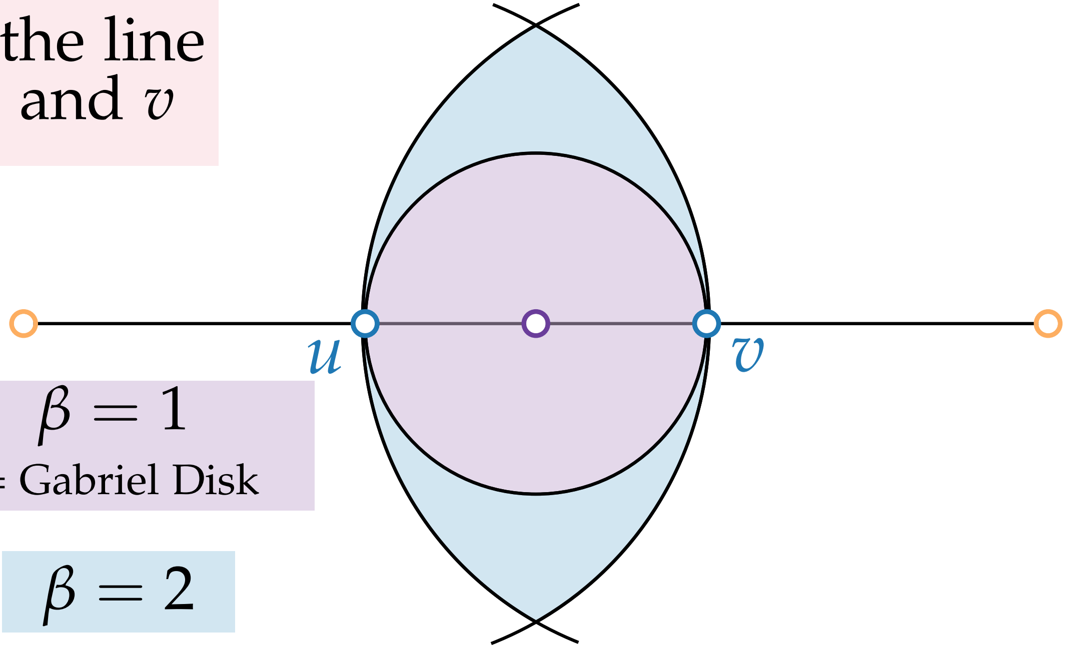


β-Proximity Drawings:

two disks:

radius := $\frac{\beta d(u,v)}{2}$

center on the line through u and v



$\beta = 1$

= Gabriel Disk

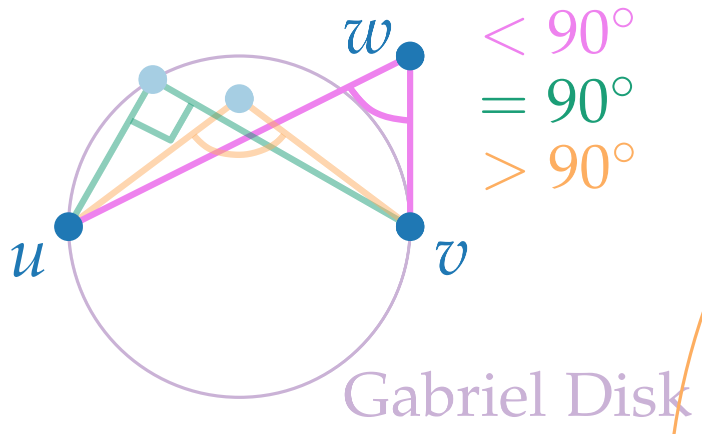
$\beta = 2$

$\beta = 4$

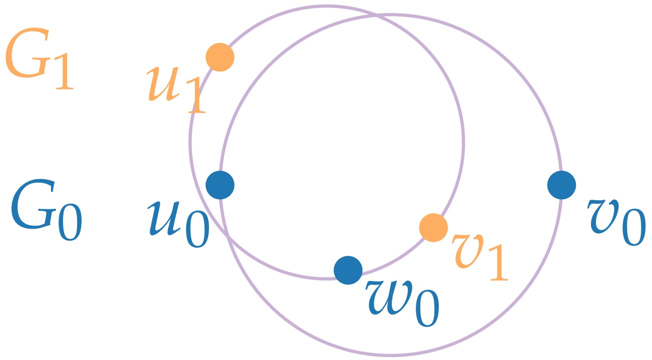
Mutual Witness Proximity Drawings

Gabriel Drawings:

[Gabriel, Sokal '69]



Mutual Witness Gabriel Drawings:



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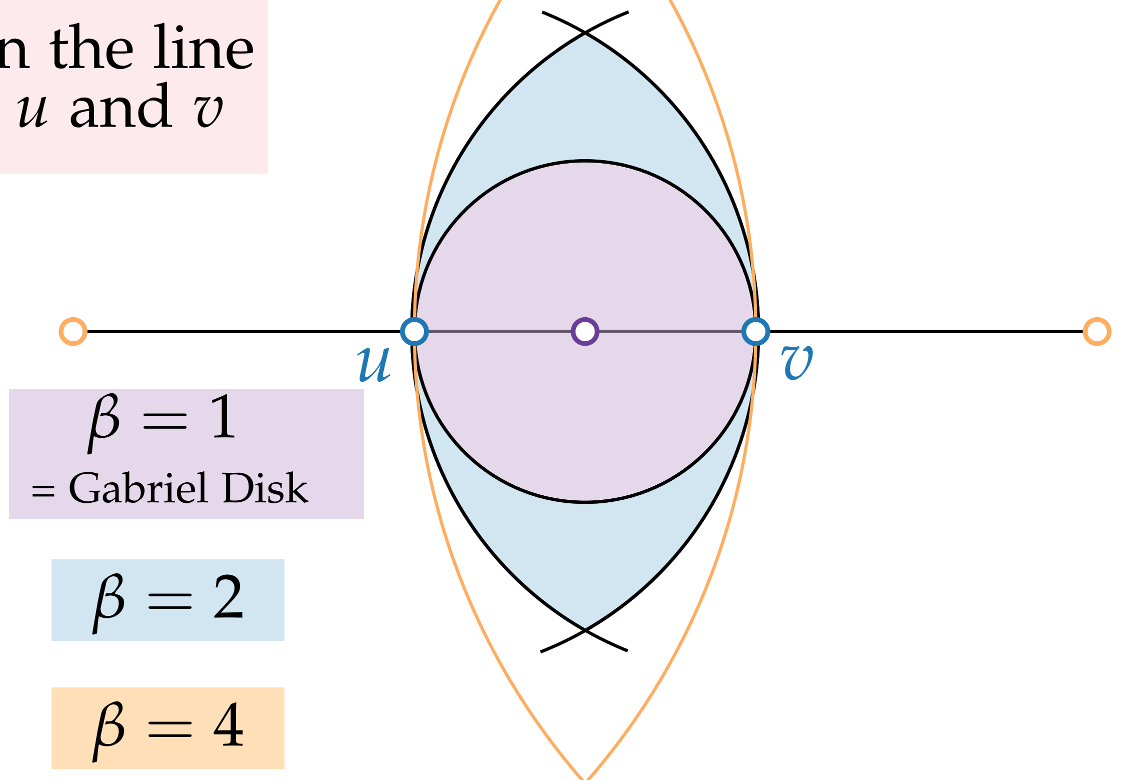
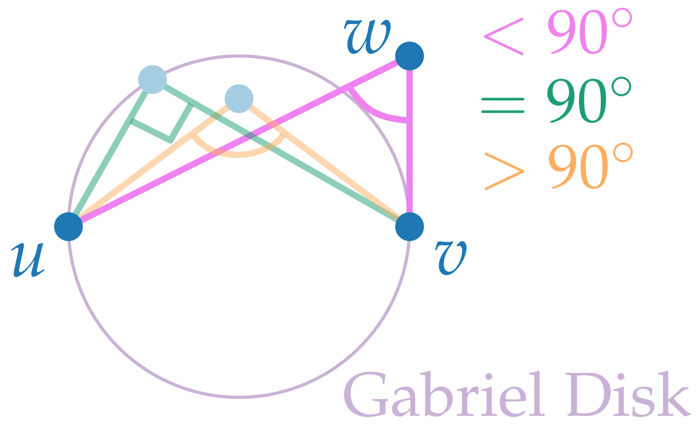


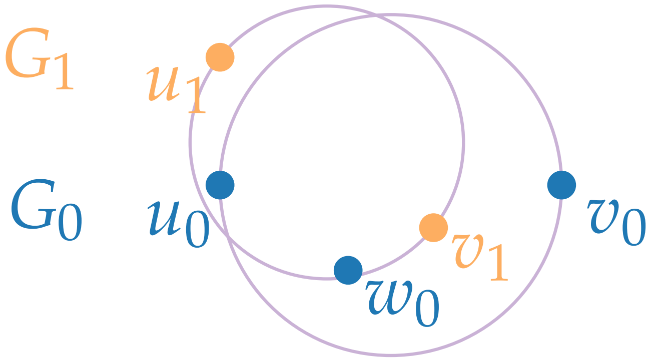
Mutual Witness Proximity Drawings

Gabriel Drawings:

[Gabriel, Sokal '69]



Mutual Witness Gabriel Drawings:



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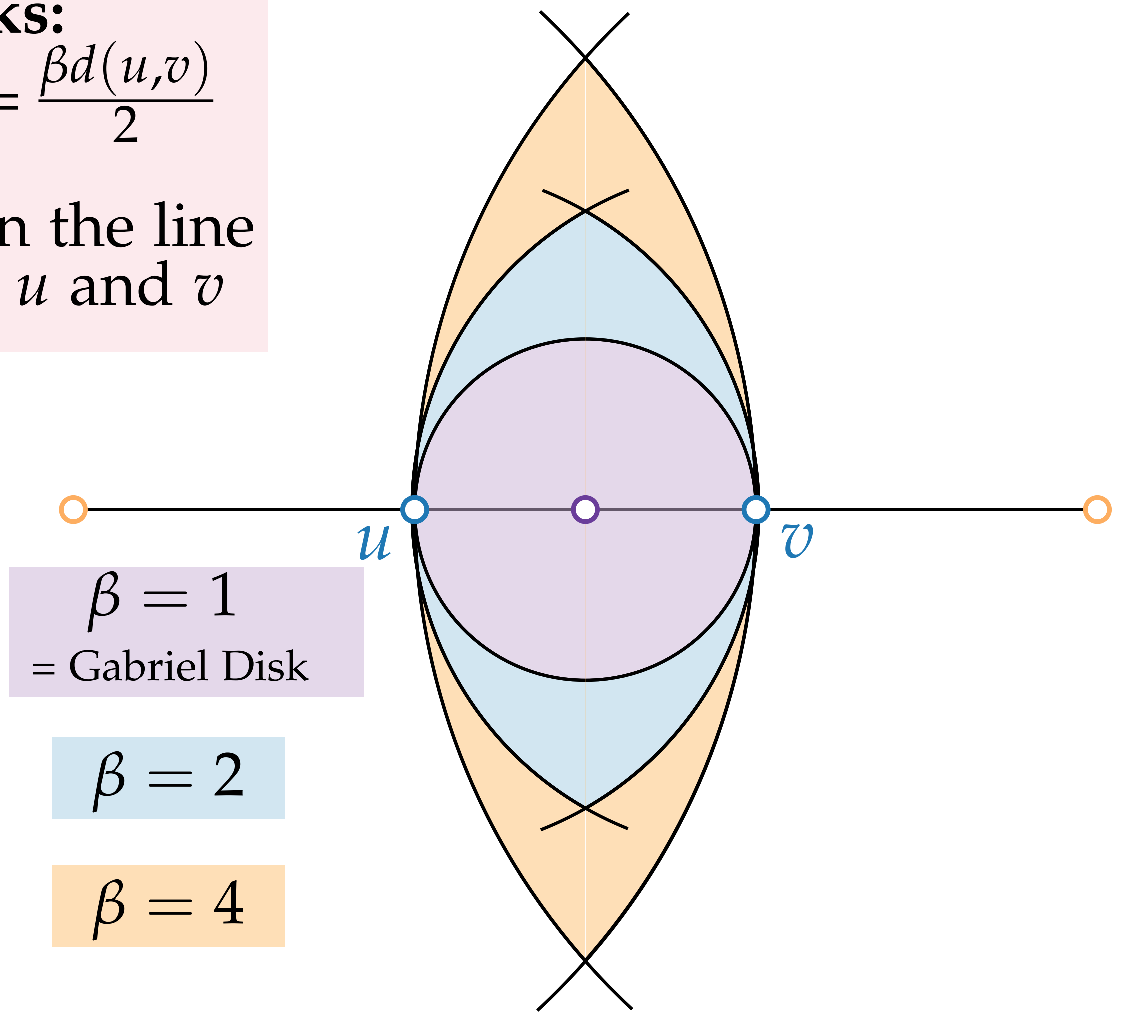
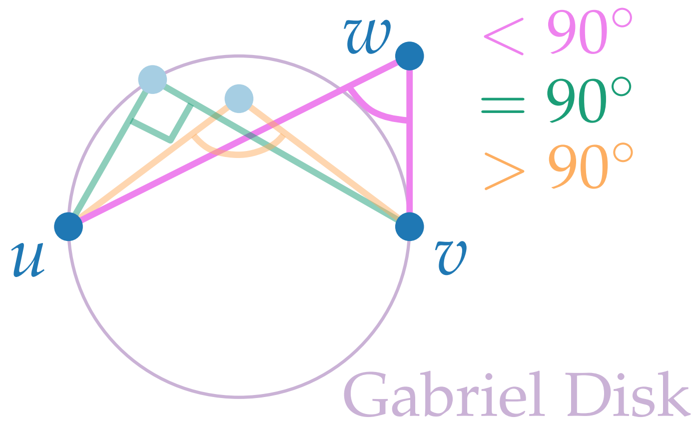


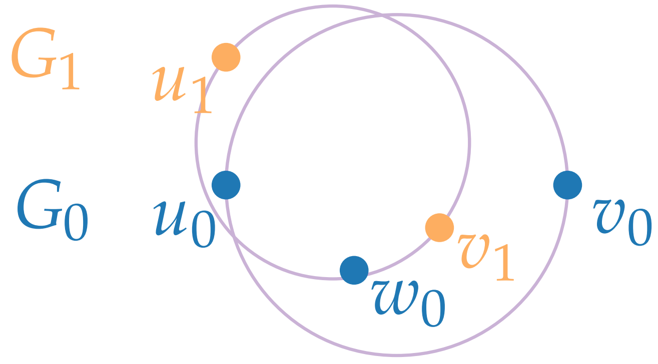
Mutual Witness Proximity Drawings

Gabriel Drawings:

[Gabriel, Sokal '69]



Mutual Witness Gabriel Drawings:



β-Proximity Drawings:

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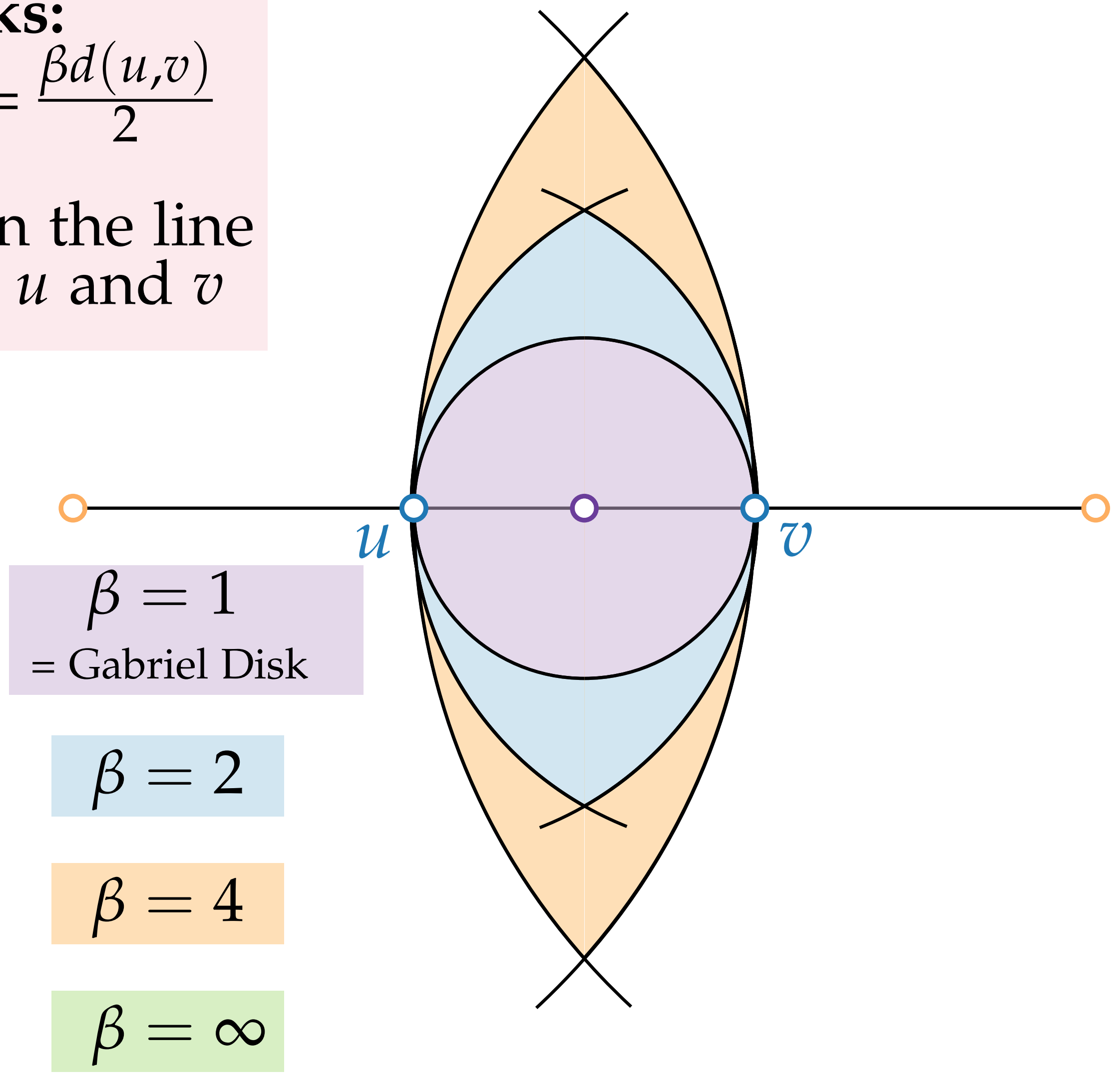
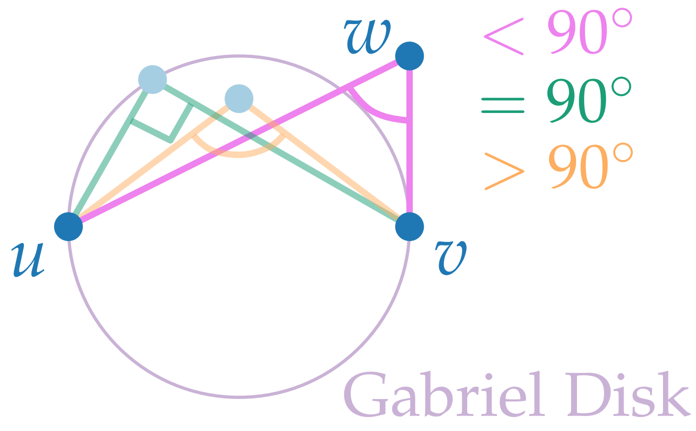


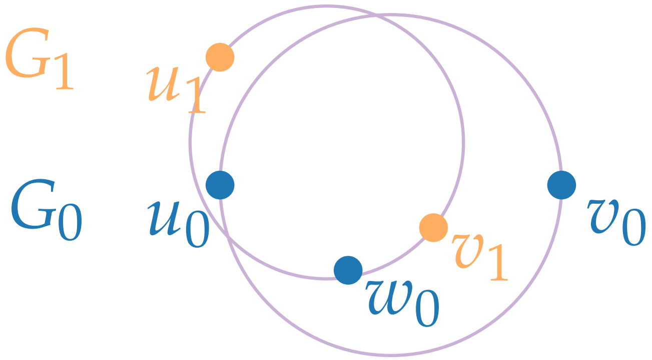
Mutual Witness Proximity Drawings

Gabriel Drawings:

[Gabriel, Sokal '69]



Mutual Witness Gabriel Drawings:



β-Proximity Drawings:

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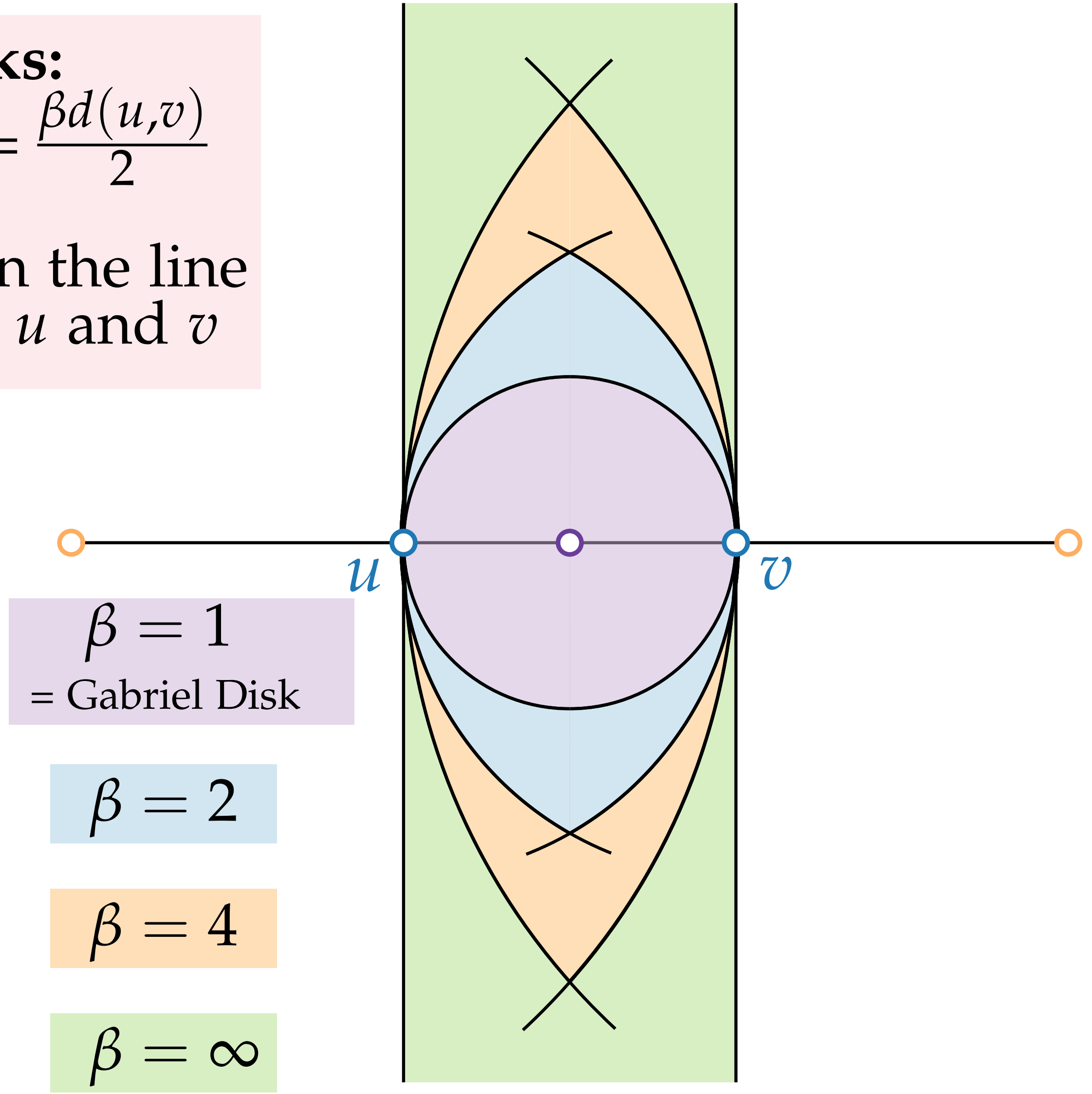

Our Contribution

Our Contribution

(1) Mutual Witness Gabriel (MWG) Drawings
of isomorphic caterpillars

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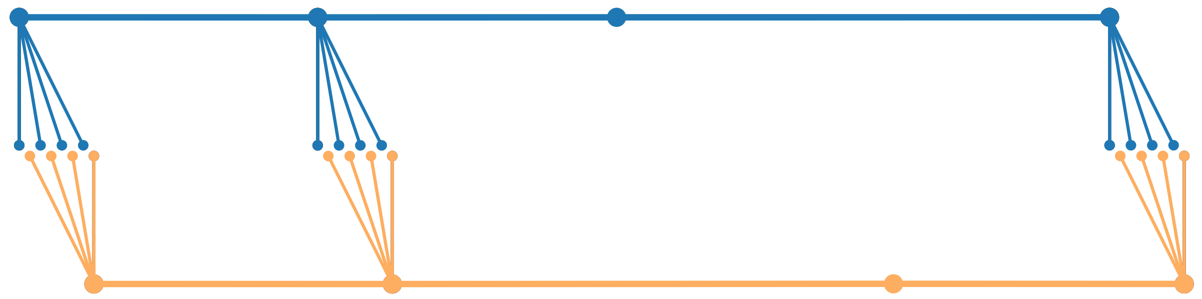
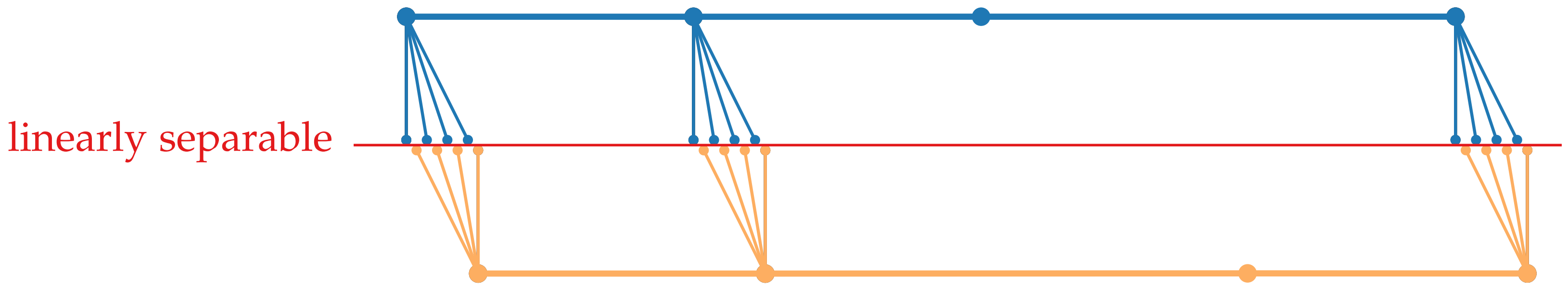


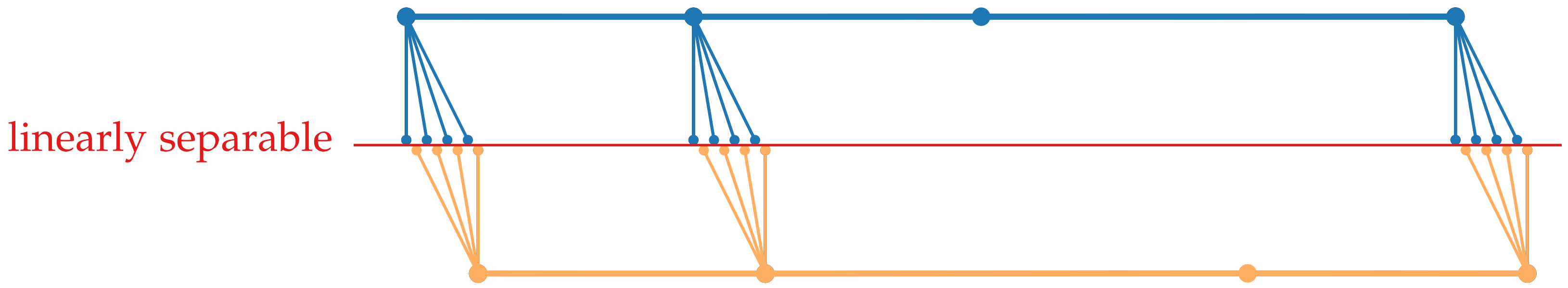
Our Contribution

(1) Mutual Witness Gabriel (MWG) Drawings
of isomorphic caterpillars



Our Contribution

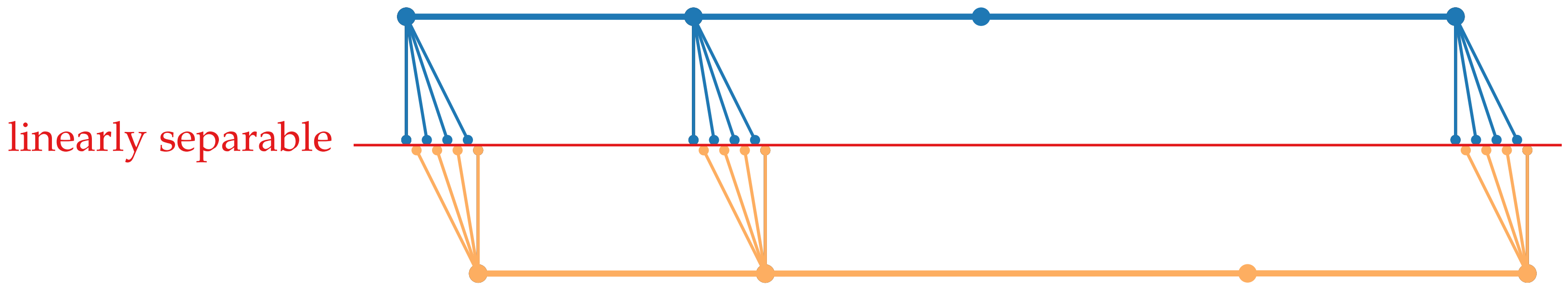
(1) Mutual Witness Gabriel (MWG) Drawings
of isomorphic caterpillars



Our Contribution

$\beta = 1$

(1) Mutual Witness Gabriel (MWG) Drawings
of isomorphic caterpillars



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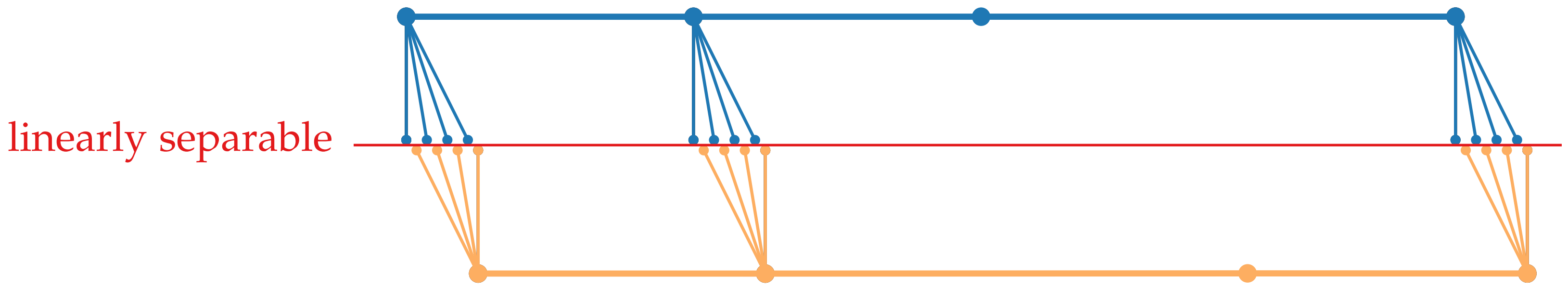
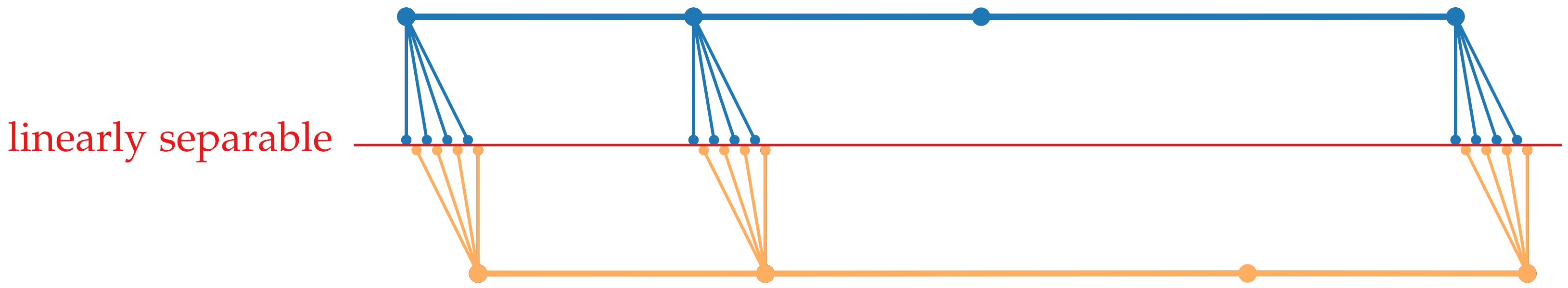


(2) Mutual Witness β Drawings
of isomorphic trees

Our Contribution

$\beta = 1$

(1) Mutual Witness Gabriel (MWG) Drawings
of isomorphic caterpillars



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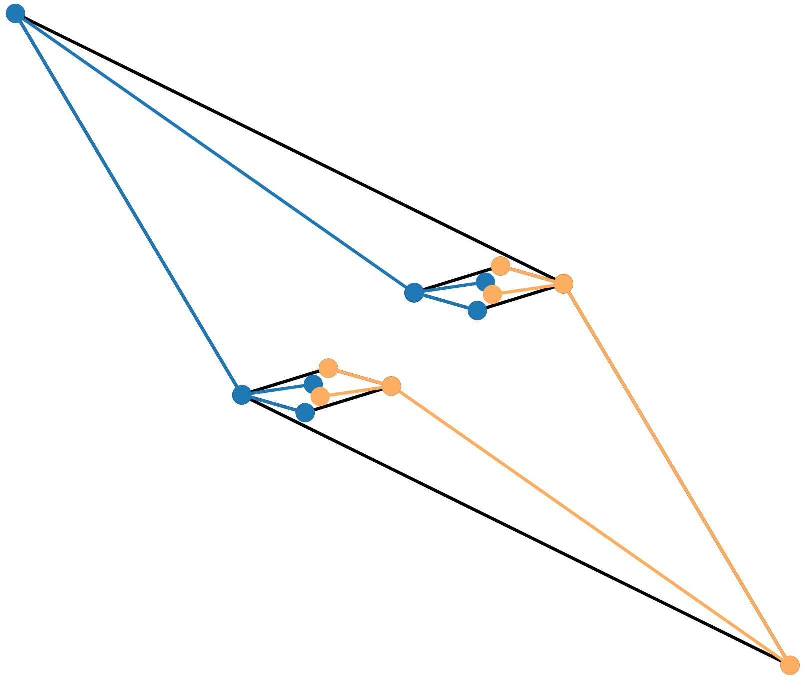
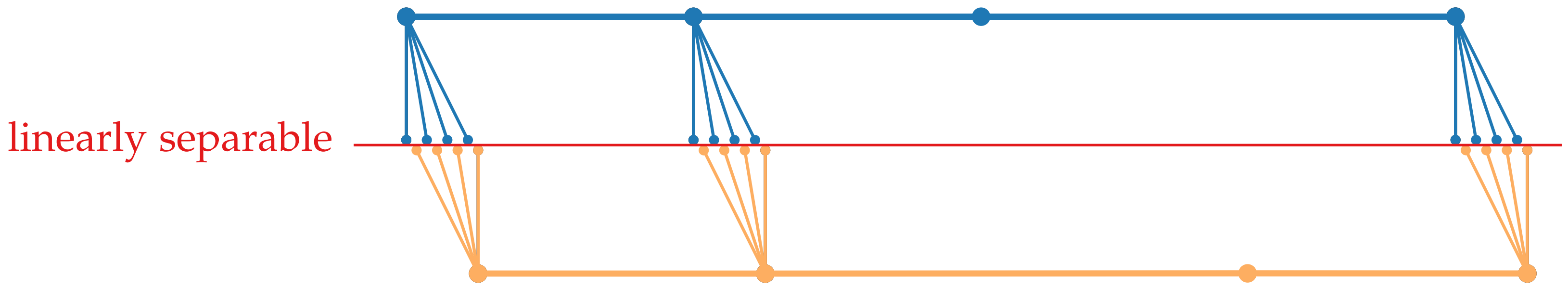
of isomorphic trees



Our Contribution

$\beta = 1$

(1) Mutual Witness Gabriel (MWG) Drawings
of isomorphic caterpillars



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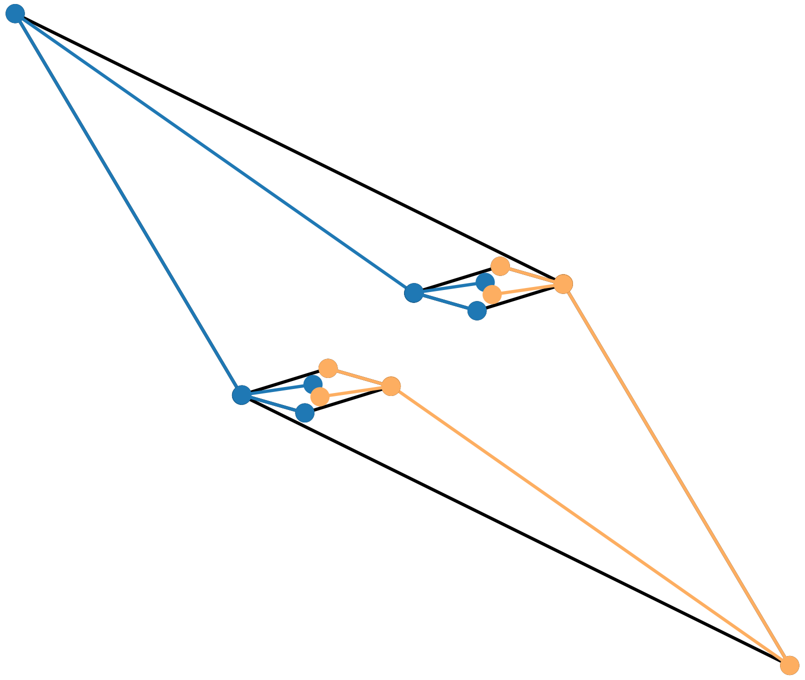
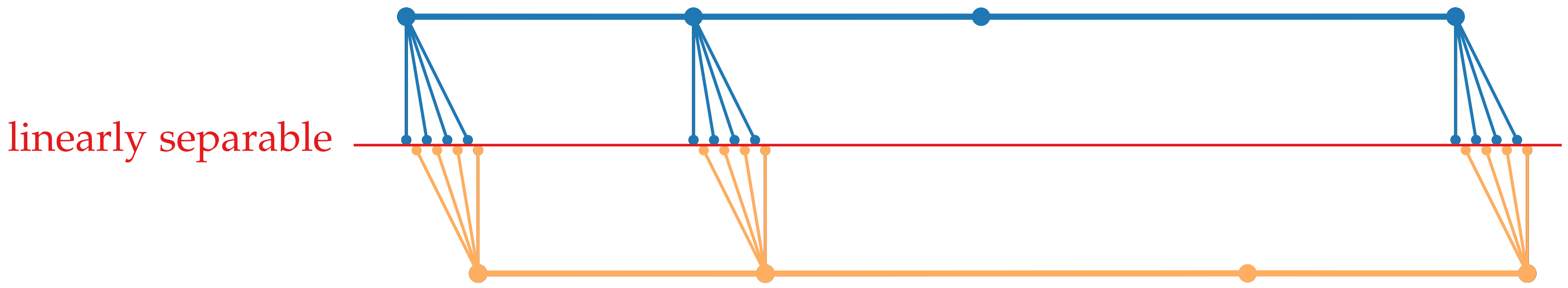


Our Contribution

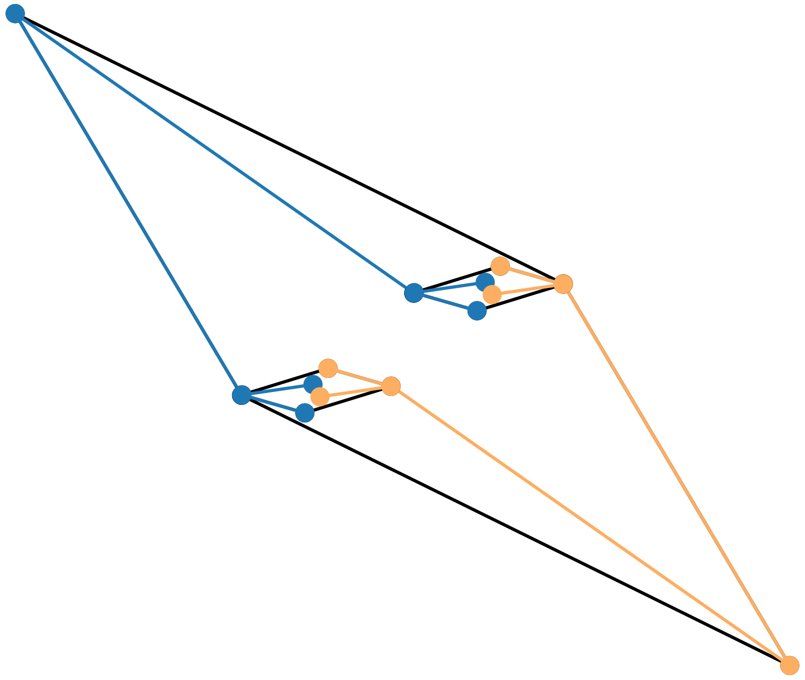
$\beta = 1$

(1) Mutual Witness Gabriel (MWG) Drawings
of isomorphic caterpillars



(2) Mutual Witness β Drawings
of isomorphic trees

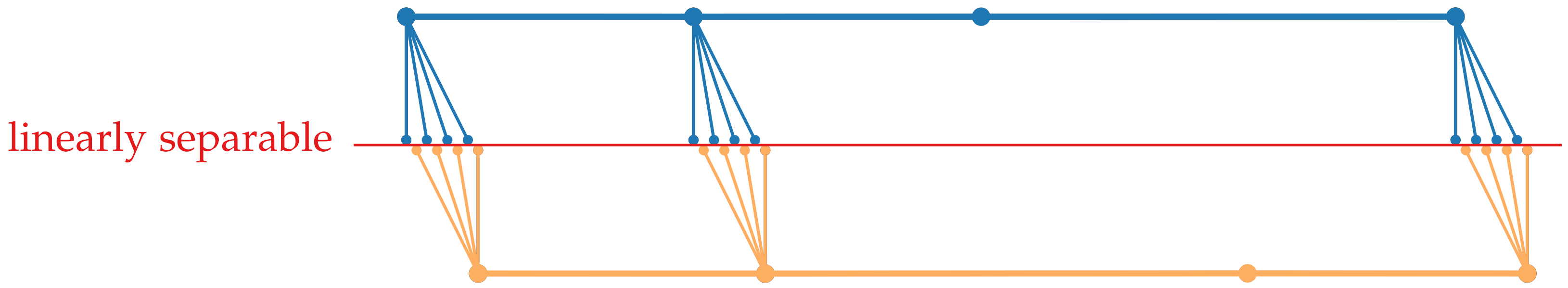
for all $\beta \geq 1$



Our Contribution

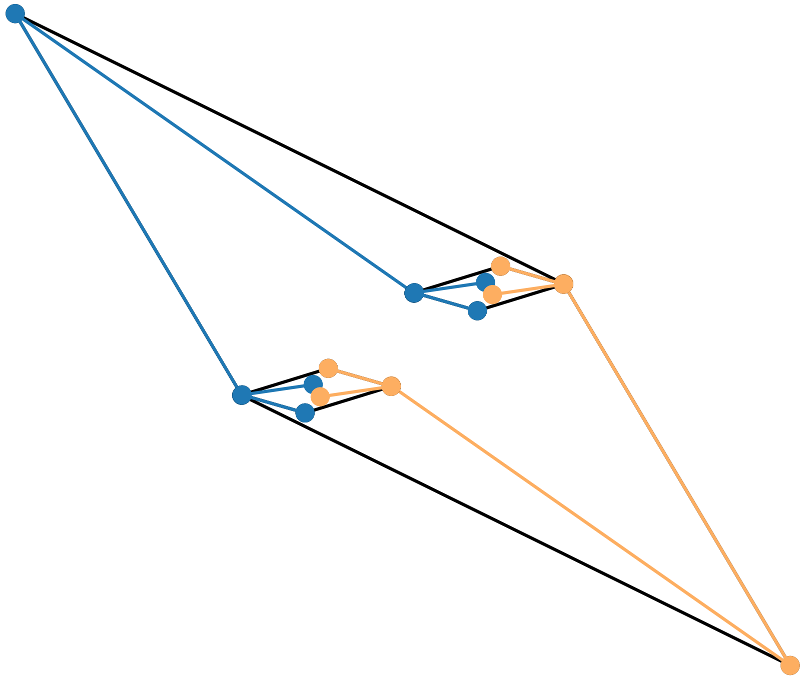
$\beta = 1$

(1) Mutual Witness Gabriel (MWG) Drawings
of isomorphic caterpillars



(2) Mutual Witness β Drawings
of isomorphic trees

for all $\beta \geq 1$

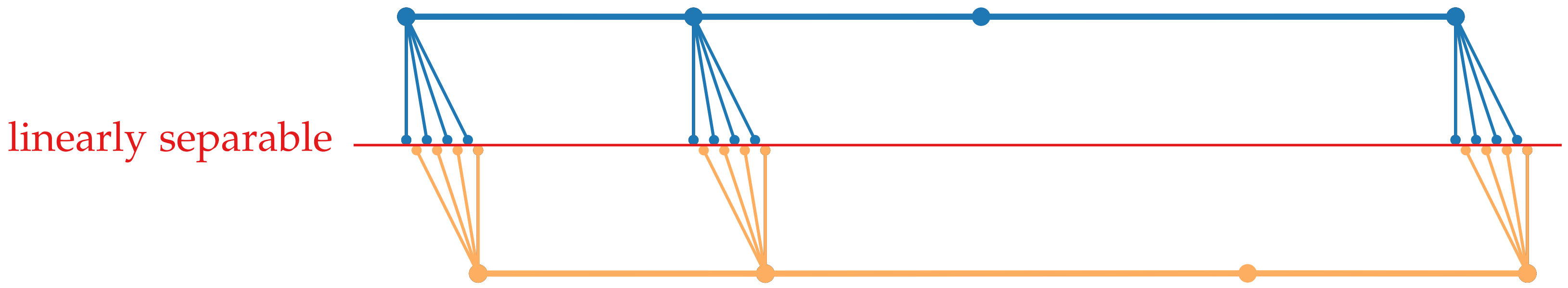


(3) Mutual Witness β Drawings
of almost isomorphic trees

Our Contribution

$\beta = 1$

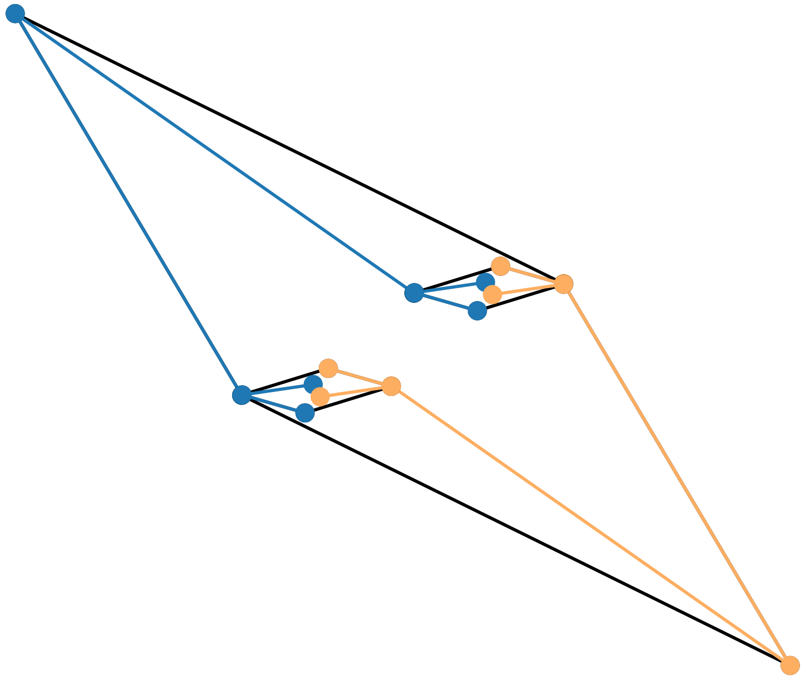
(1) Mutual Witness Gabriel (MWG) Drawings
of isomorphic caterpillars



(2) Mutual Witness β Drawings
of isomorphic trees

for all $\beta \geq 1$

(3) Mutual Witness β Drawings
of almost isomorphic trees



Our Contribution

$\beta = 1$

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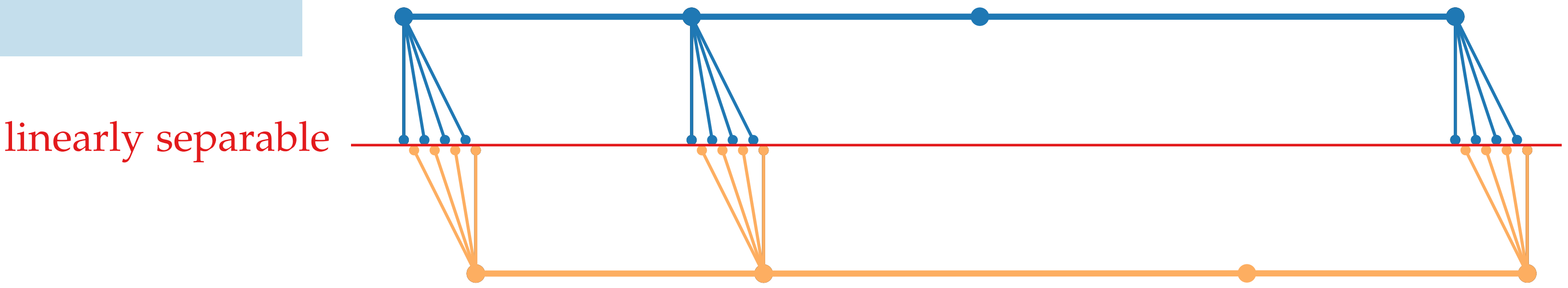
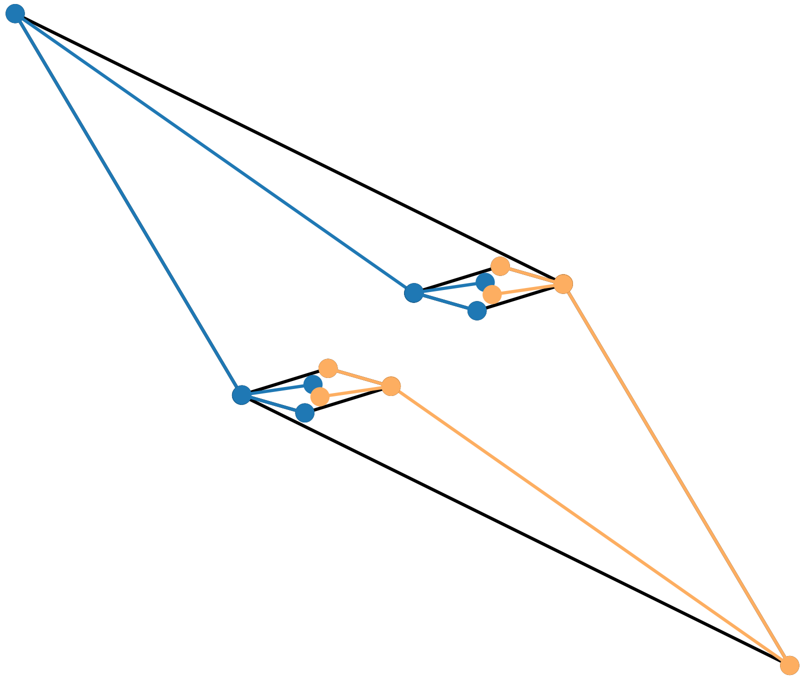
of isomorphic caterpillars



(2) Mutual Witness β Drawings
of isomorphic trees

for all $\beta \geq 1$

(3) Mutual Witness β Drawings
of almost isomorphic trees



Winged Parallelograms

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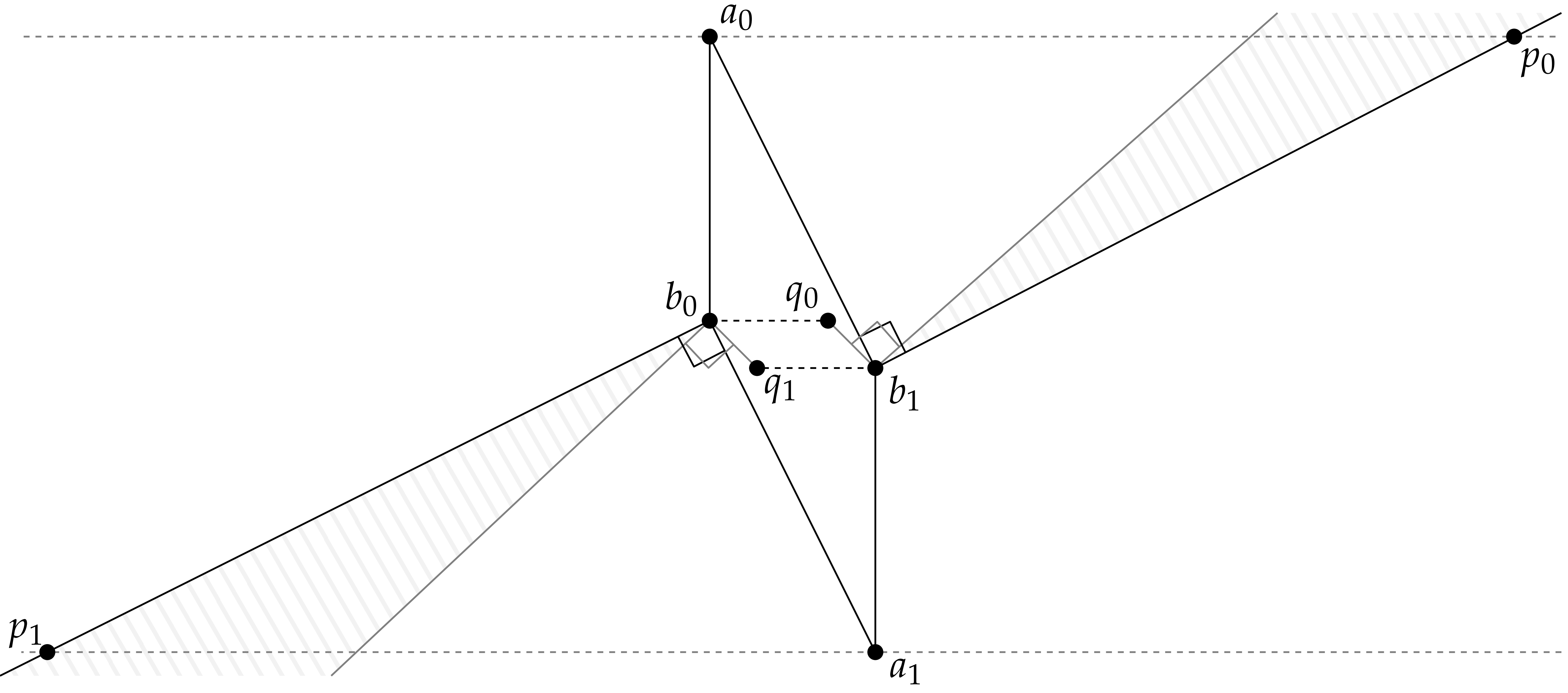

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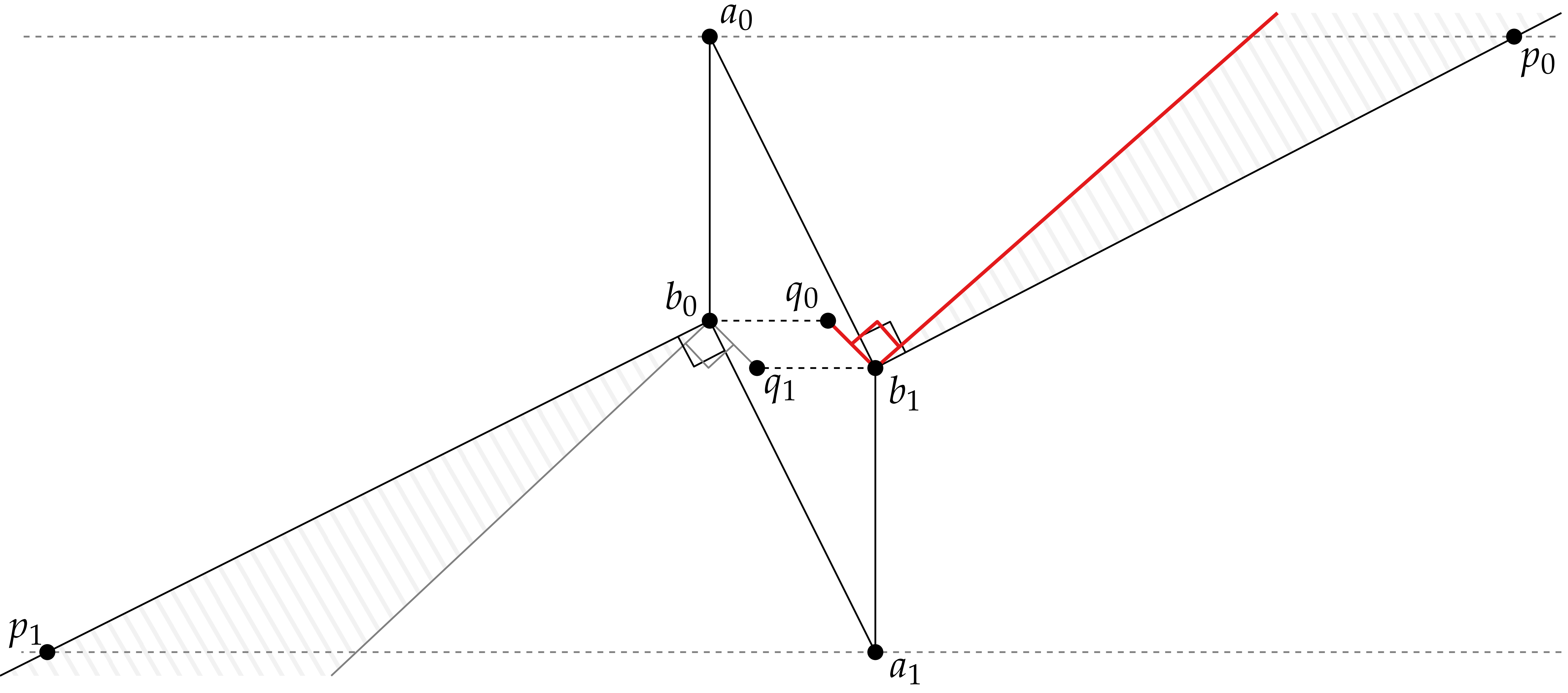
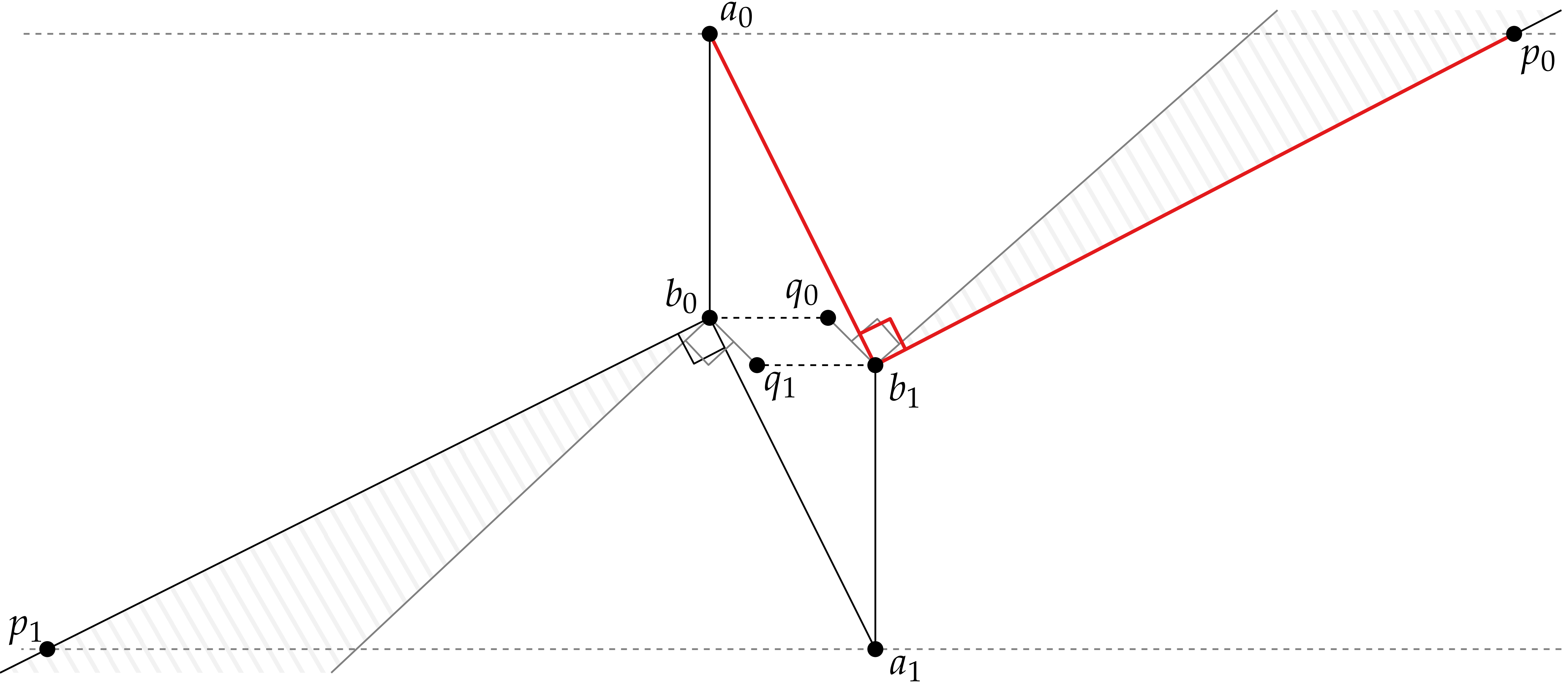
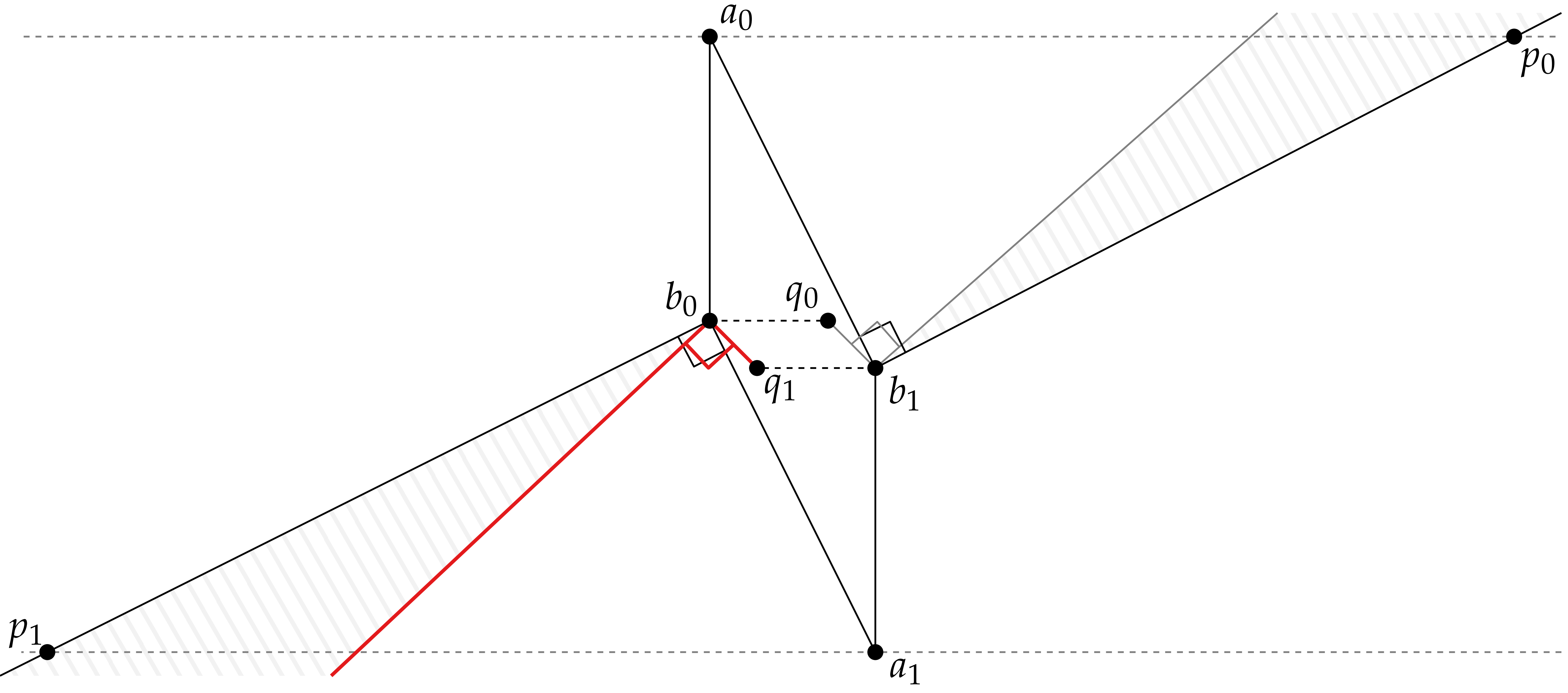


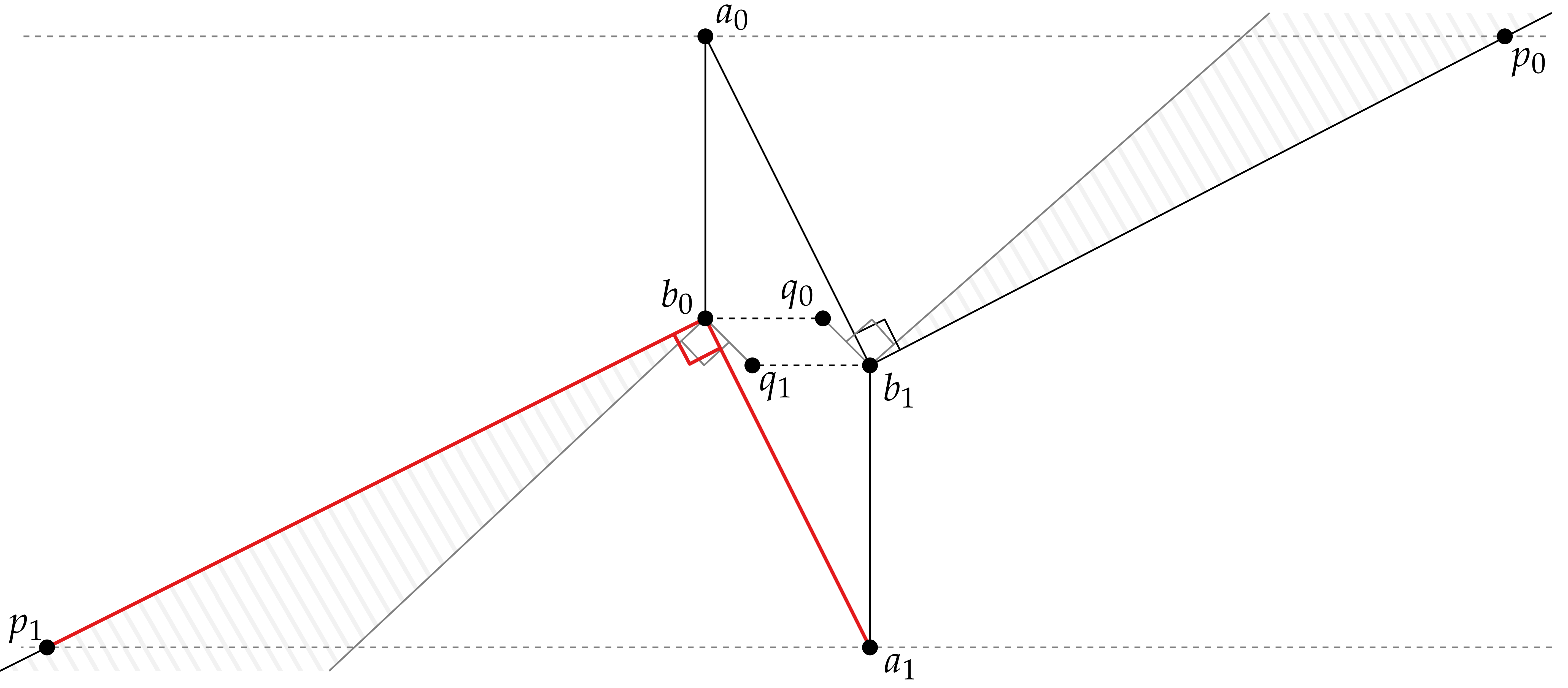
Winged Parallelograms



Winged Parallelograms



Winged Parallelograms



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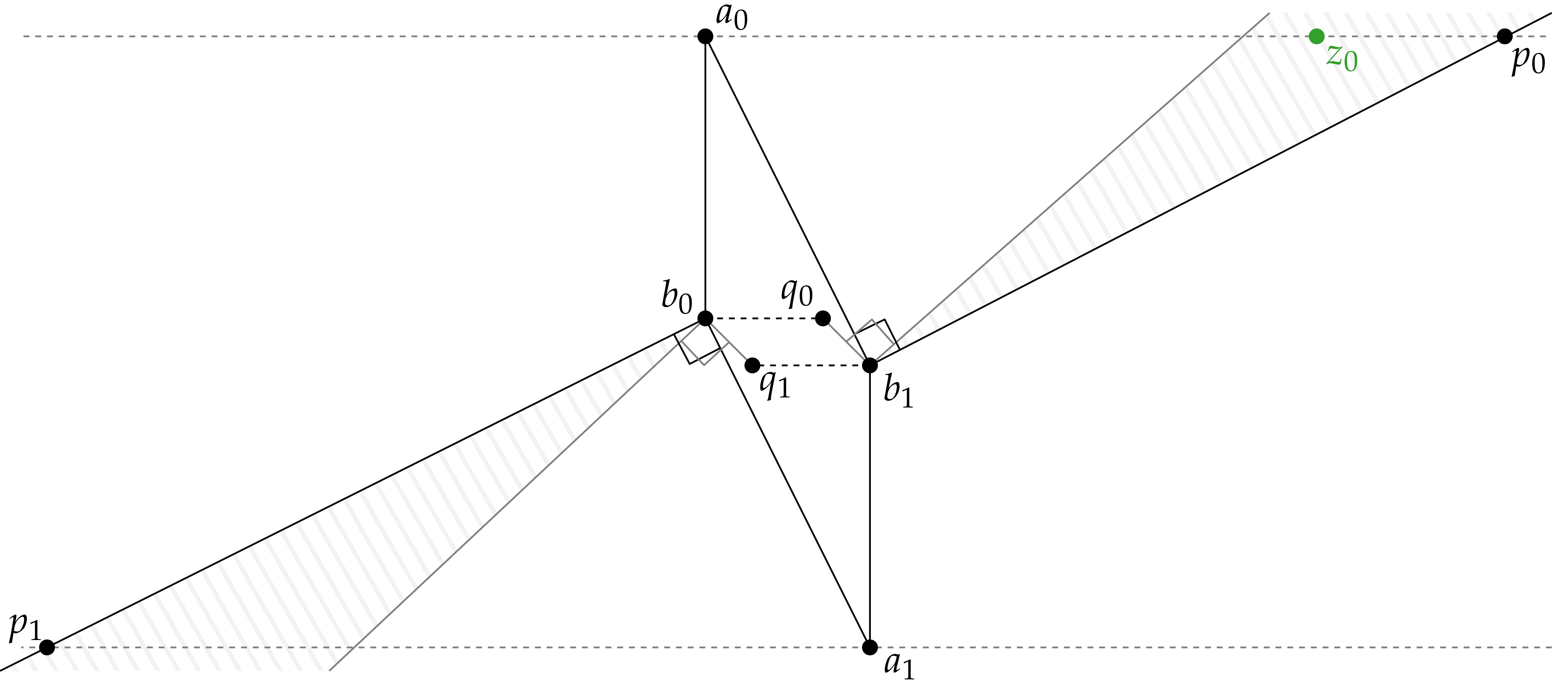

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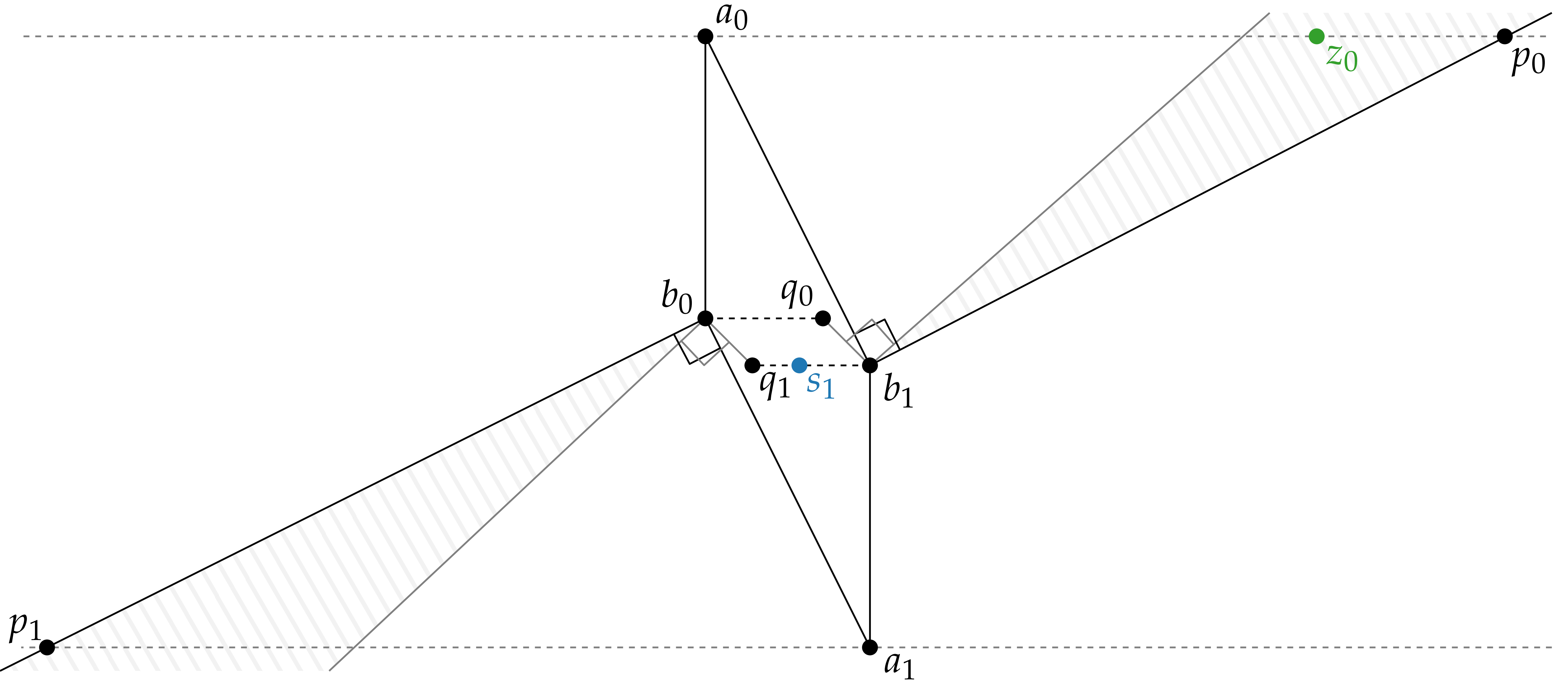
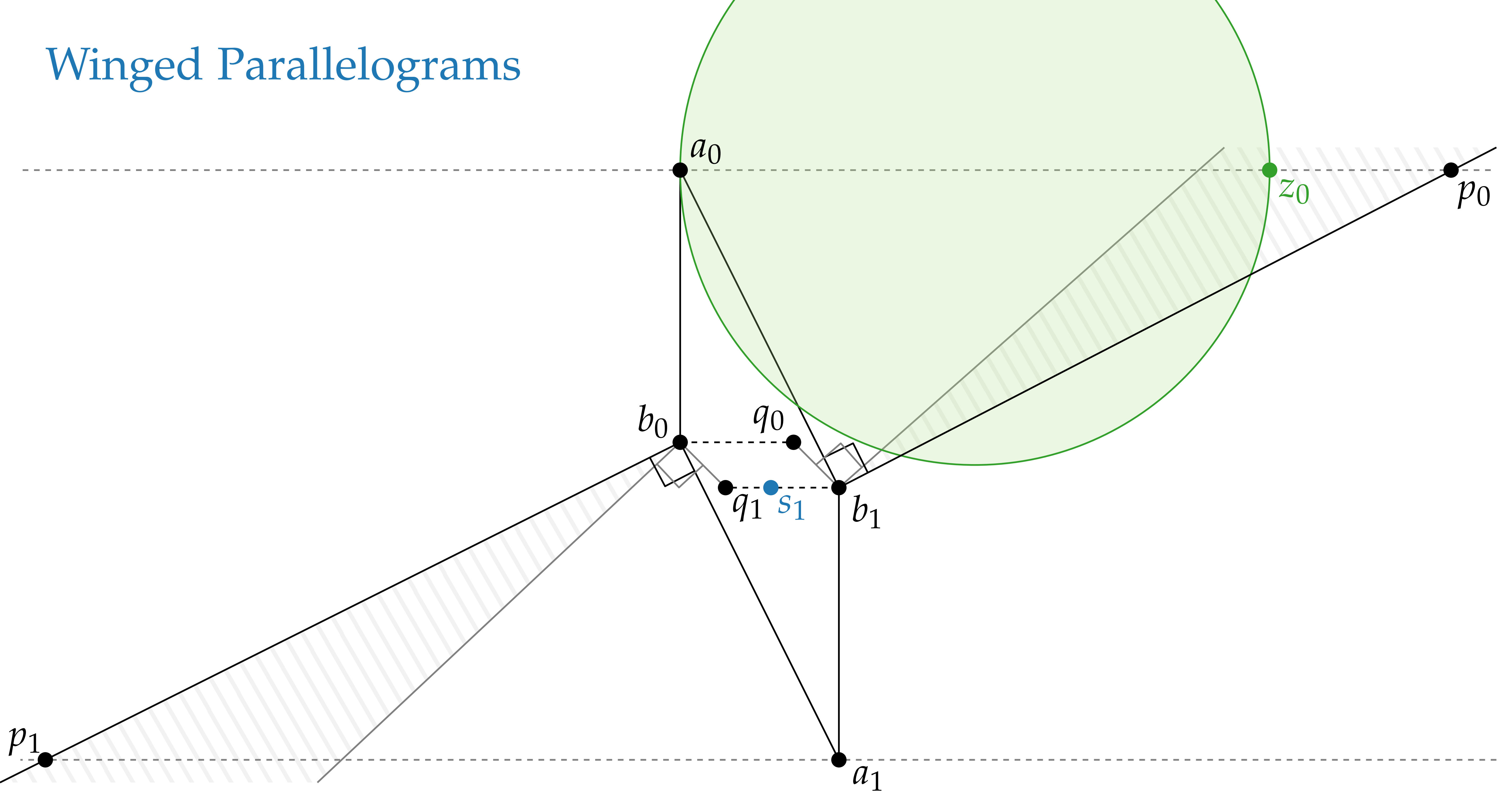


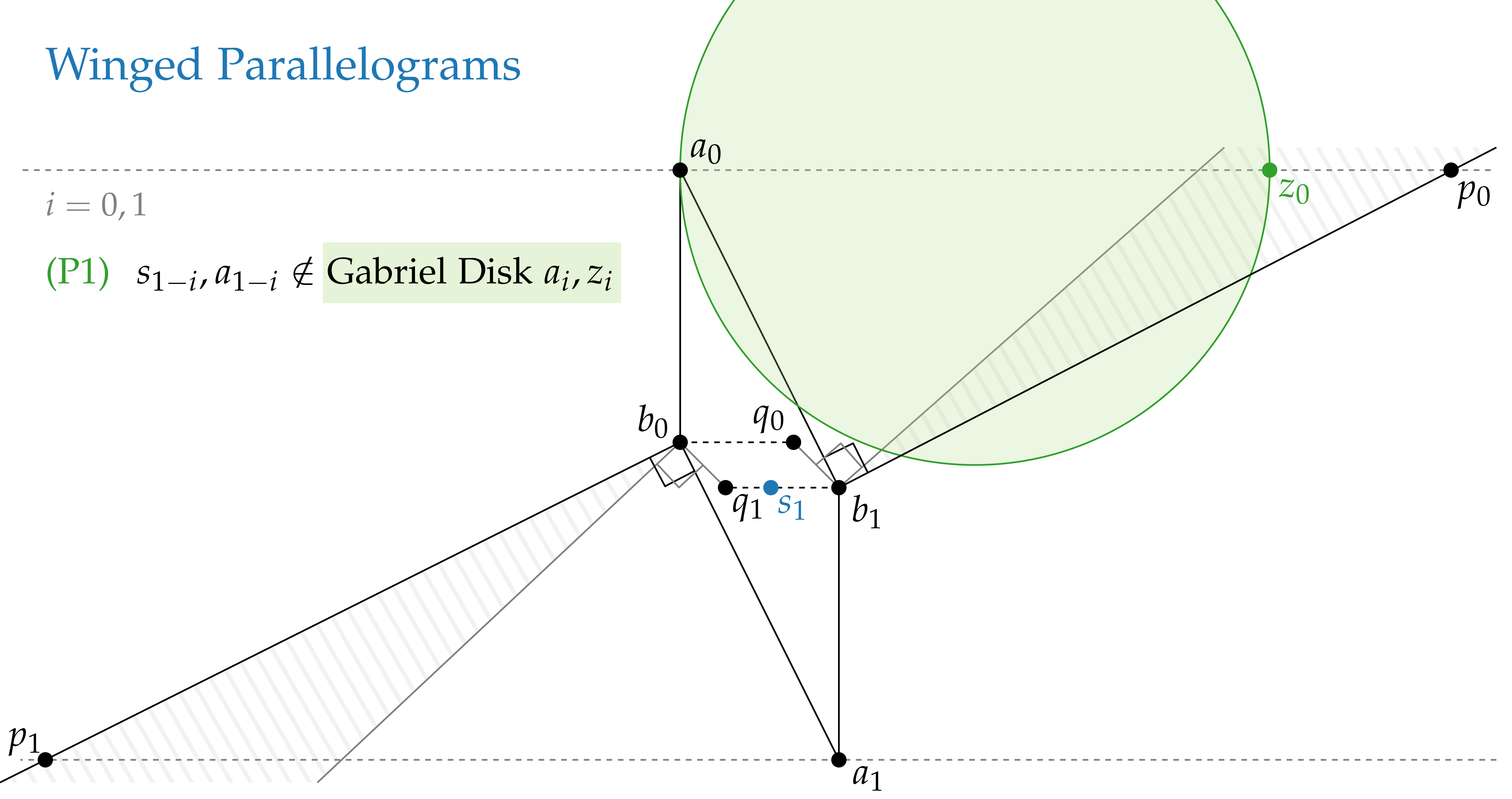
Winged Parallelograms



Winged Parallelograms

$i = 0, 1$

(P1) $s_{1-i}, a_{1-i} \notin$ Gabriel Disk a_i, z_i



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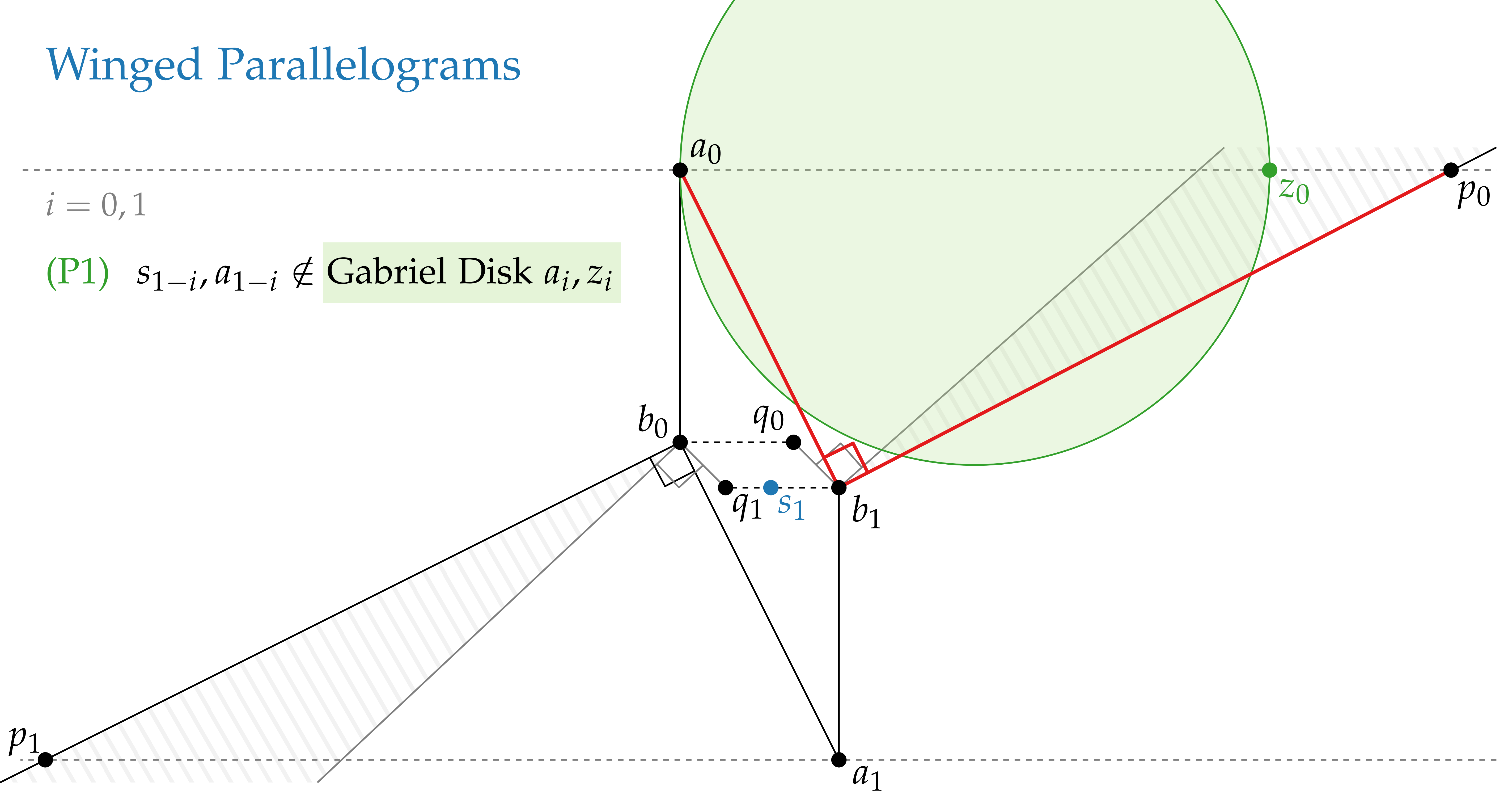

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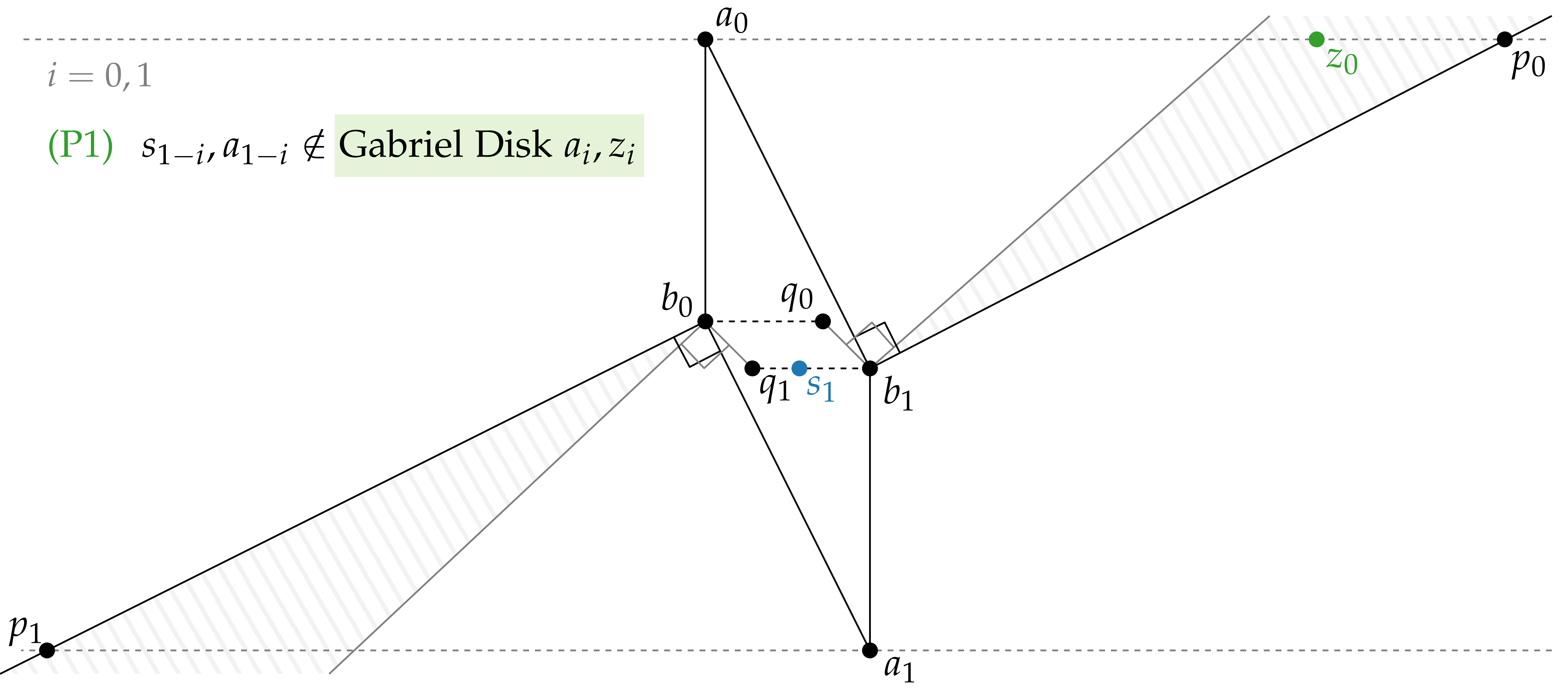

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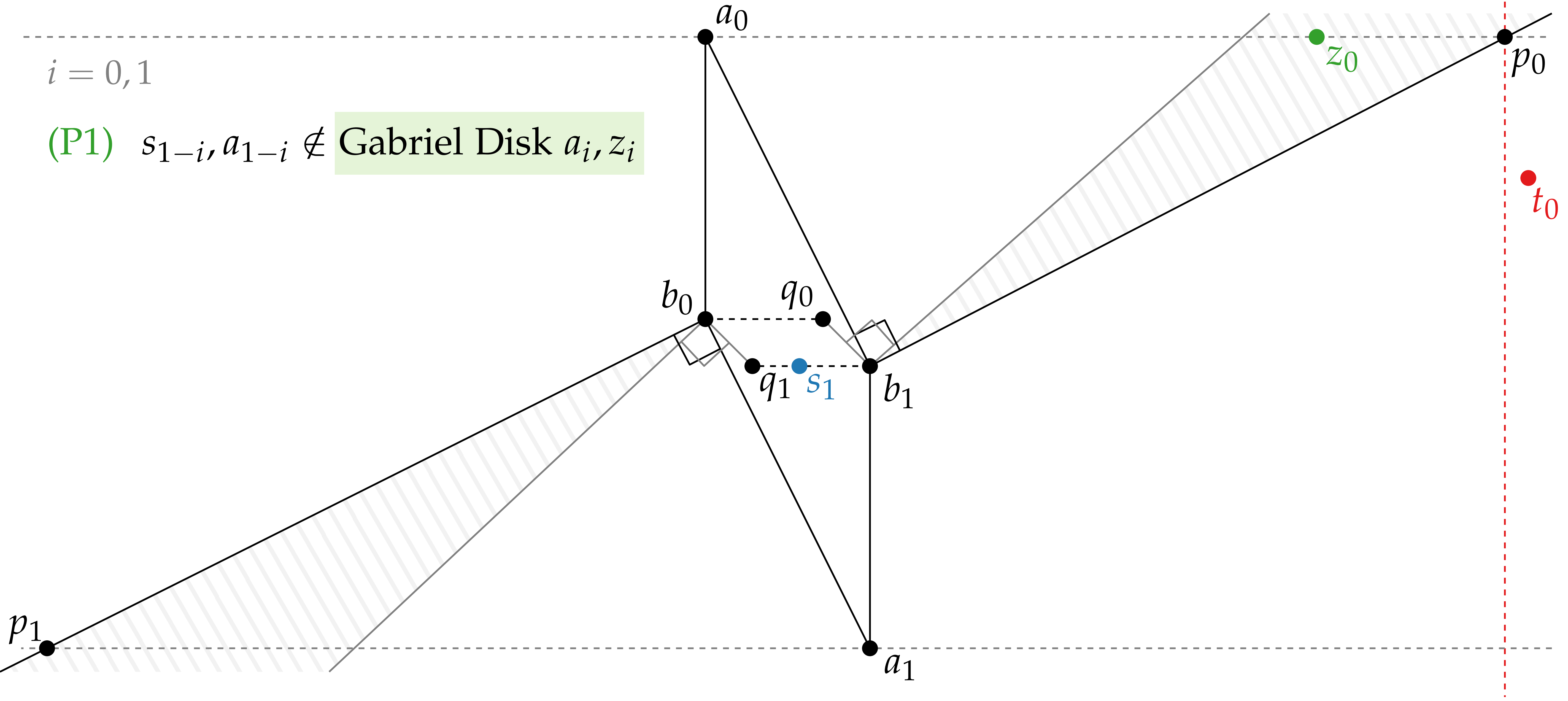

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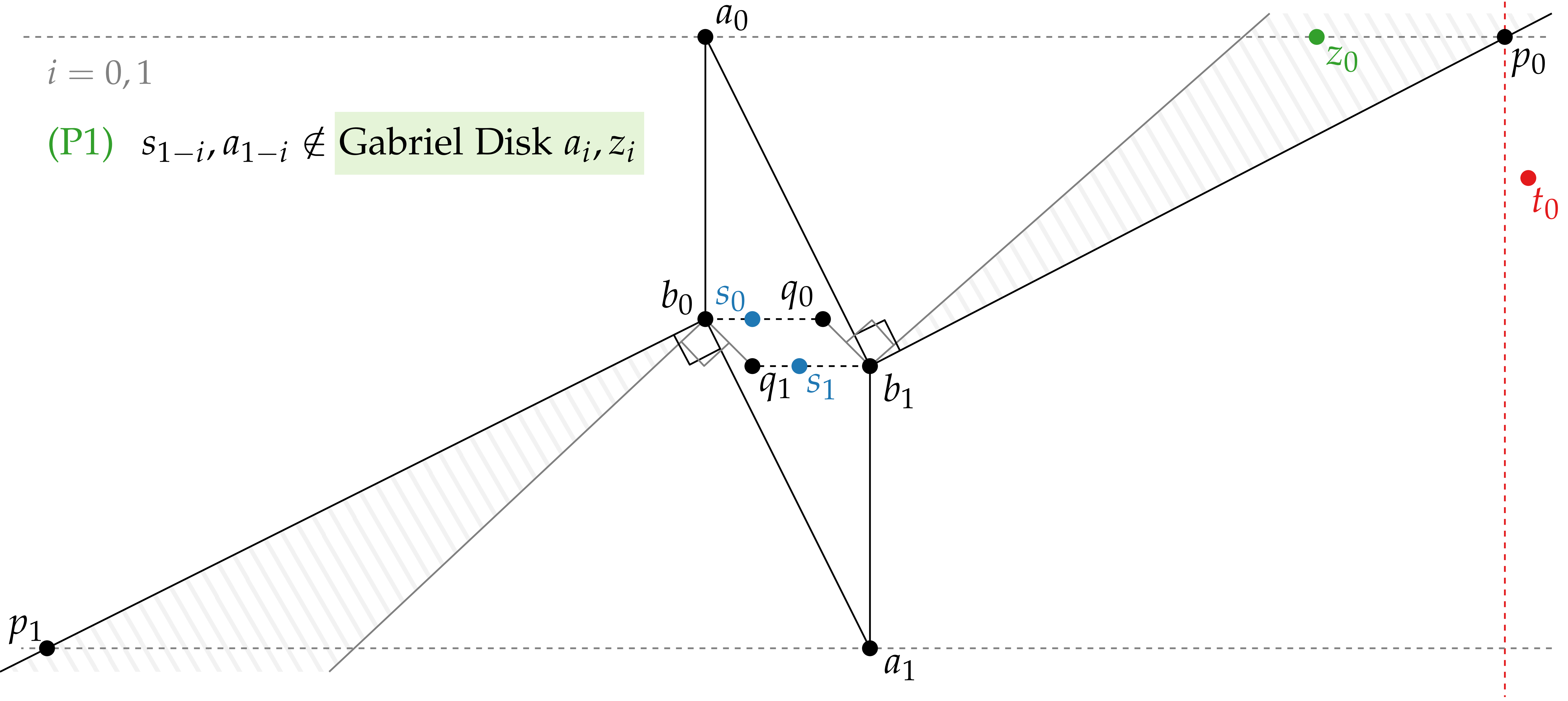

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$i = 0, 1$

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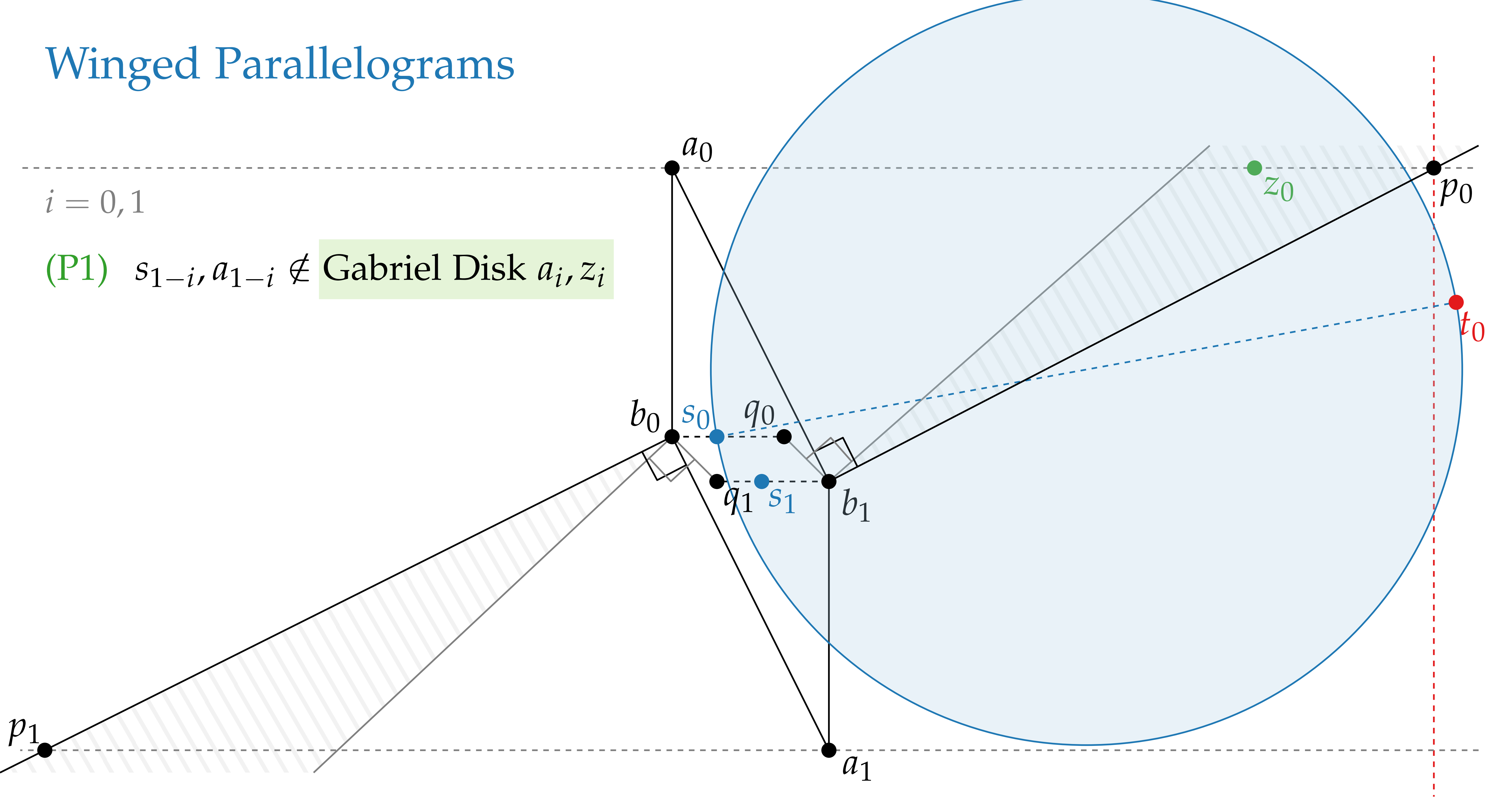
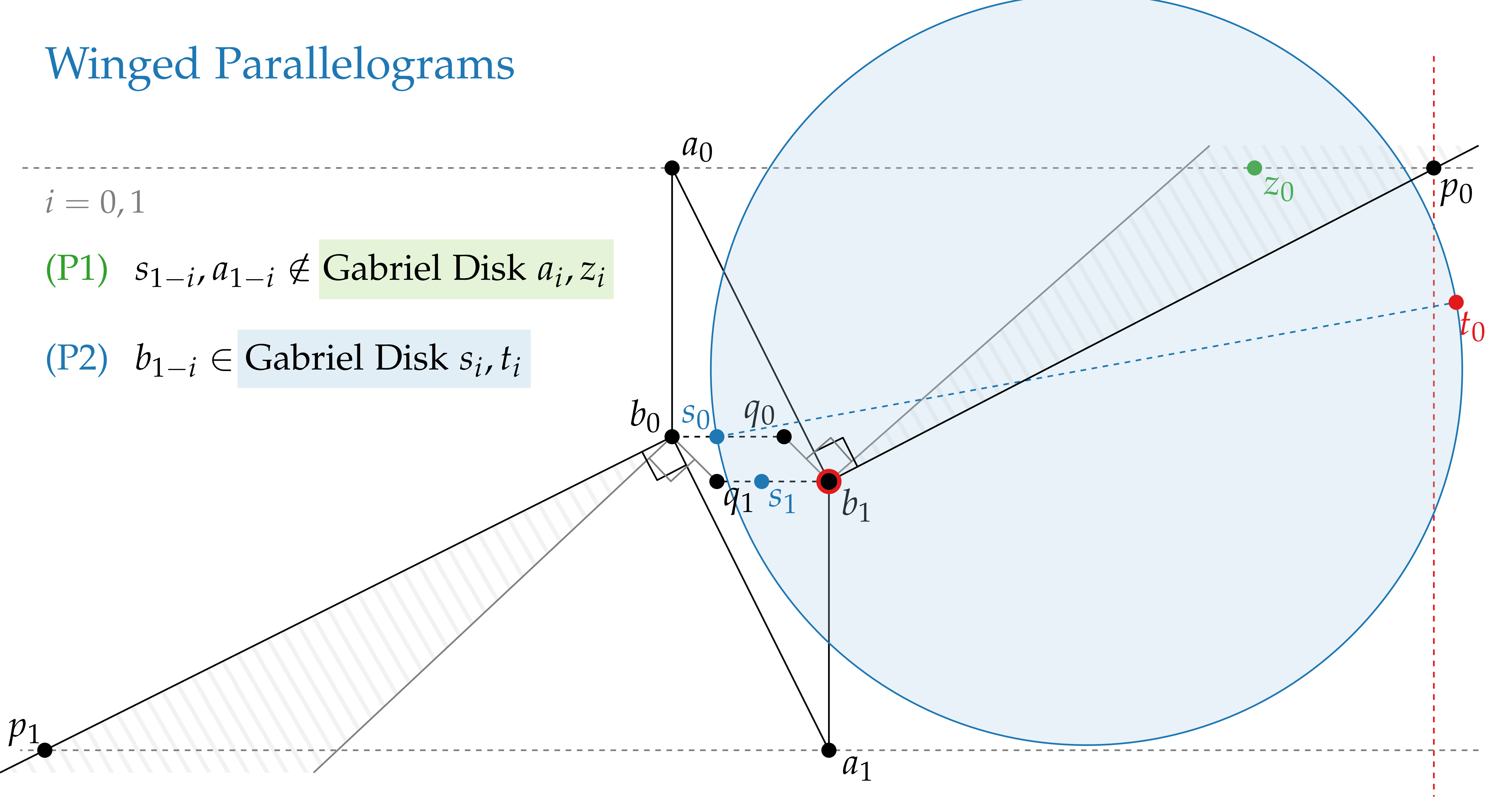


Winged Parallelograms

$i = 0, 1$

(P1) $s_{1-i}, a_{1-i} \notin$ Gabriel Disk a_i, z_i

(P2) $b_{1-i} \in$ Gabriel Disk s_i, t_i

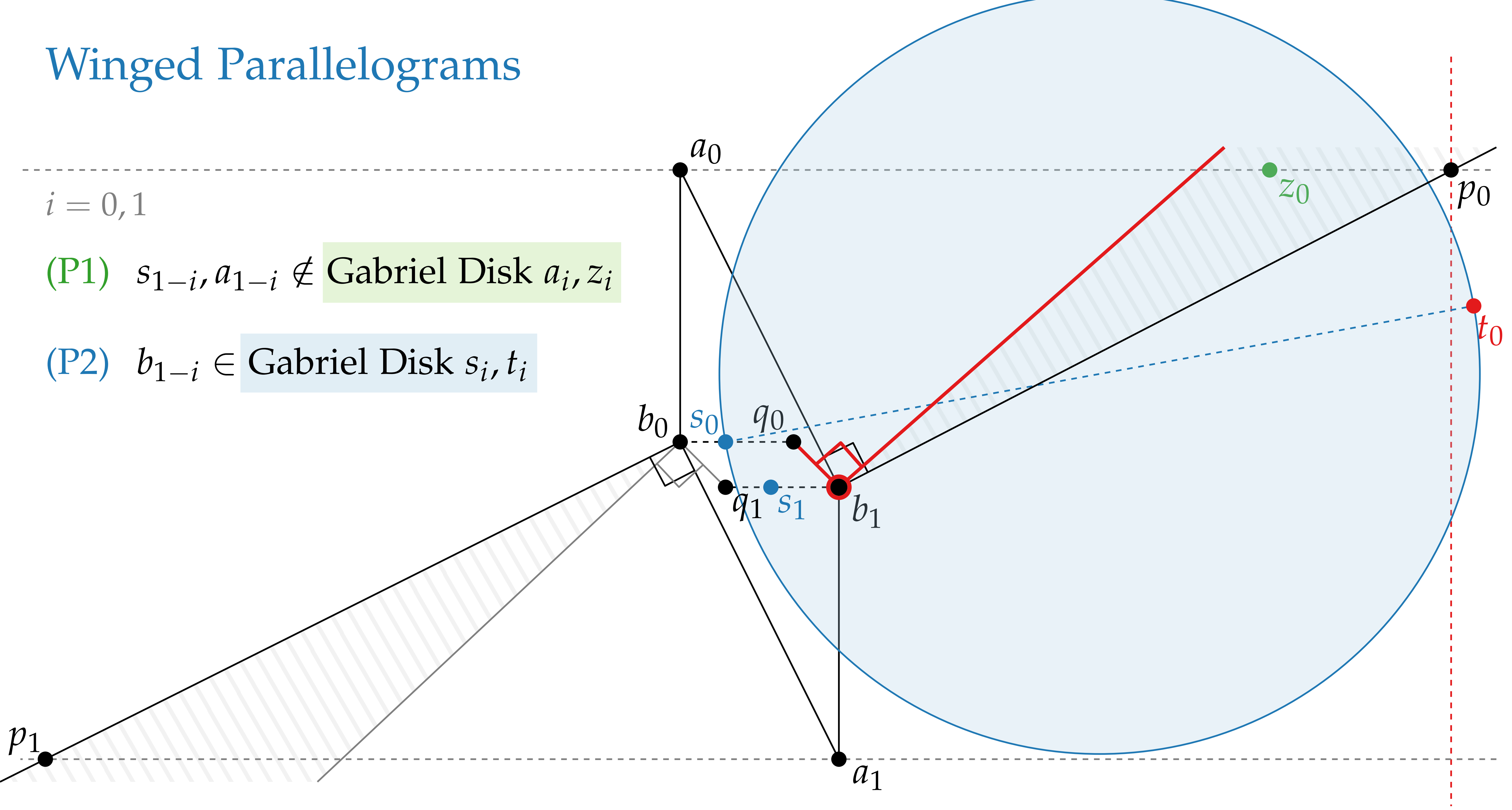


Winged Parallelograms

$i = 0, 1$

(P1) $s_{1-i}, a_{1-i} \notin$ Gabriel Disk a_i, z_i

(P2) $b_{1-i} \in$ Gabriel Disk s_i, t_i



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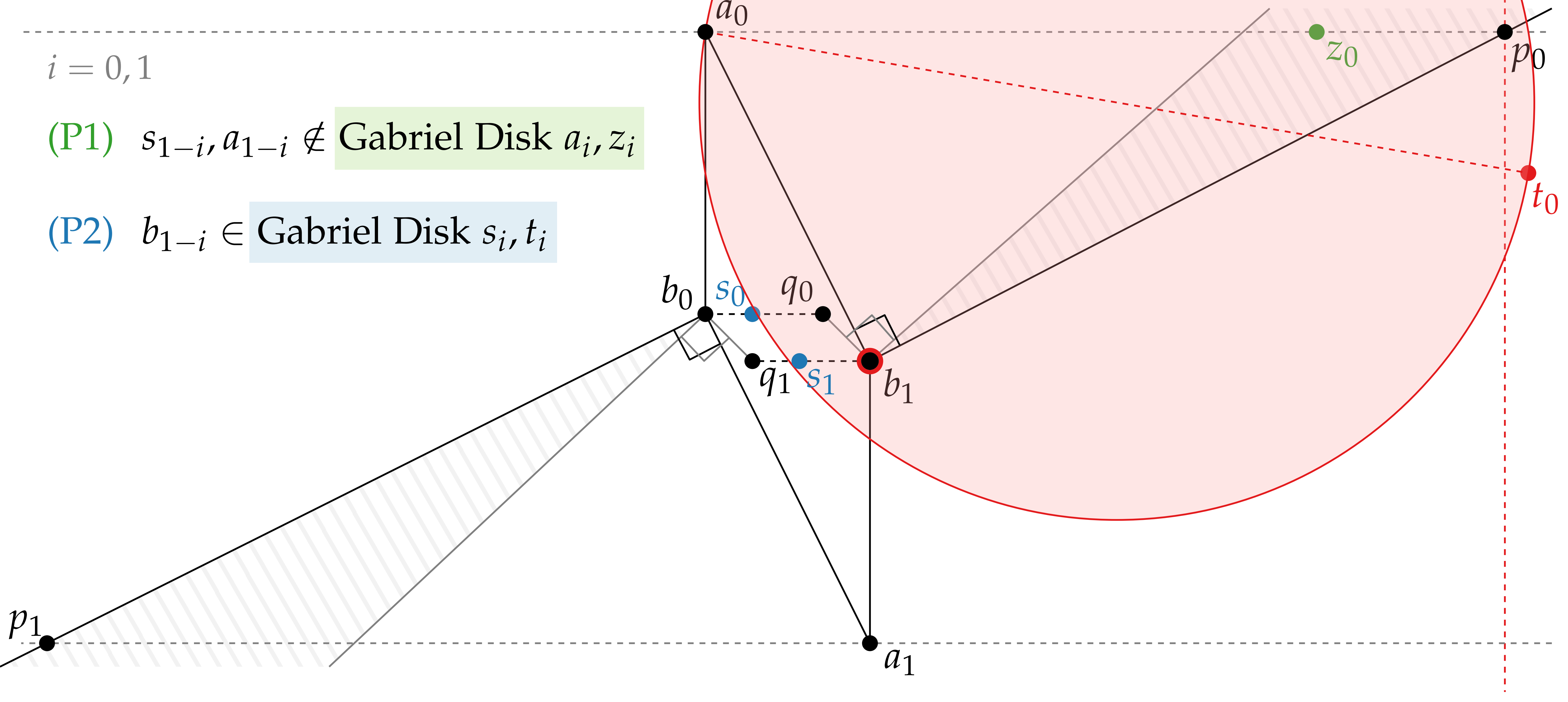
$i = 0, 1$

(P1) $s_{1-i}, a_{1-i} \notin$ Gabriel Disk a_i, z_i

(P2) $b_{1-i} \in$ Gabriel Disk s_i, t_i



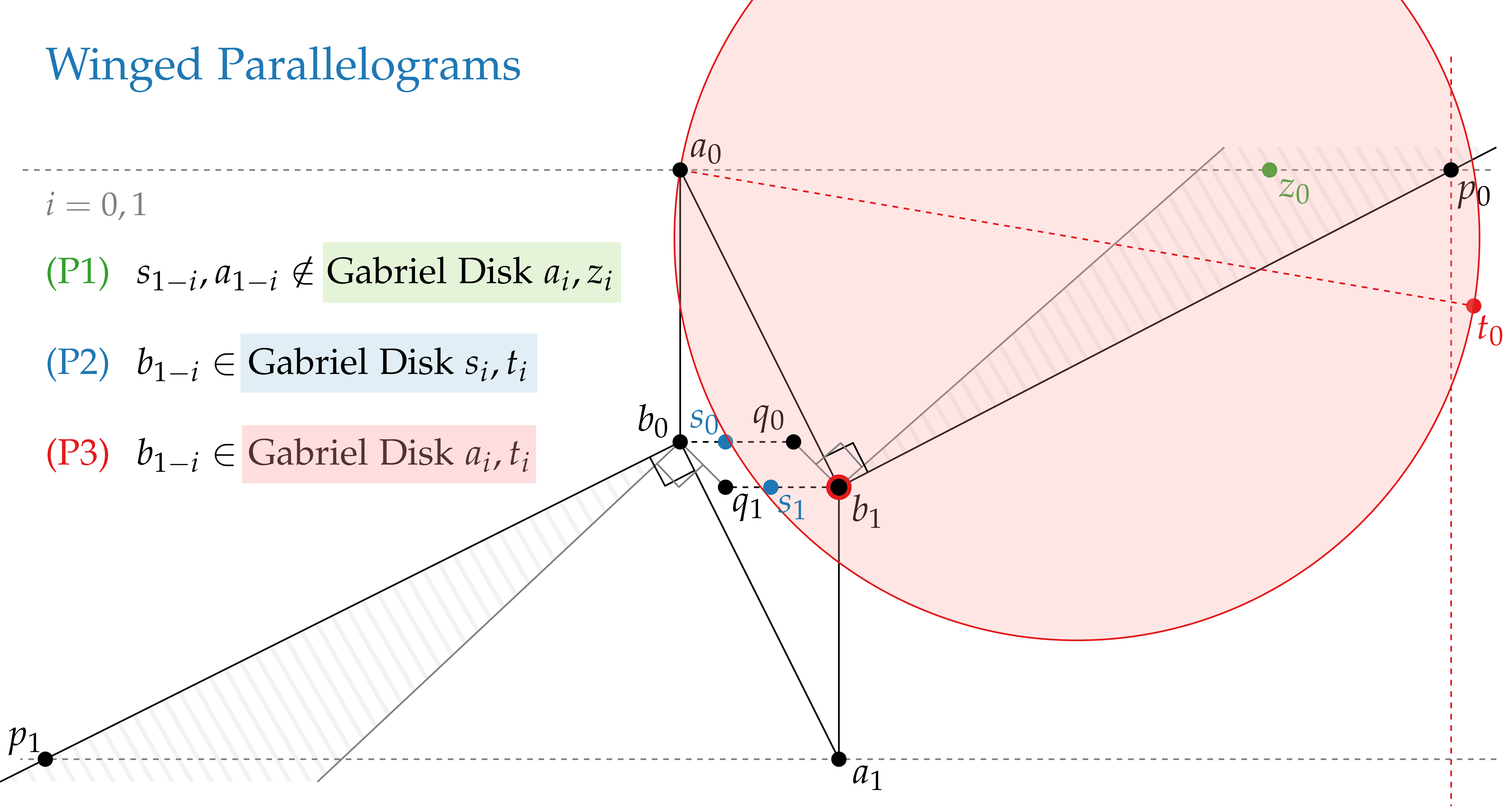
Winged Parallelograms

$i = 0, 1$

(P1) $s_{1-i}, a_{1-i} \notin$ Gabriel Disk a_i, z_i

(P2) $b_{1-i} \in$ Gabriel Disk s_i, t_i

(P3) $b_{1-i} \in$ Gabriel Disk a_i, t_i



Winged Parallelograms

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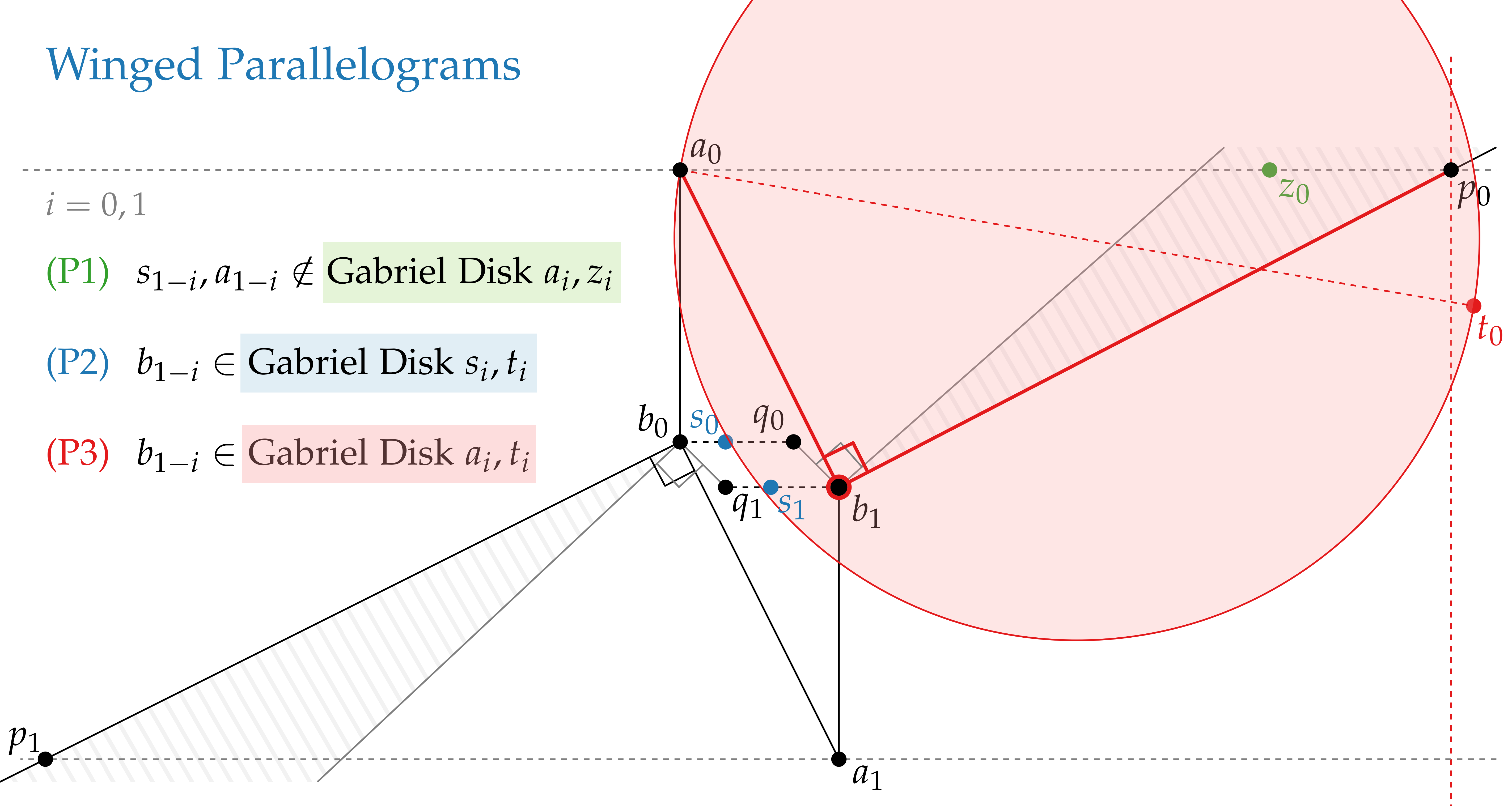
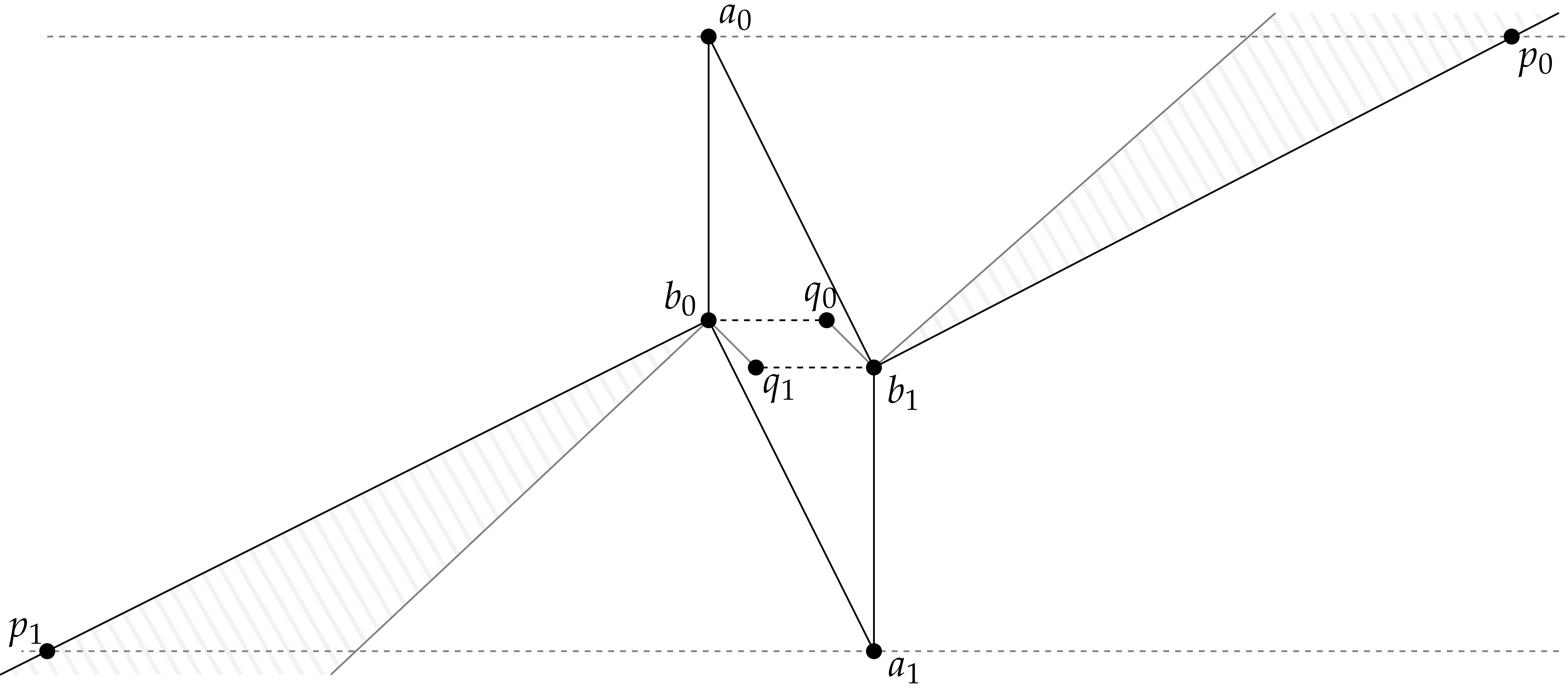
(P2) $b_{1-i} \in$ Gabriel Disk s_i, t_i

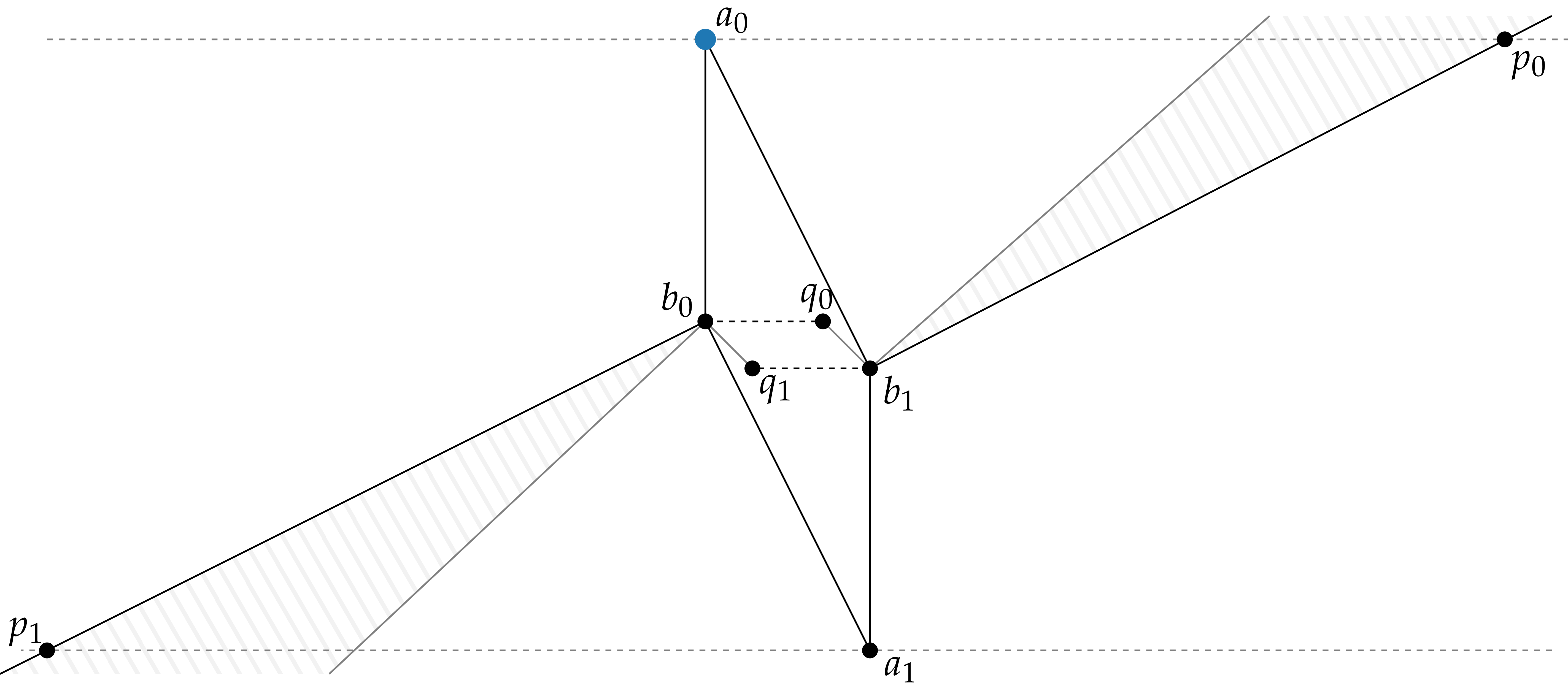
(P3) $b_{1-i} \in$ Gabriel Disk a_i, t_i



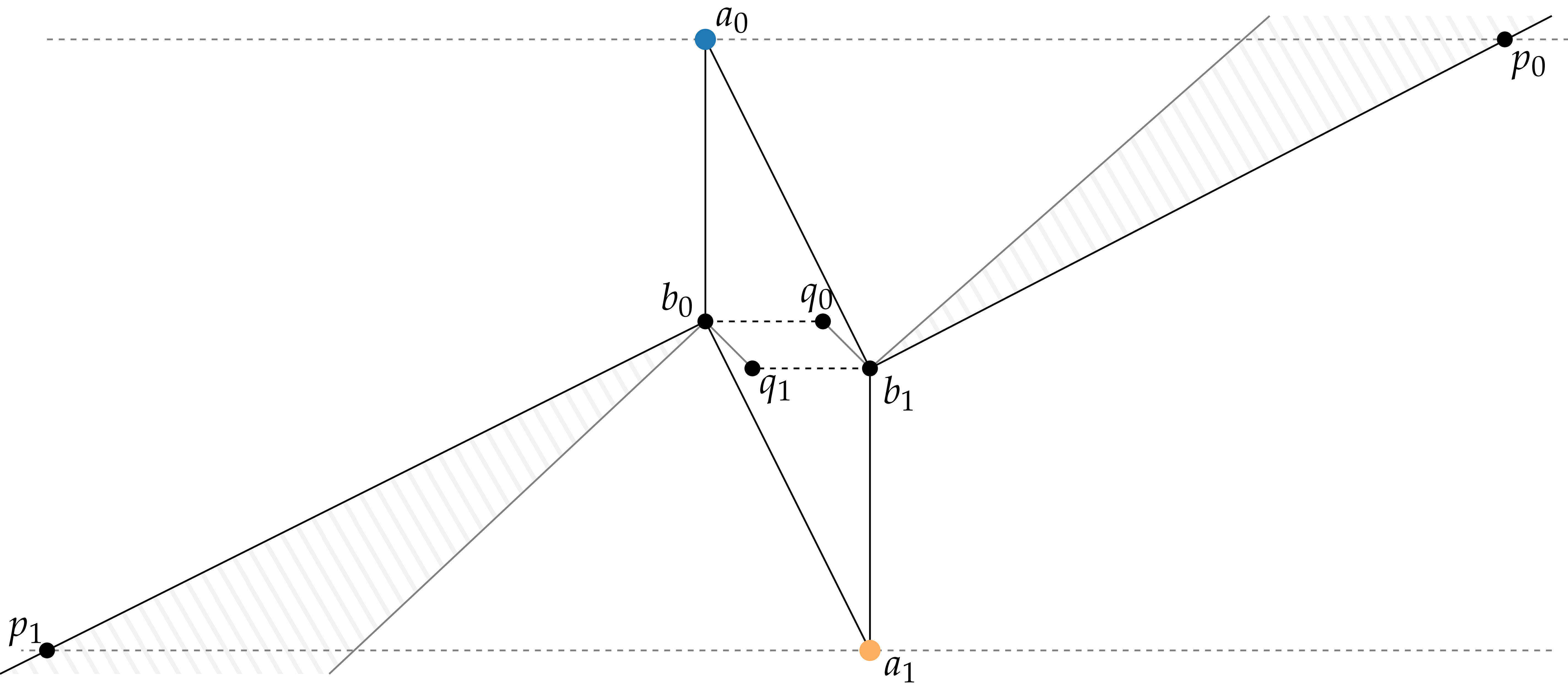
Winged Parallelogram Drawings



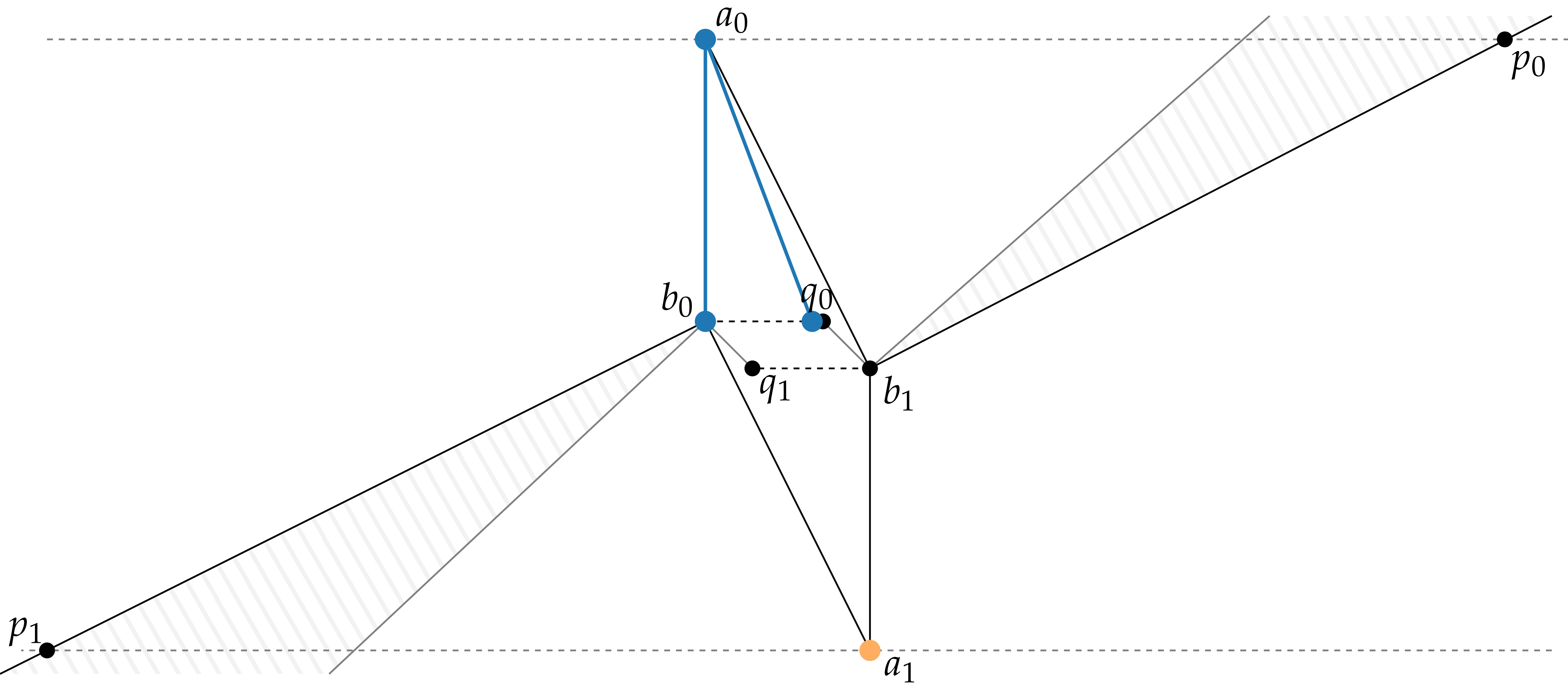
Winged Parallelogram Drawings



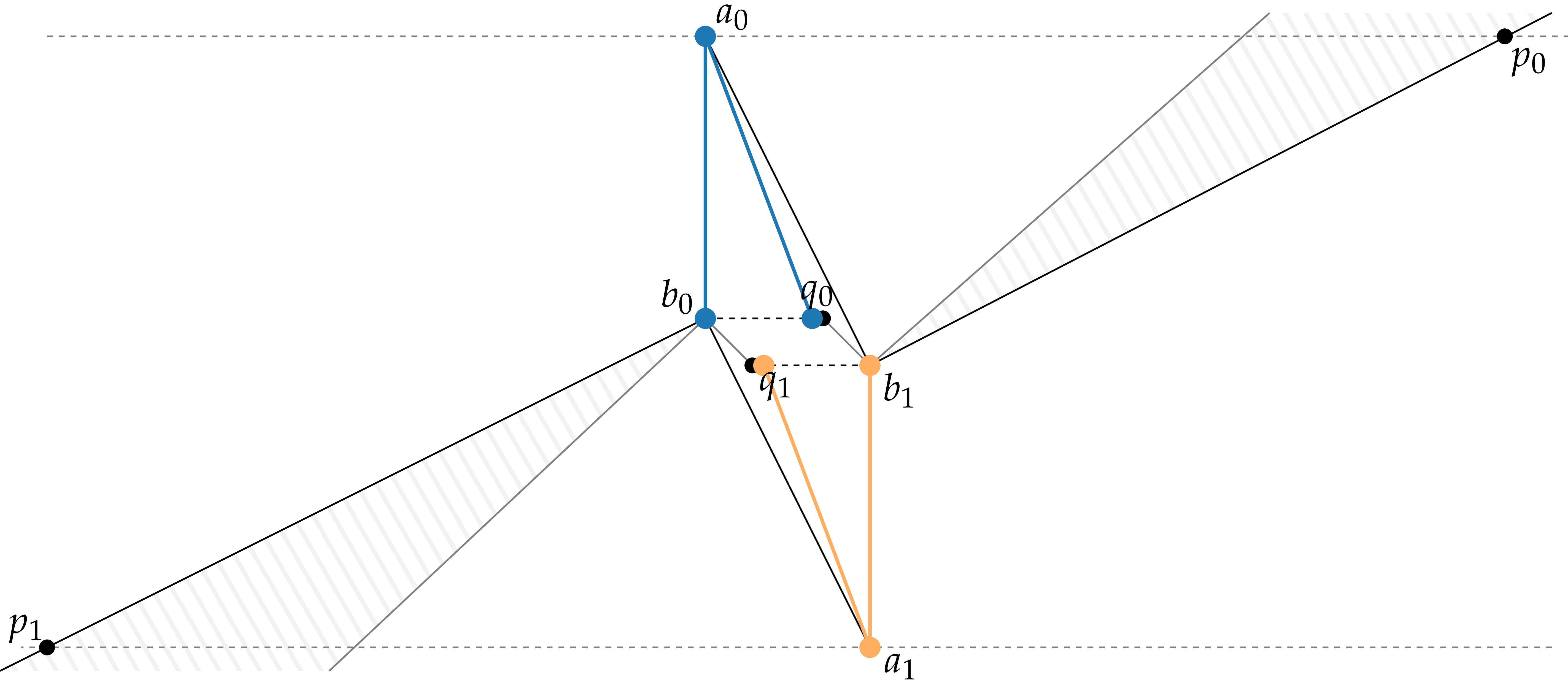
Winged Parallelogram Drawings



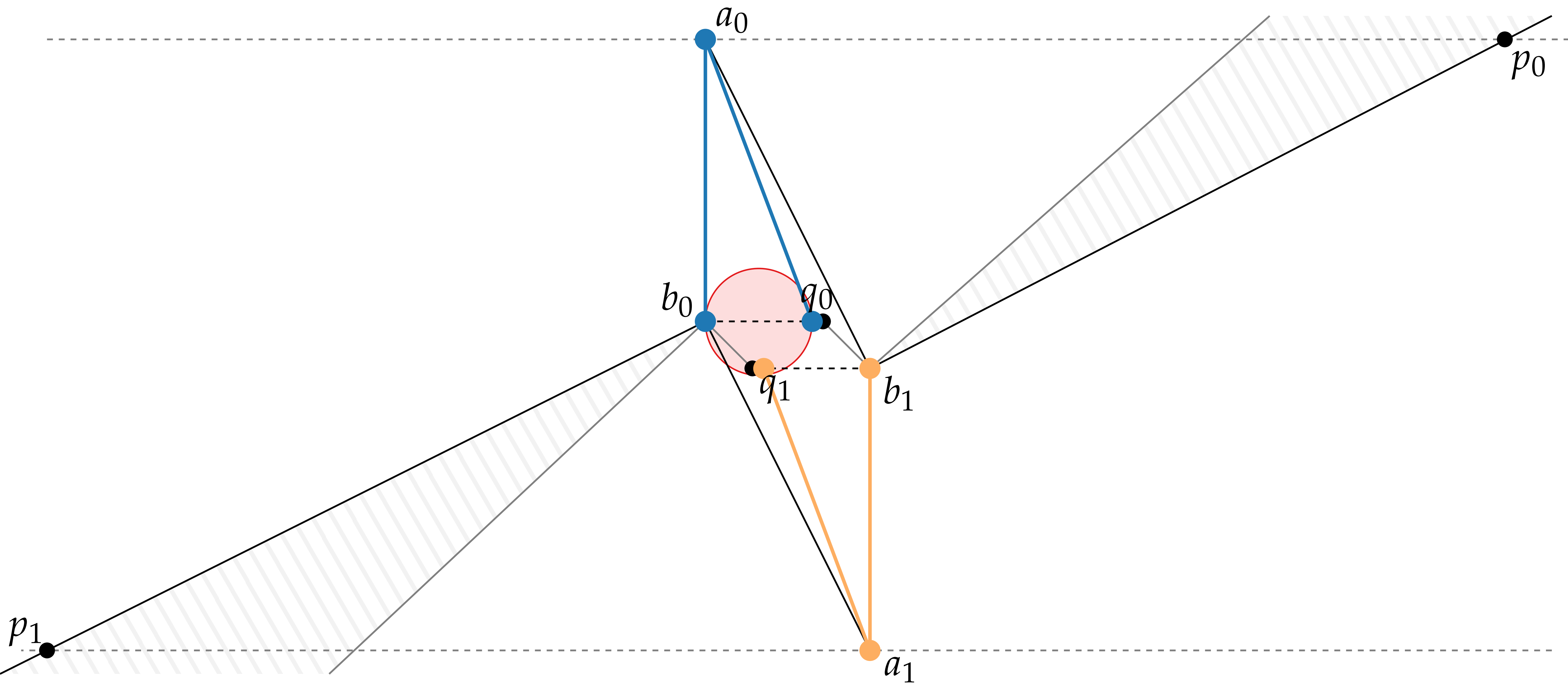
Winged Parallelogram Drawings



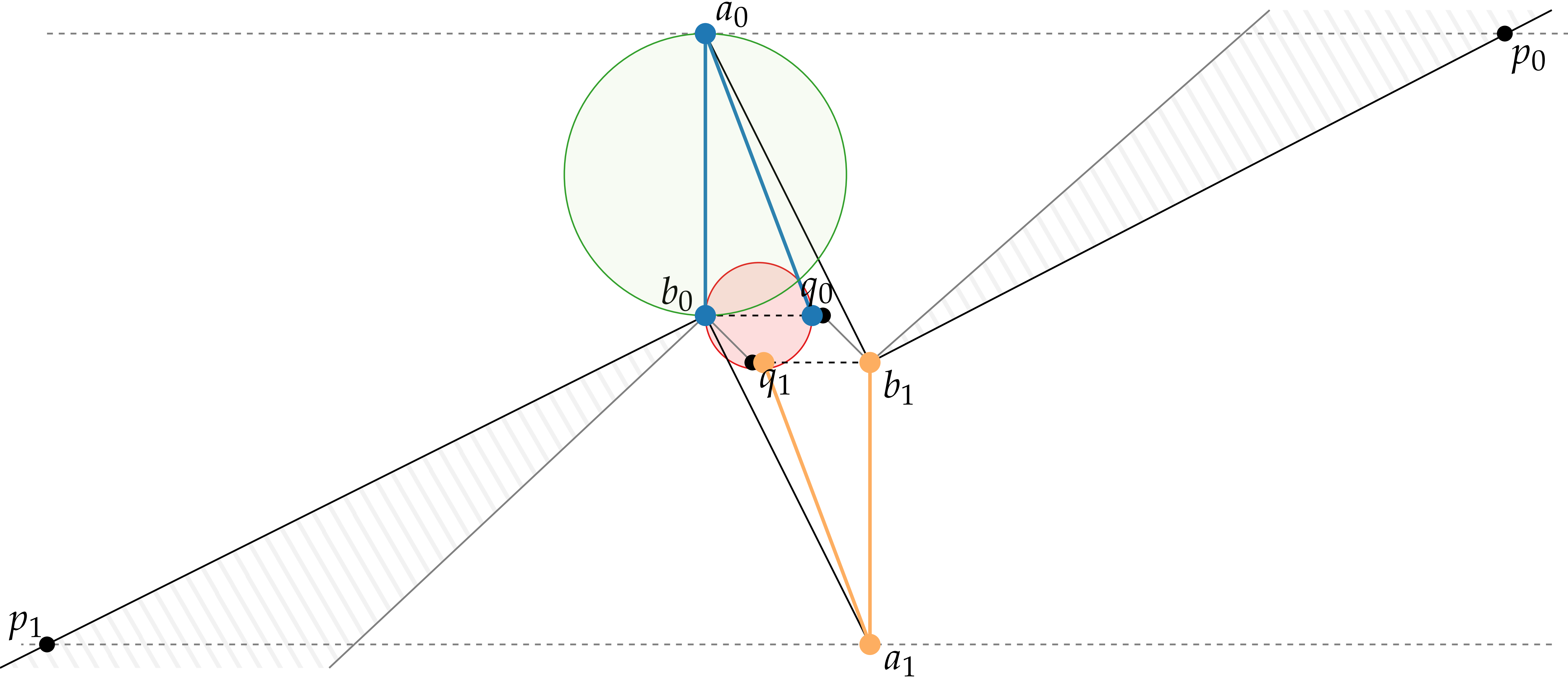
Winged Parallelogram Drawings



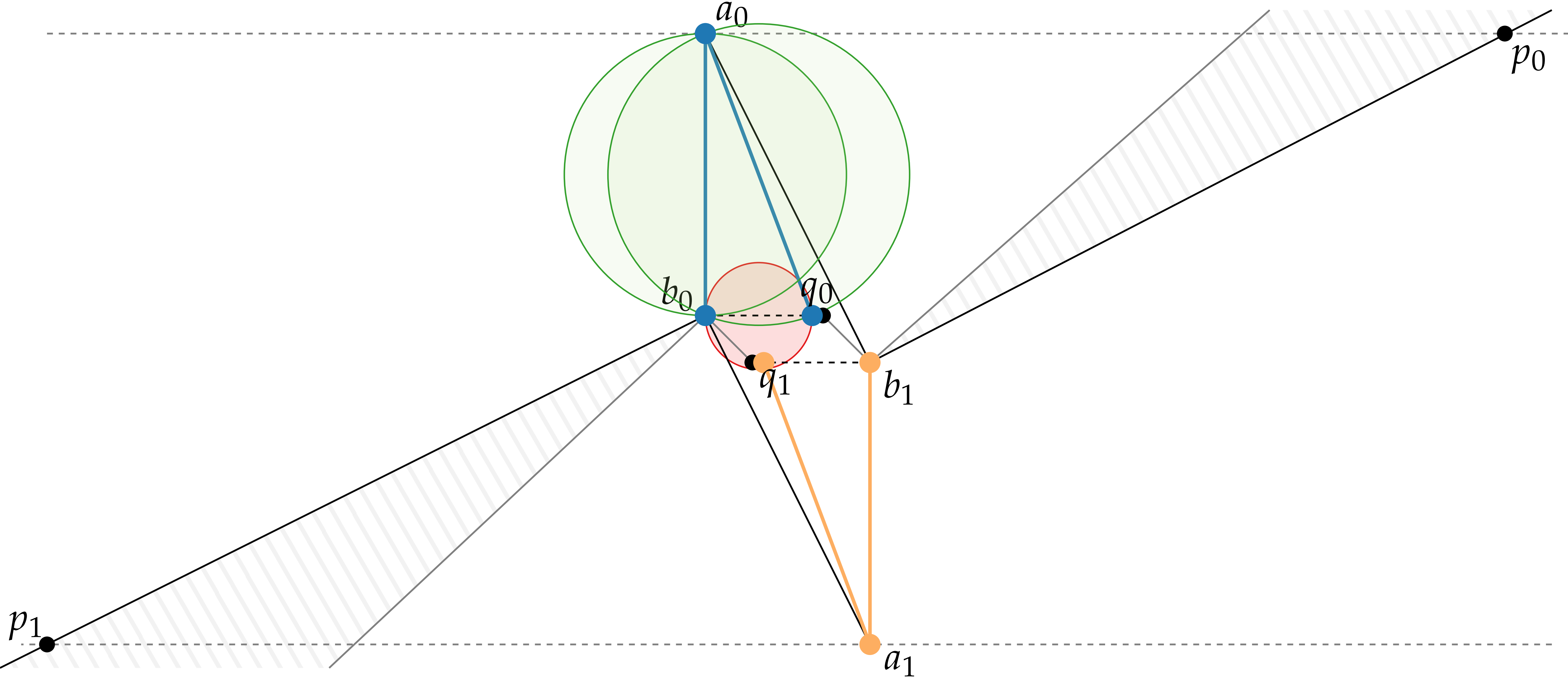
Winged Parallelogram Drawings



Winged Parallelogram Drawings



Winged Parallelogram Drawings



MWG Drawings of Isomorphic Caterpillars

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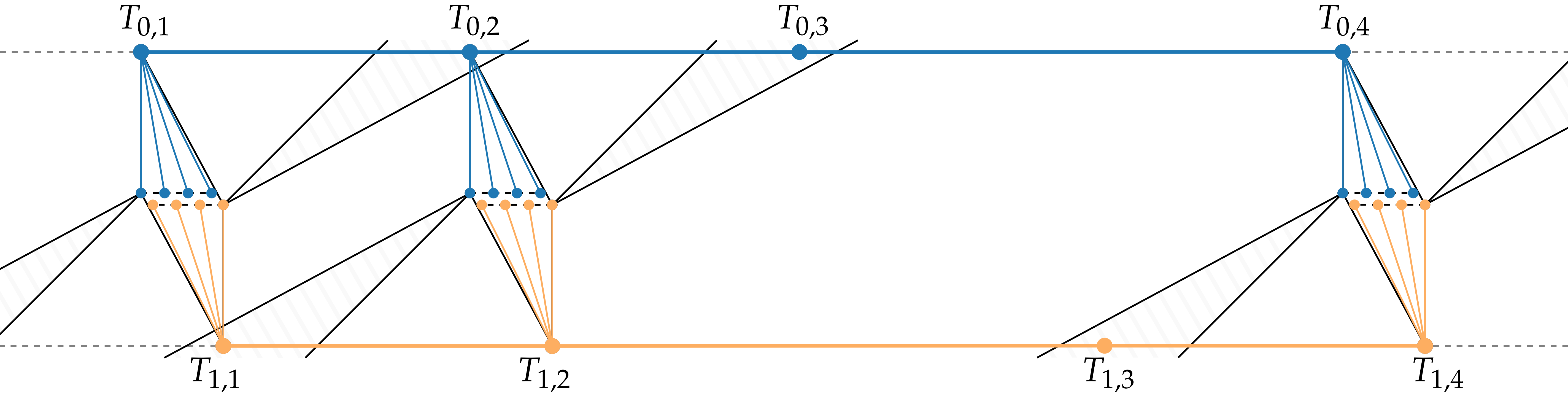
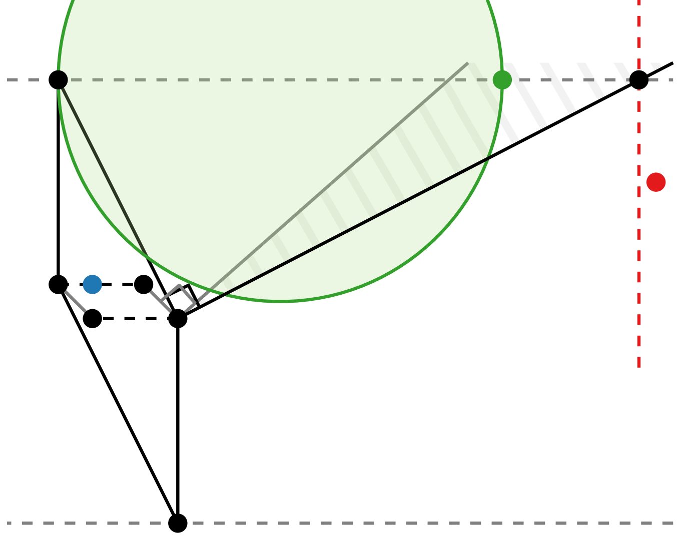
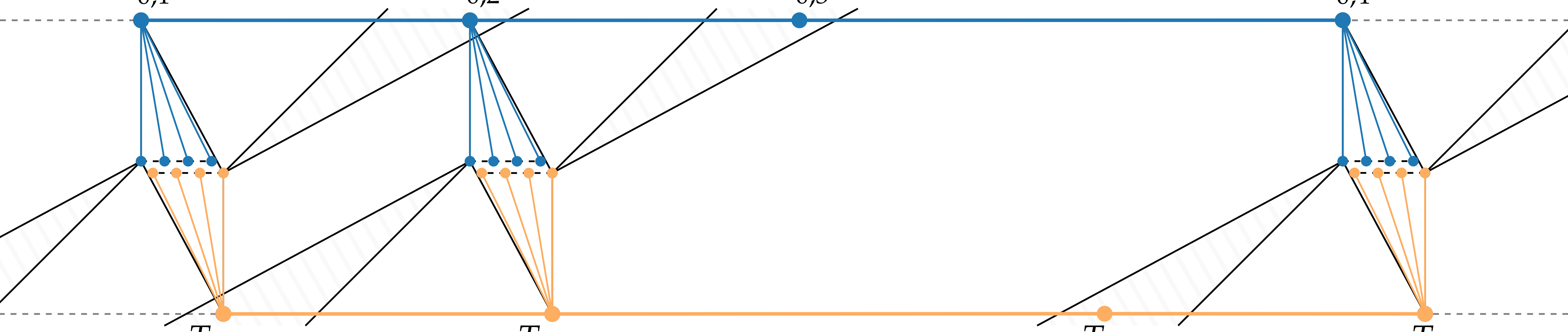


MWG Drawings of Isomorphic Caterpillars

(P1) ensures that there **are edges** between the root of $T_{i,j}$ and $T_{i,j+1}$.

(P1)





MWG Drawings of Isomorphic Caterpillars

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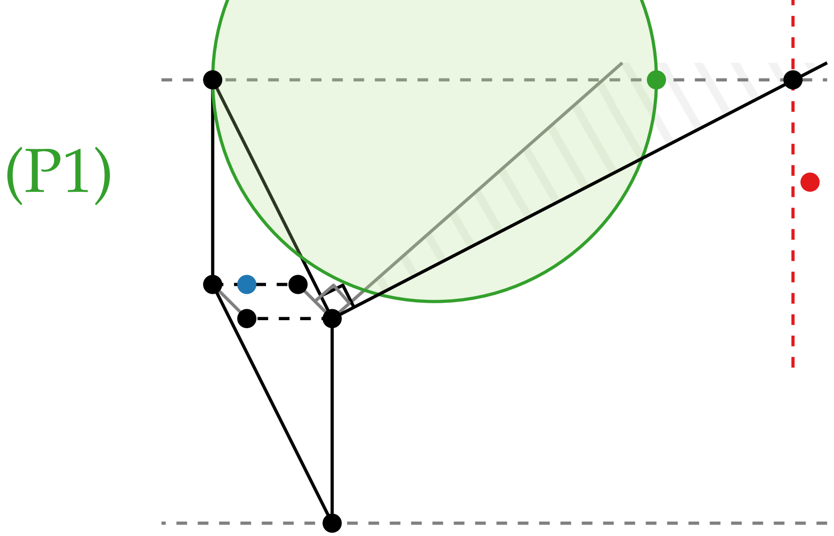
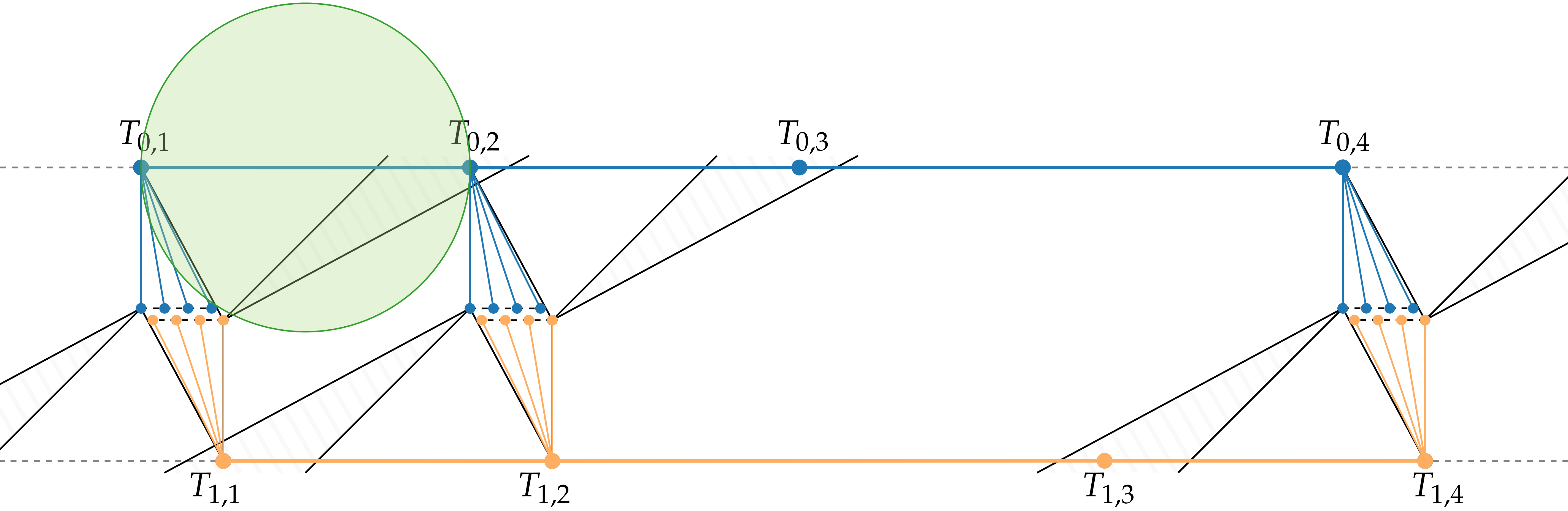


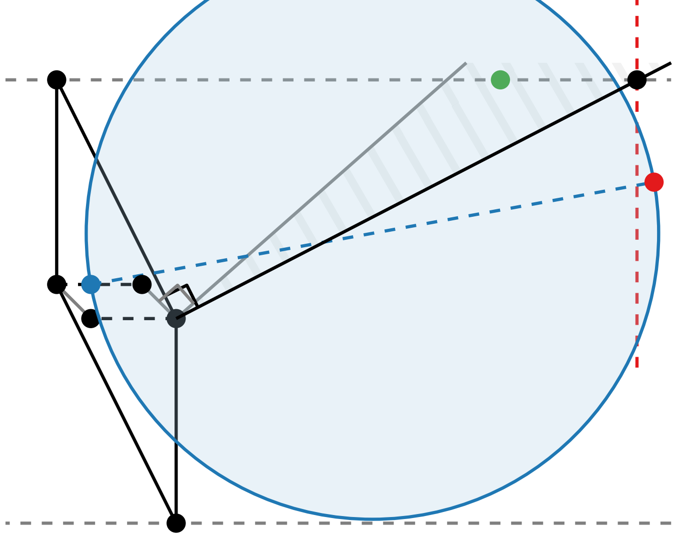


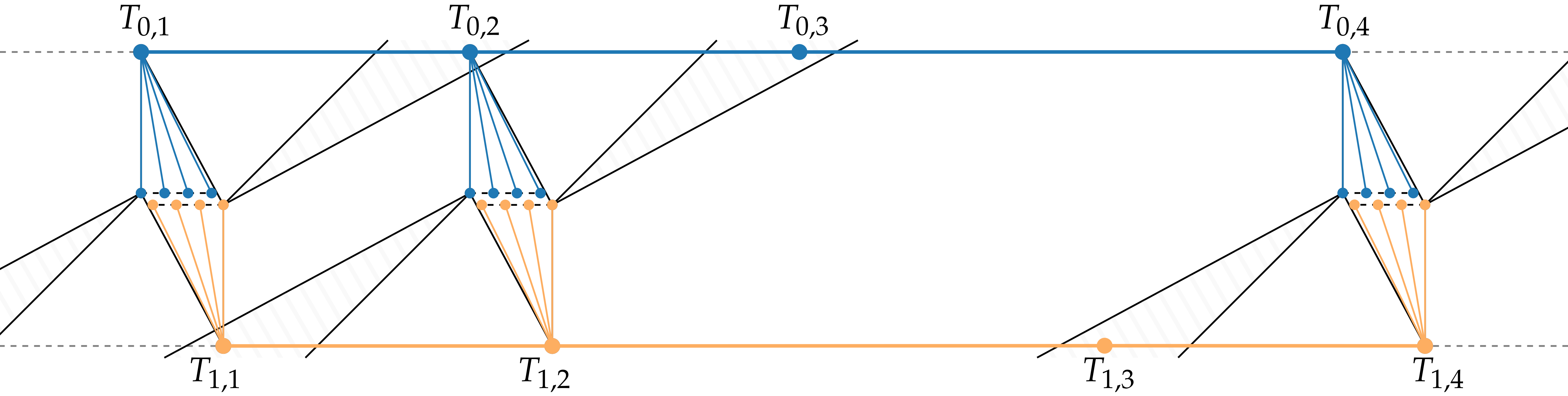
MWG Drawings of Isomorphic Caterpillars

(P1) ensures that there **are edges** between the root of $T_{i,j}$ and $T_{i,j+1}$.

(P2) ensures that there are **no edges** between leaves of $T_{i,j}$ and any vertex of $T_{i,j+1}$.

(P2)





MWG Drawings of Isomorphic Caterpillars

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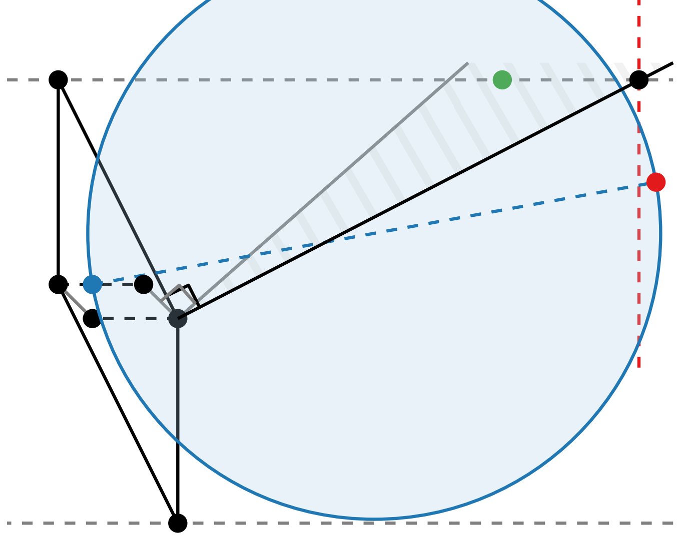

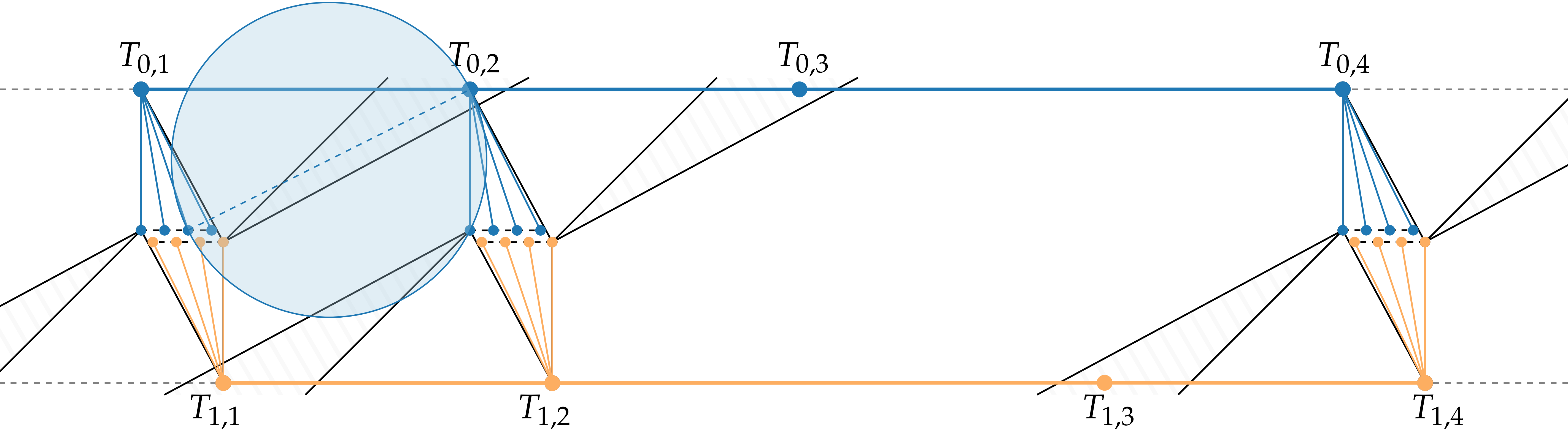

MWG Drawings of Isomorphic Caterpillars

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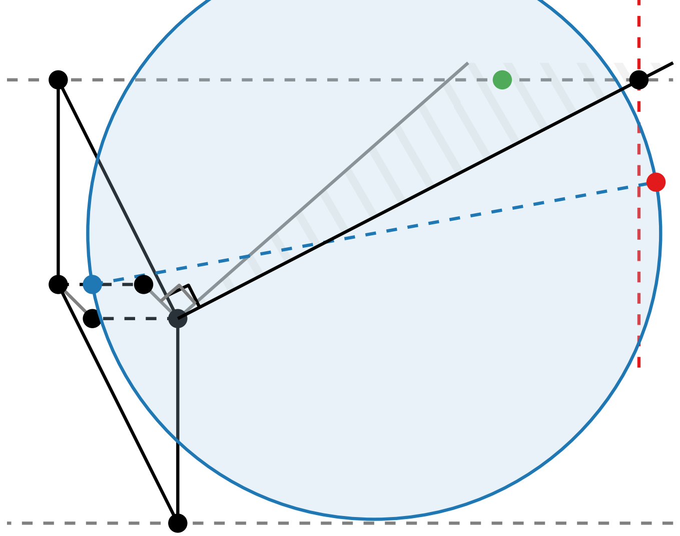

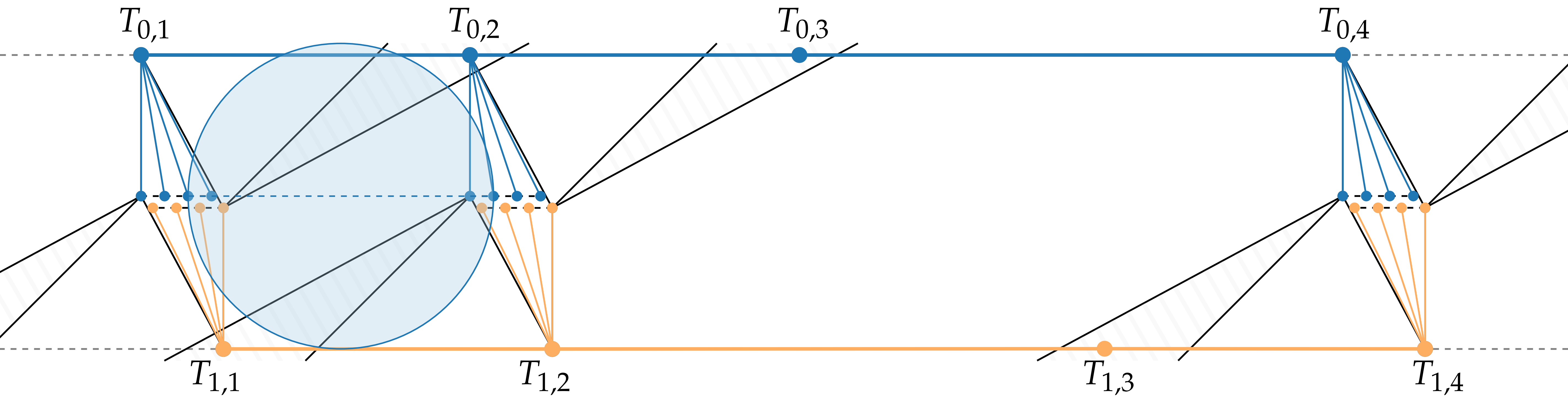

MWG Drawings of Isomorphic Caterpillars

(P1) ensures that there **are edges** between the root of $T_{i,j}$ and $T_{i,j+1}$.

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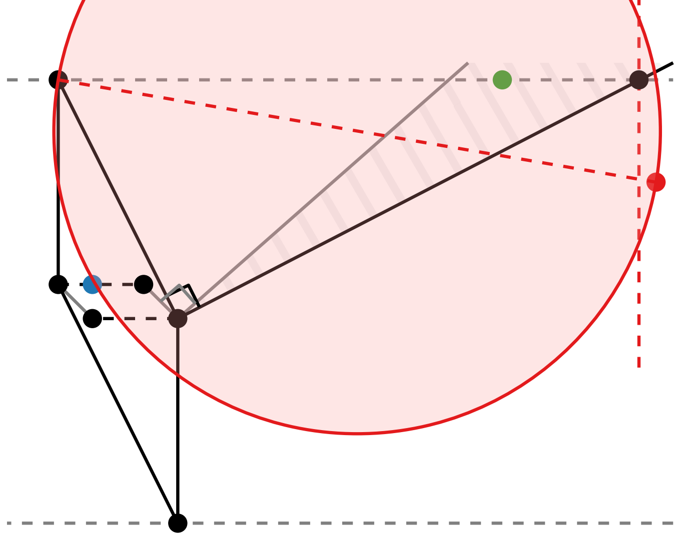
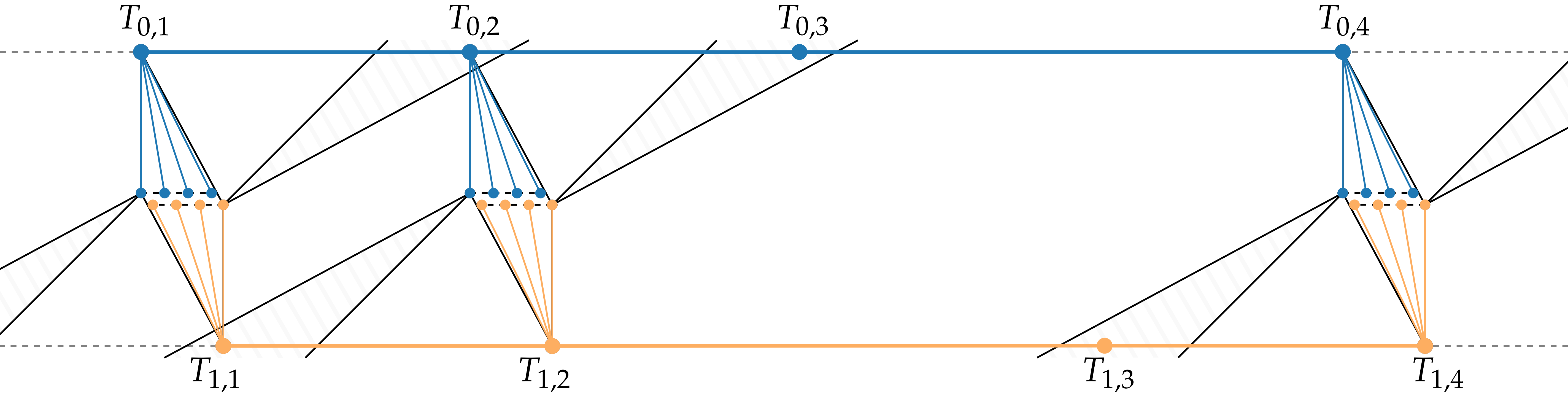
(P3) ensures that there are **no edges** between the root of $T_{i,j}$ and leaves of $T_{i,j+1}$.

(P3)





MWG Drawings of Isomorphic Caterpillars

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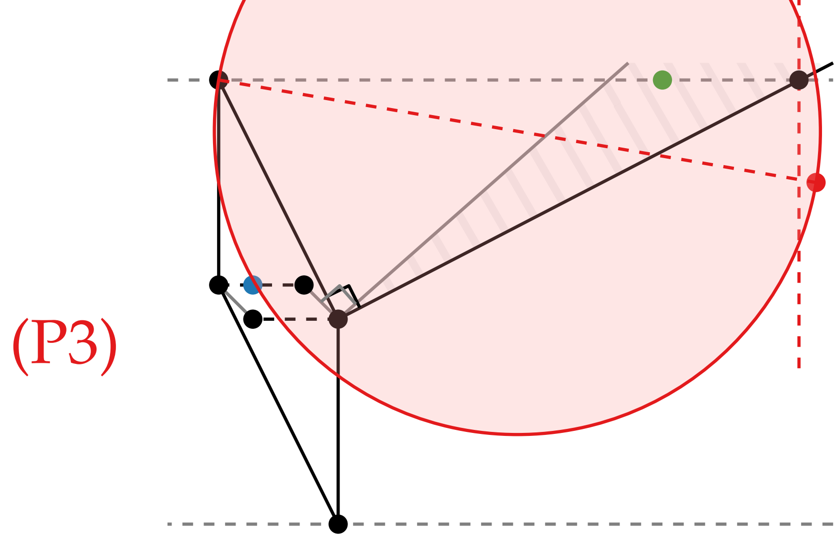
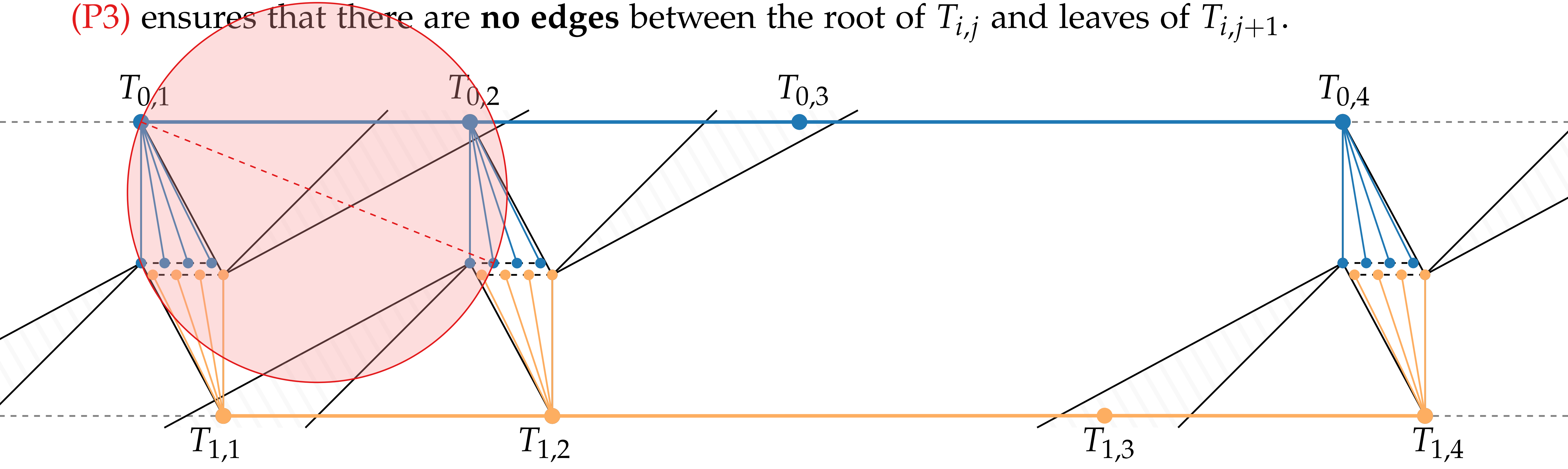
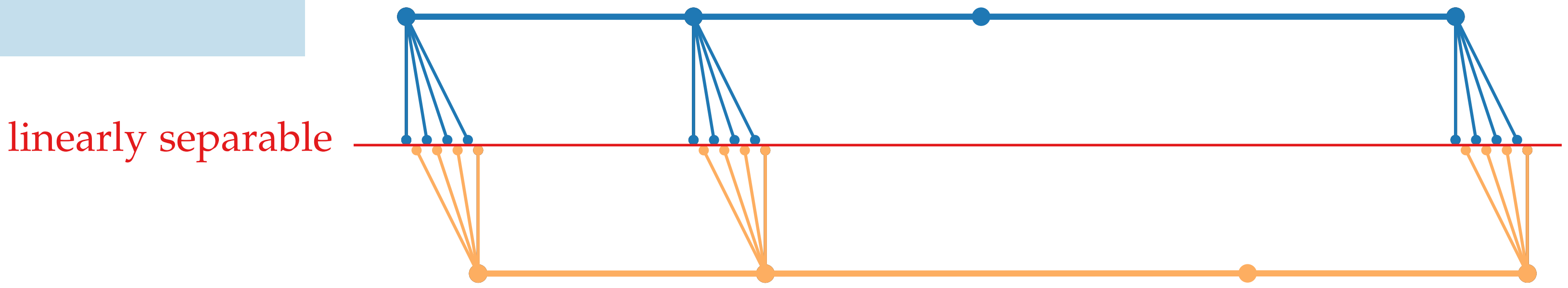




Our Contribution

(1) Mutual Witness Gabriel (MWG) Drawings of isomorphic caterpillars

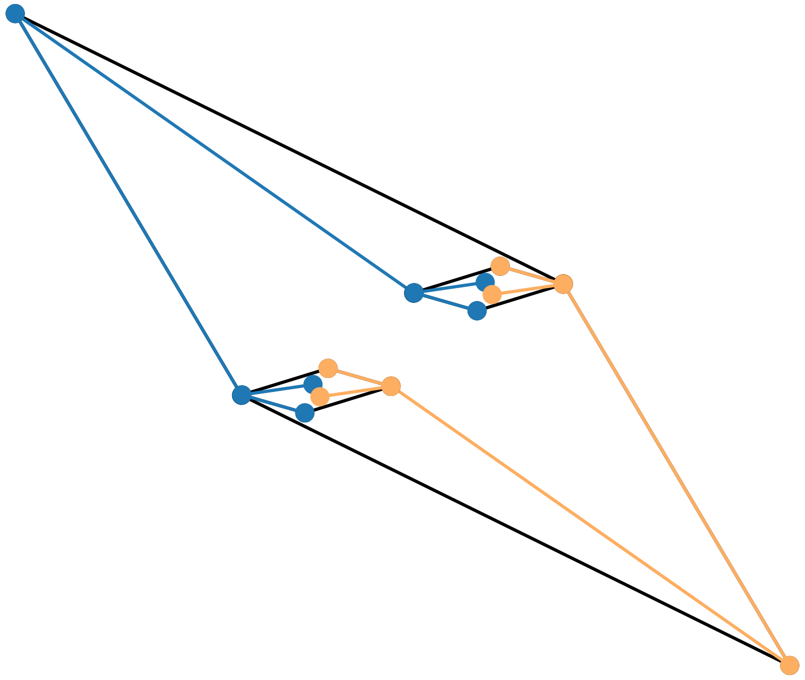
$\beta = 1$



(2) Mutual Witness β Drawings of isomorphic trees

for all $\beta \geq 1$

(3) Mutual Witness β Drawings of *almost* isomorphic trees



Our Contribution

$\beta = 1$

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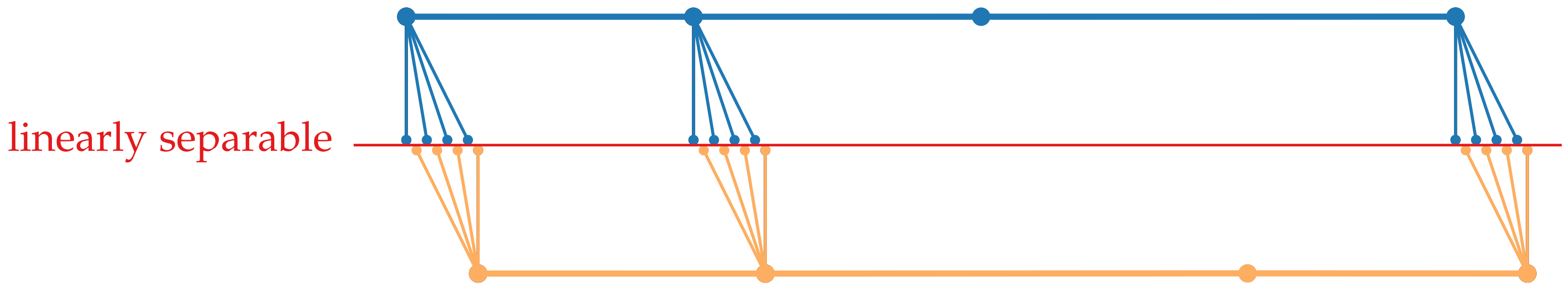
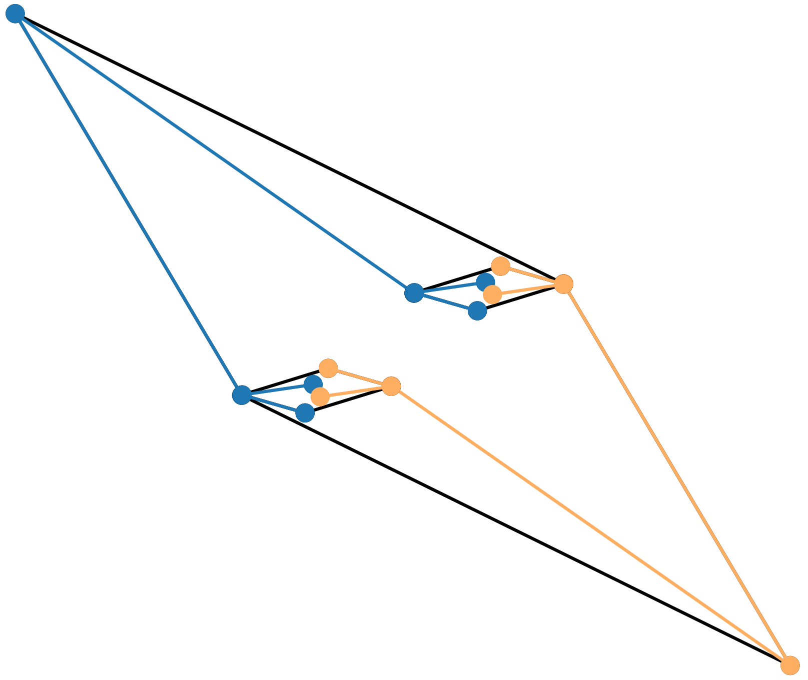
of isomorphic caterpillars



(2) Mutual Witness β Drawings
of isomorphic trees

for all $\beta \geq 1$



(3) Mutual Witness β Drawings
of almost isomorphic trees

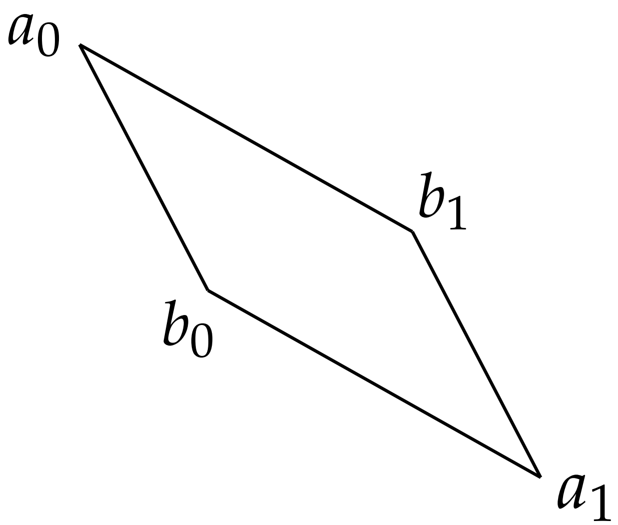
MW β-Proximity Drawings of Isomorphic Trees

MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:

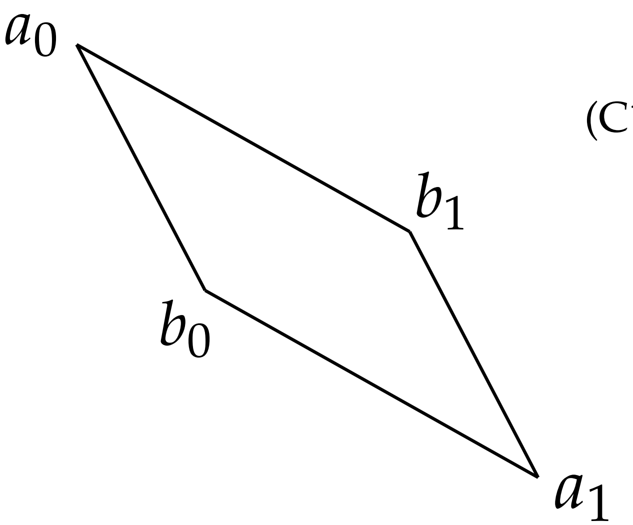
MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



MW β-Proximity Drawings of Isomorphic Trees

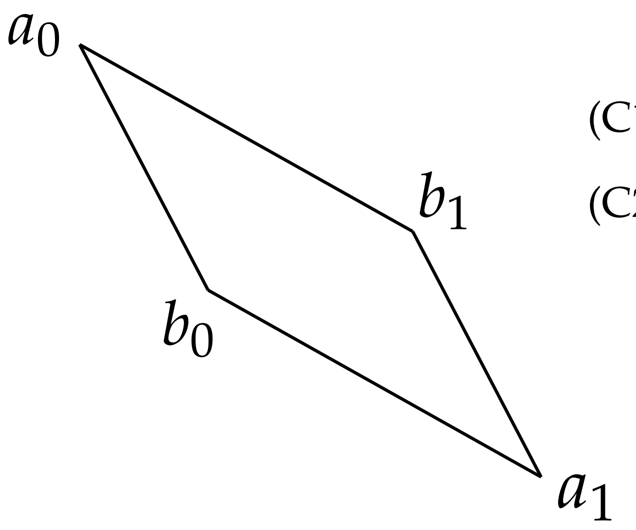
Subtrees inside parallelograms:



(C1) $x(a_0) < x(b_0) < x(b_1) < x(a_1)$

MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:

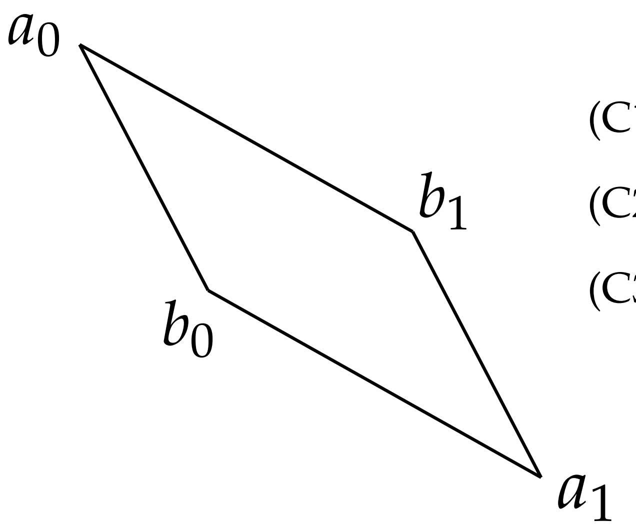


(C1) $x(a_0) < x(b_0) < x(b_1) < x(a_1)$

(C2) $y(a_0) > y(b_1) > y(b_0) > y(a_1)$

MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



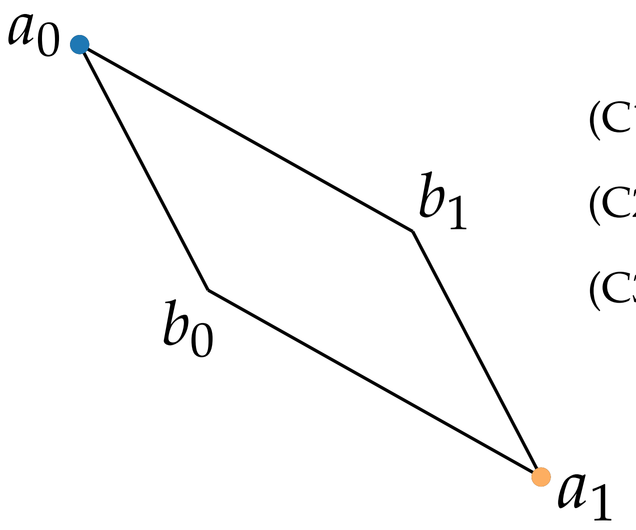
(C1) $x(a_0) < x(b_0) < x(b_1) < x(a_1)$

(C2) $y(a_0) > y(b_1) > y(b_0) > y(a_1)$

(C3) no vertical edges

MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



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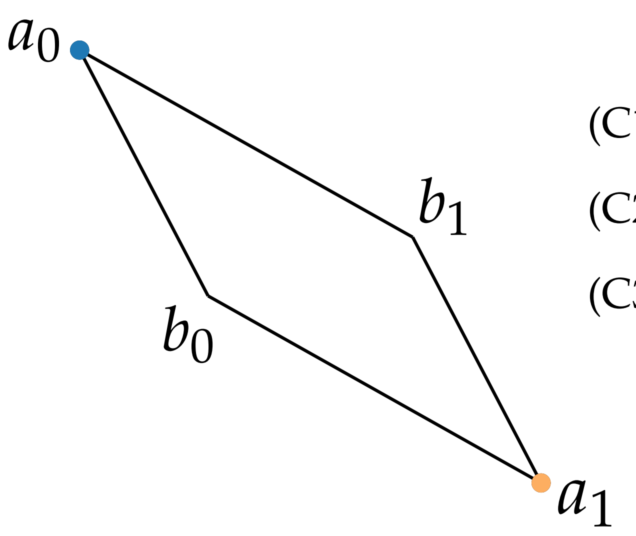

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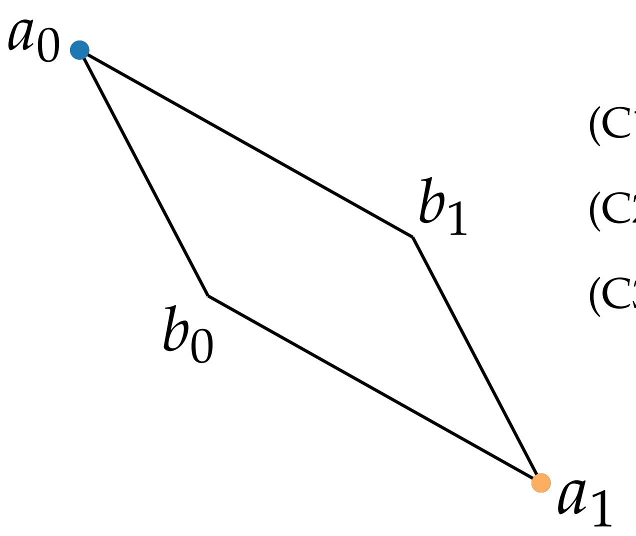
Step 1:

MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



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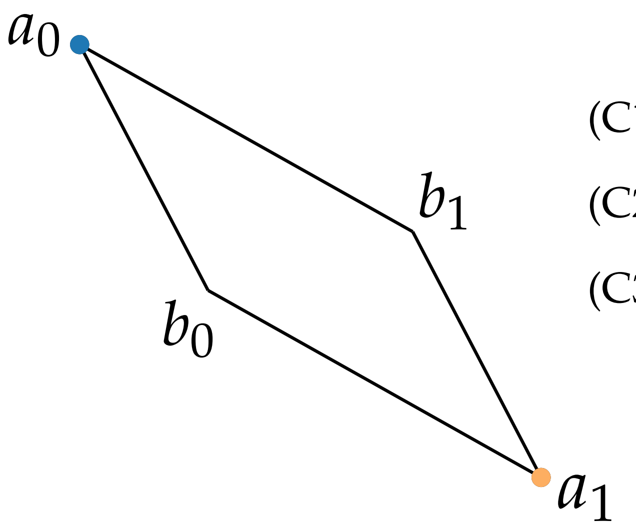
Step 1: place subtrees next to each other

MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



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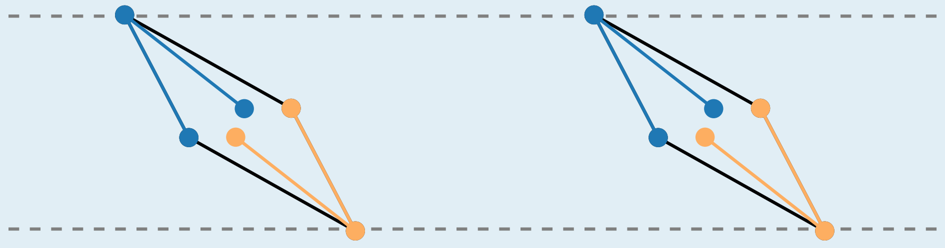
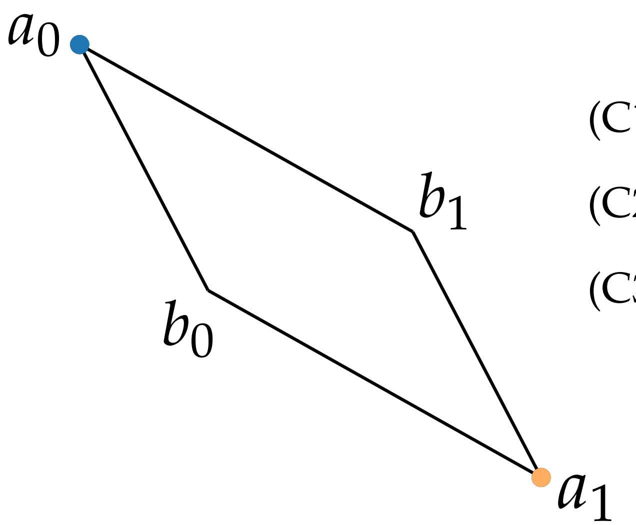


MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



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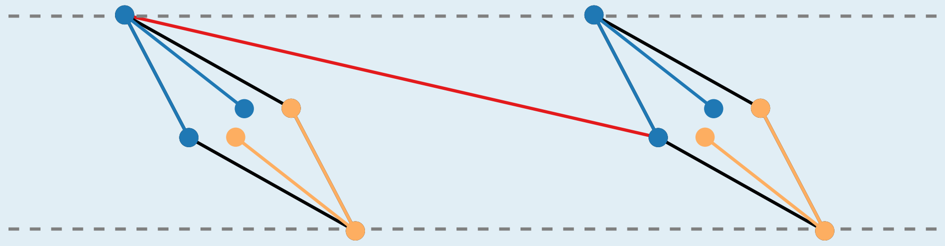

MW β-Proximity Drawings of Isomorphic Trees

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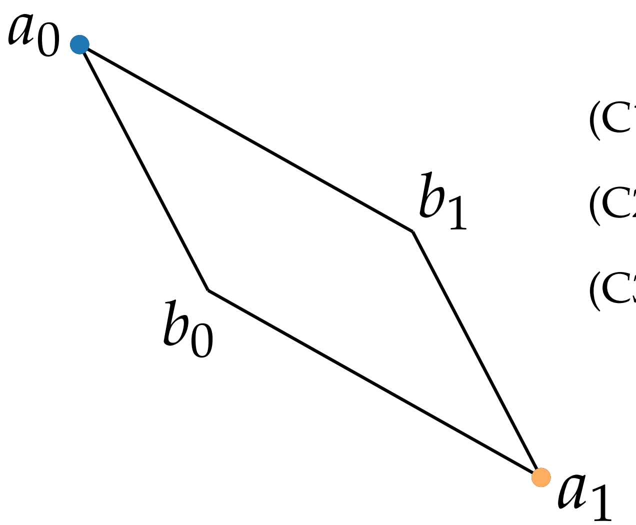

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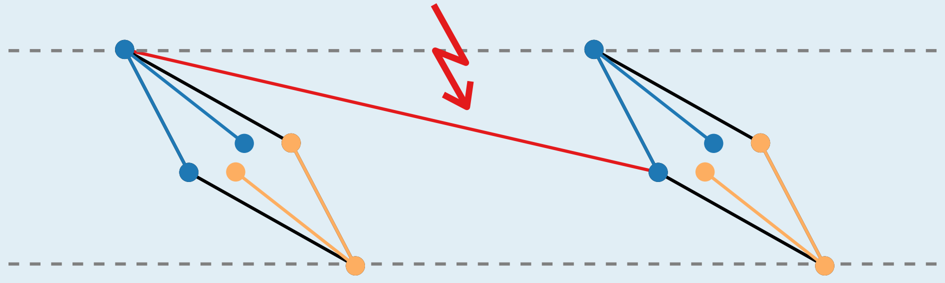
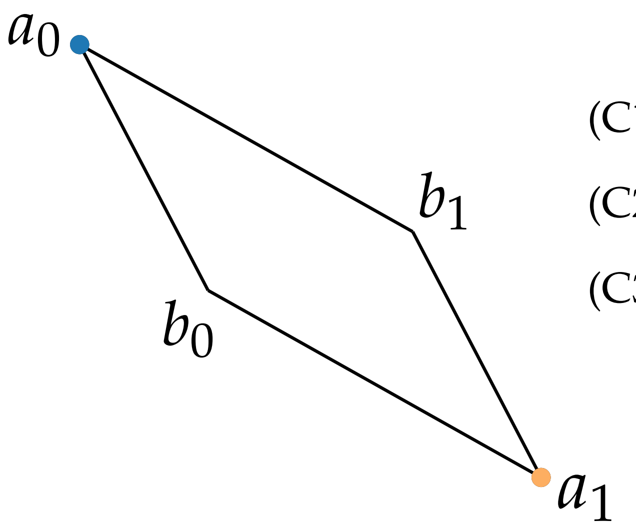


MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



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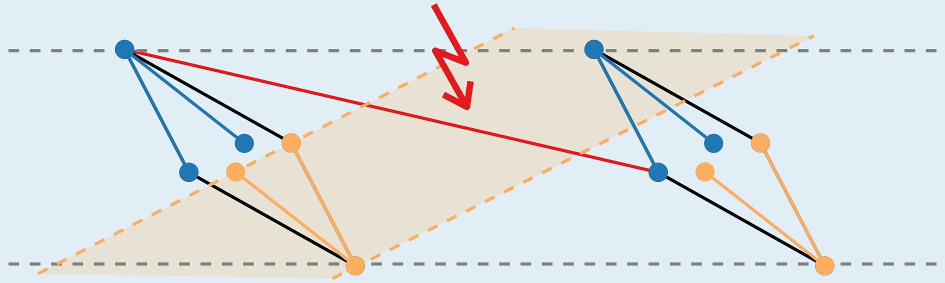
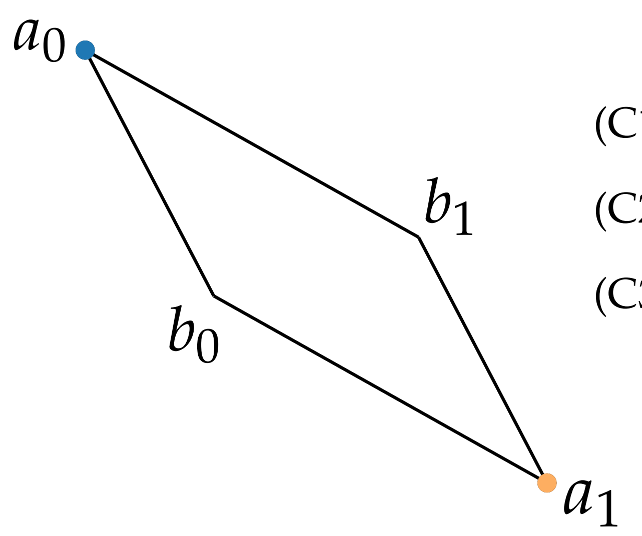


MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



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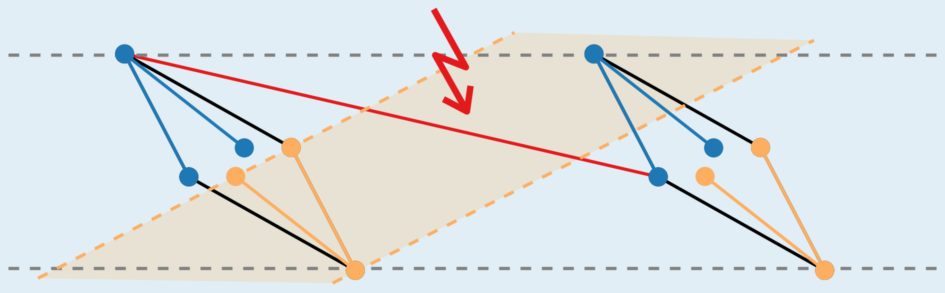
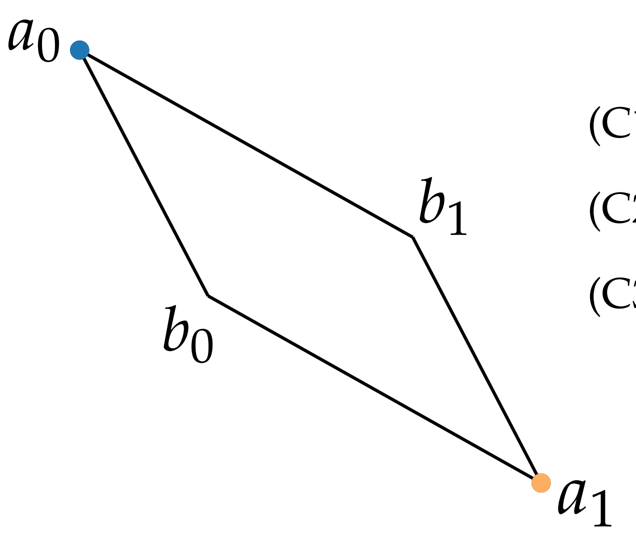


Step 2:

MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



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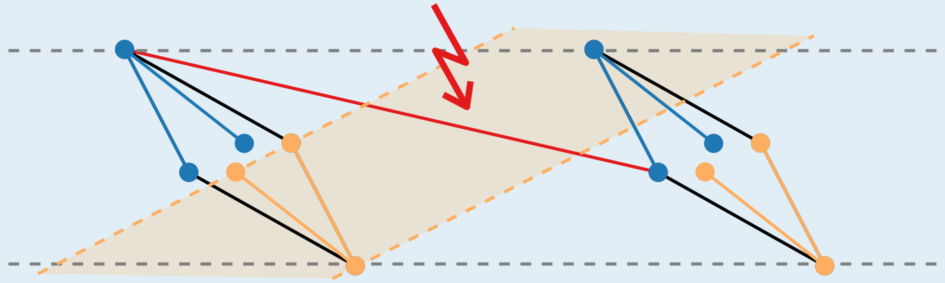
Step 1: place subtrees next to each other

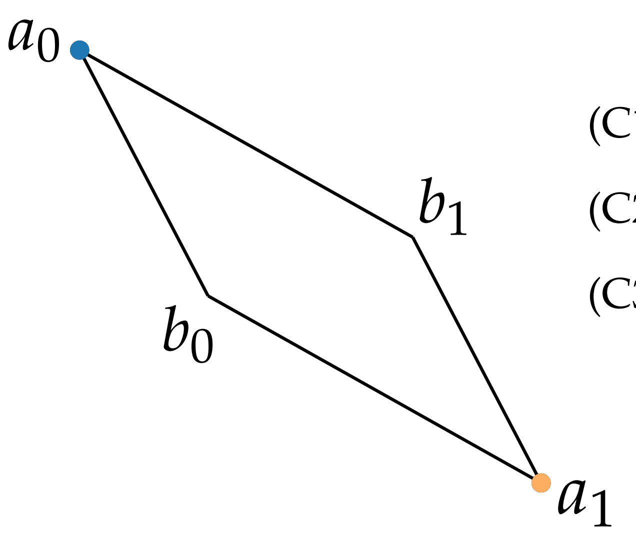


Step 2:

add root to subtrees

MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



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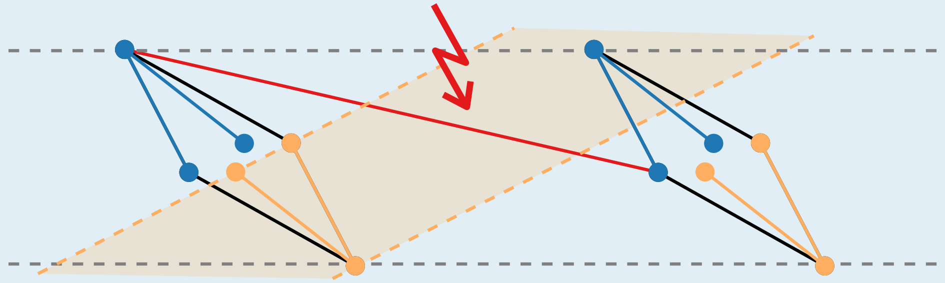

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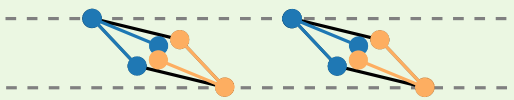
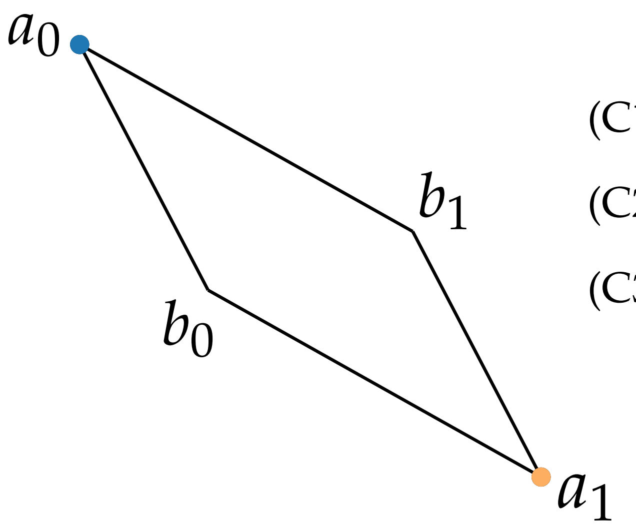


MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



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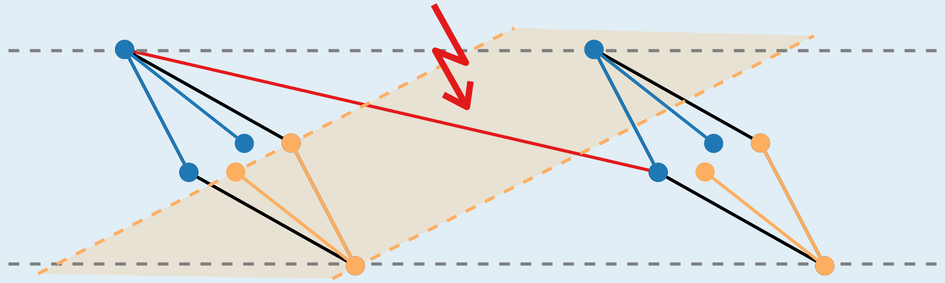
Step 1: place subtrees next to each other



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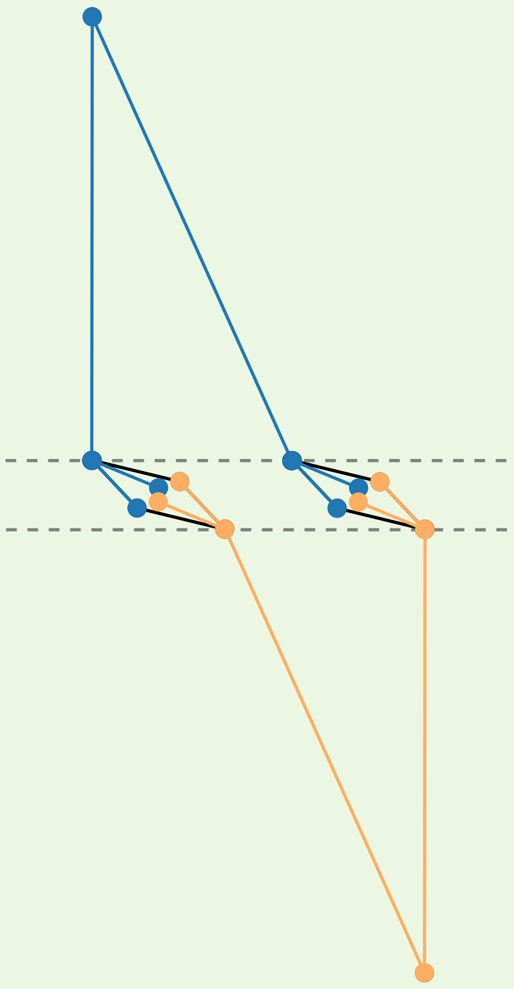
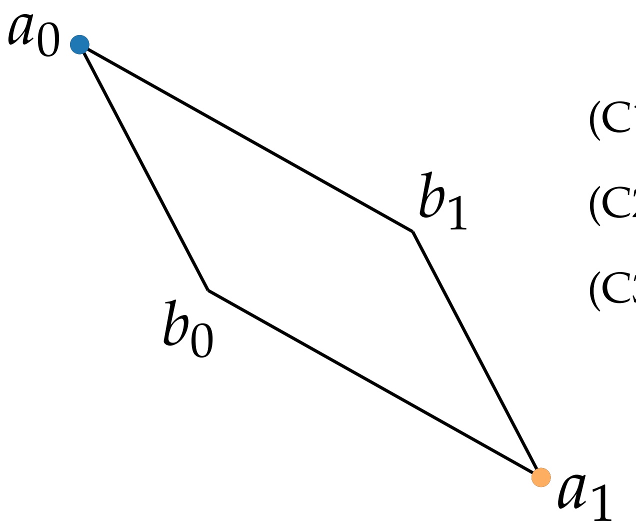


MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



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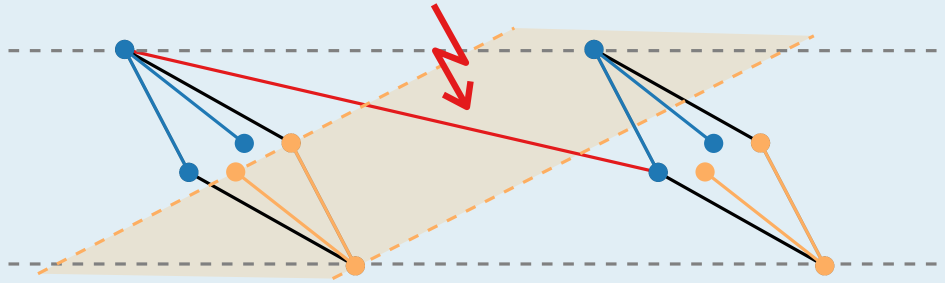
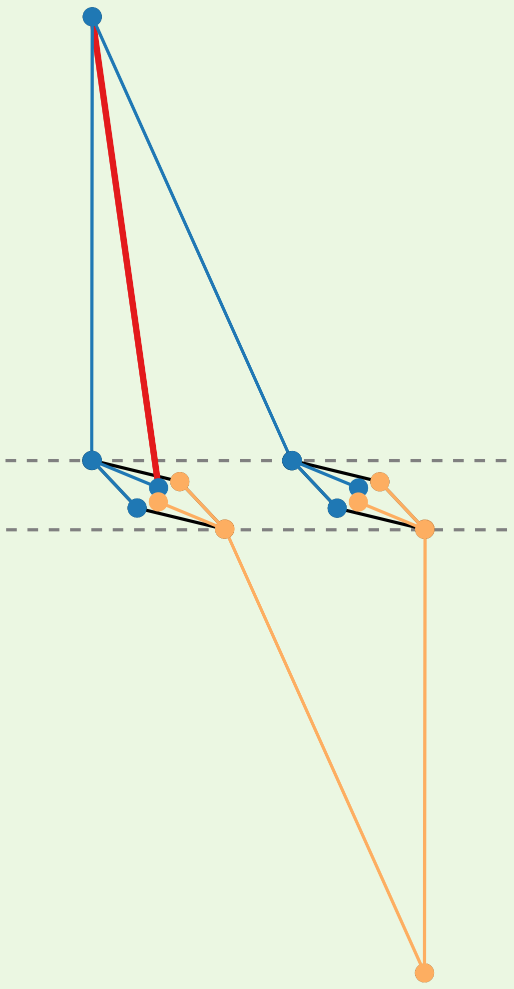
Step 1: place subtrees next to each other



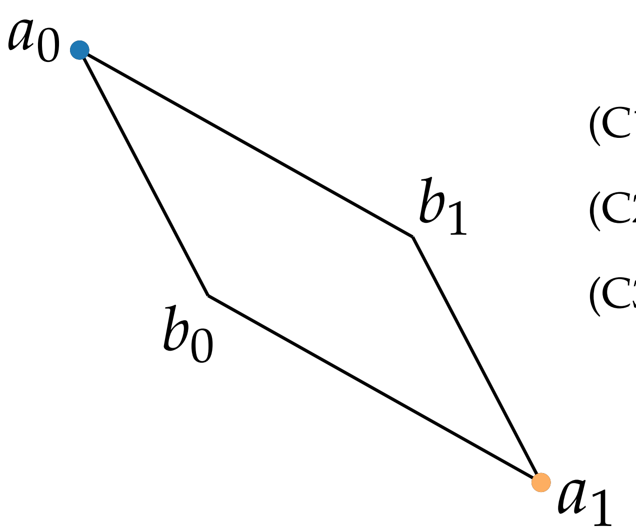
Step 2:

add root to subtrees



MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



(C1) $x(a_0) < x(b_0) < x(b_1) < x(a_1)$

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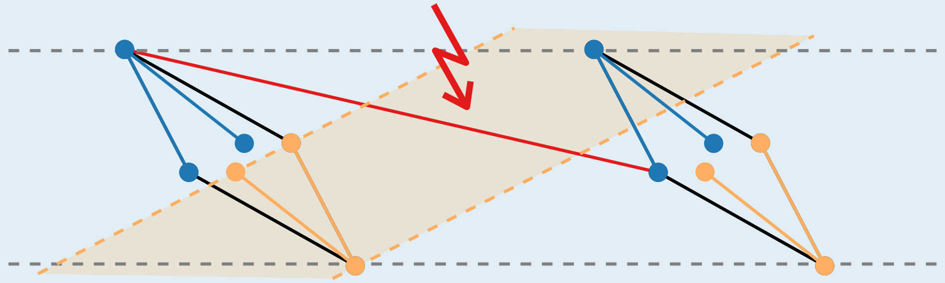
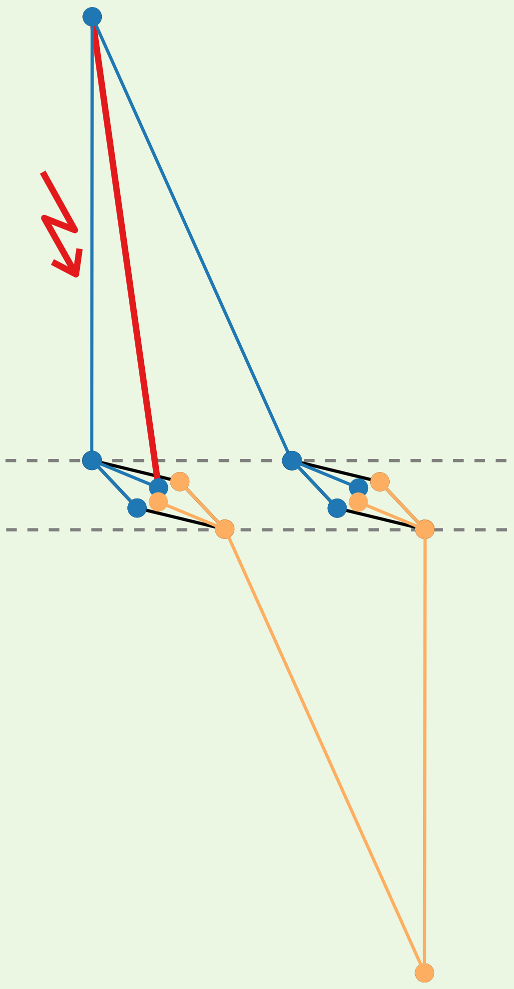
(C3) no vertical edges

Step 1: place subtrees next to each other



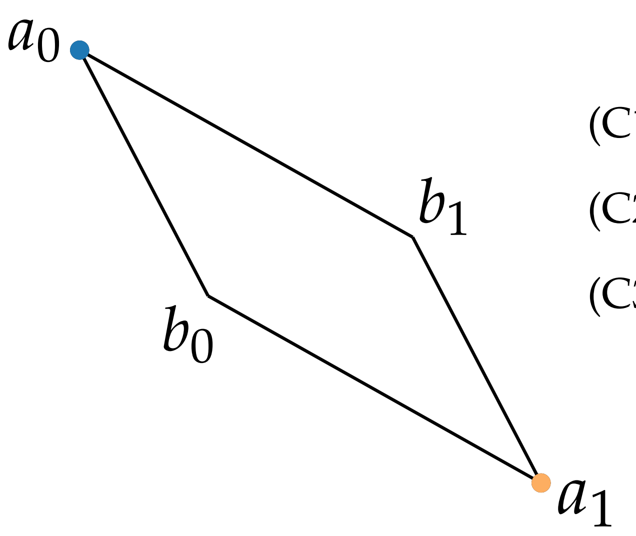
Step 2:

add root to subtrees



MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



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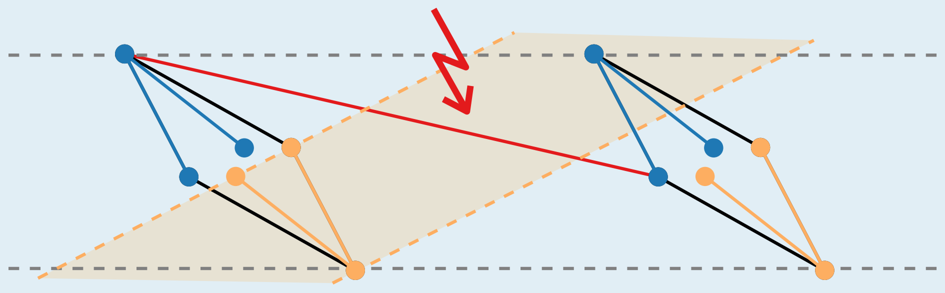
Step 1: place subtrees next to each other



Step 2:

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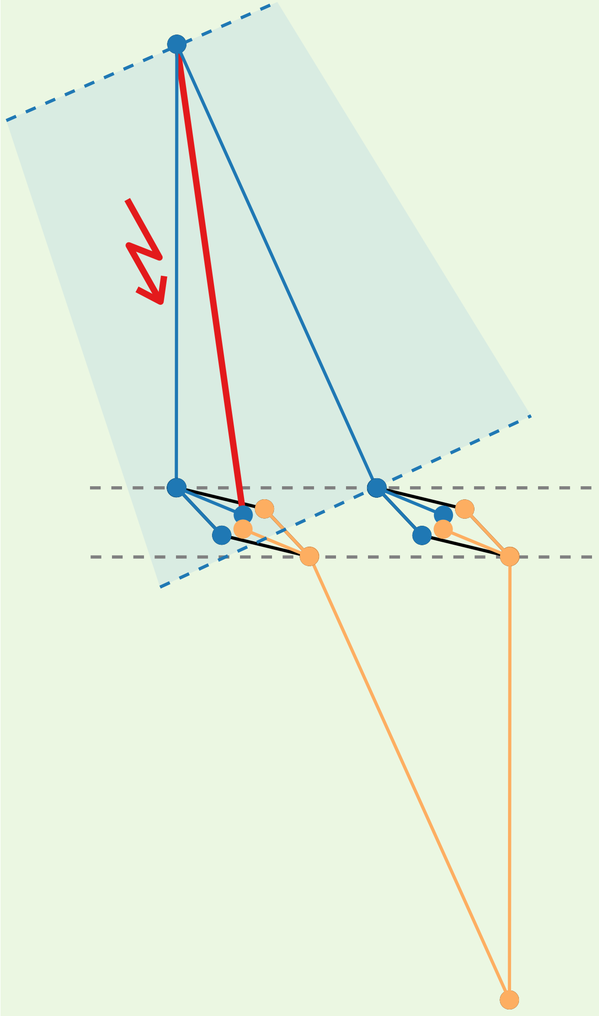
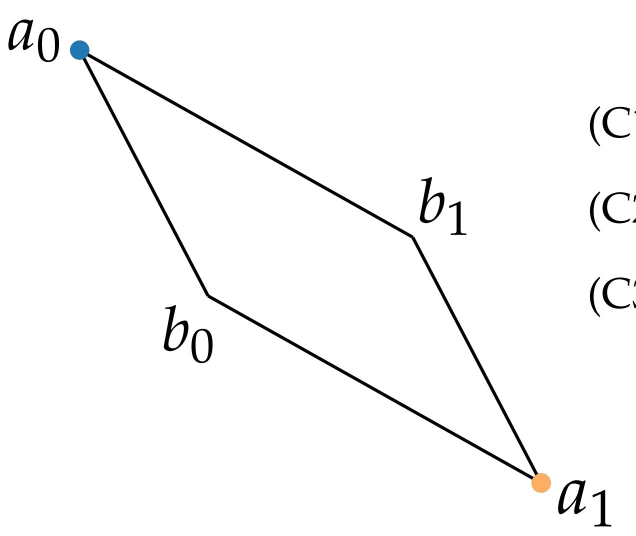


MW β-Proximity Drawings of Isomorphic Trees

Subtrees inside parallelograms:



(C1) $x(a_0) < x(b_0) < x(b_1) < x(a_1)$

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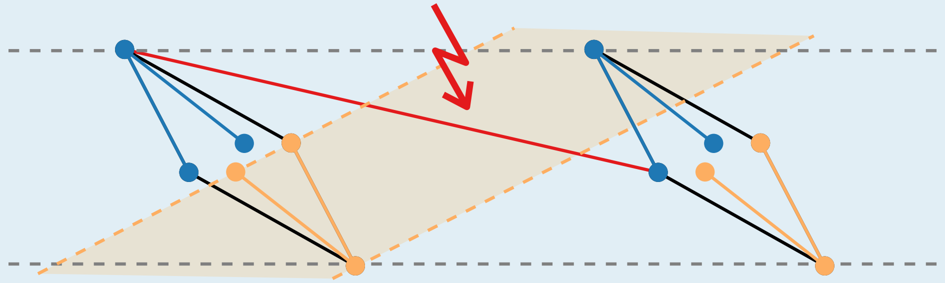
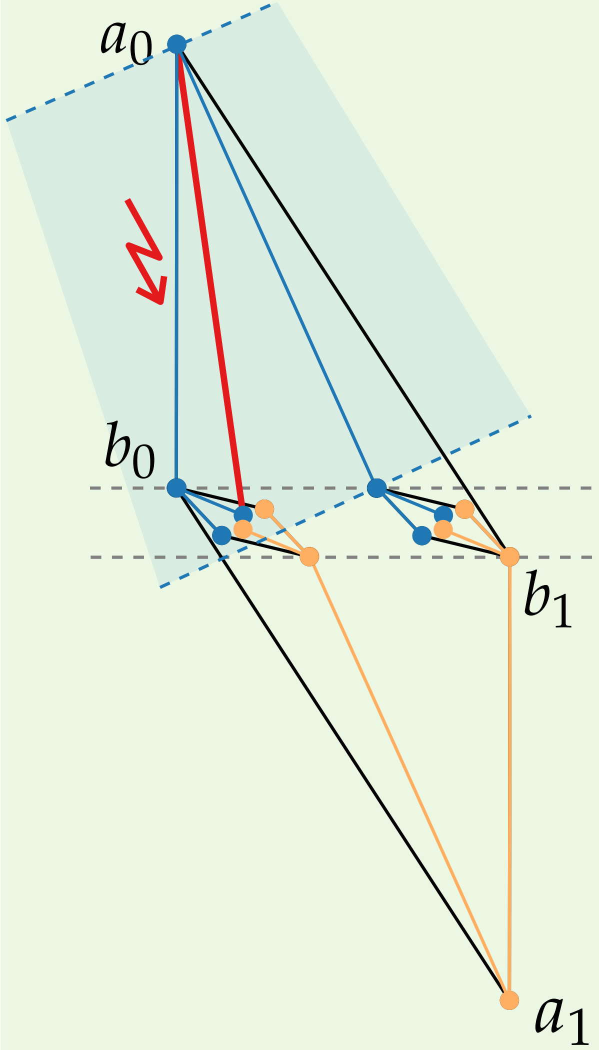
(C3) no vertical edges

Step 1: place subtrees next to each other



Step 2:

add root to subtrees



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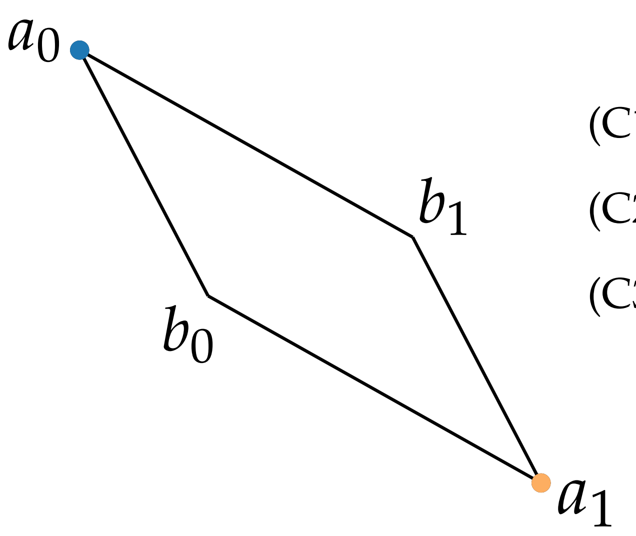

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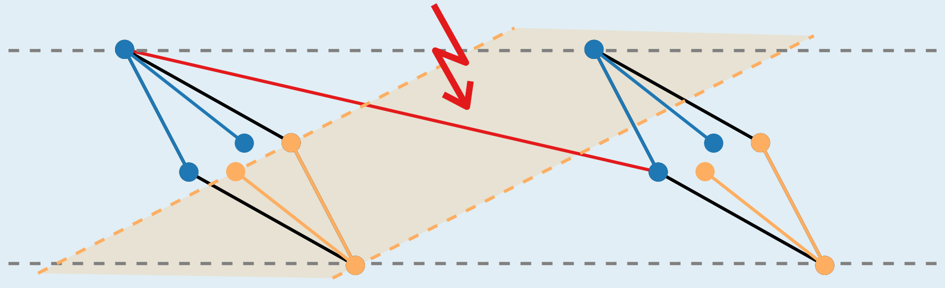

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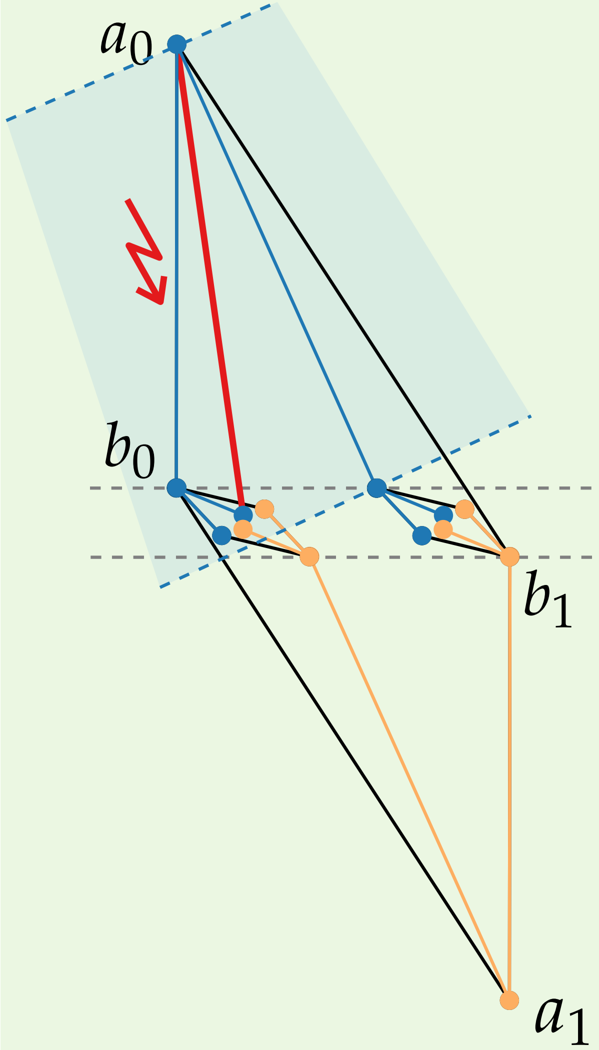

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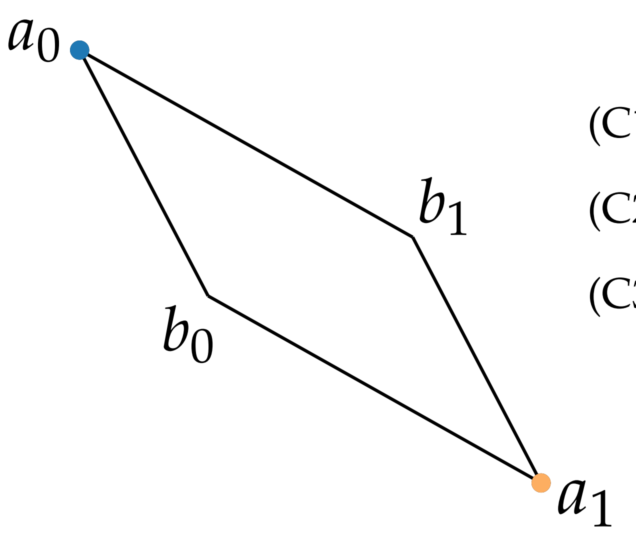

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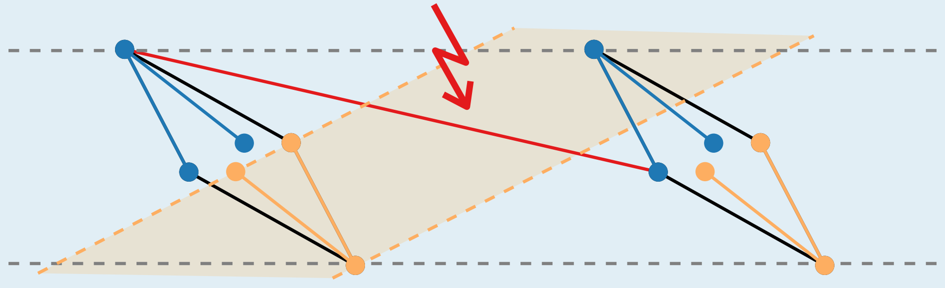

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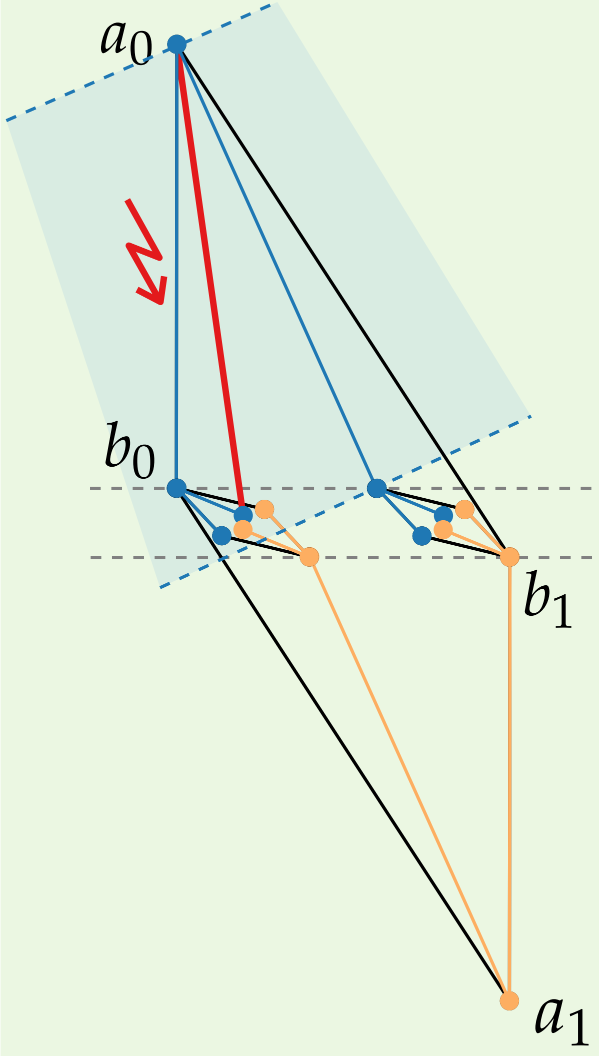

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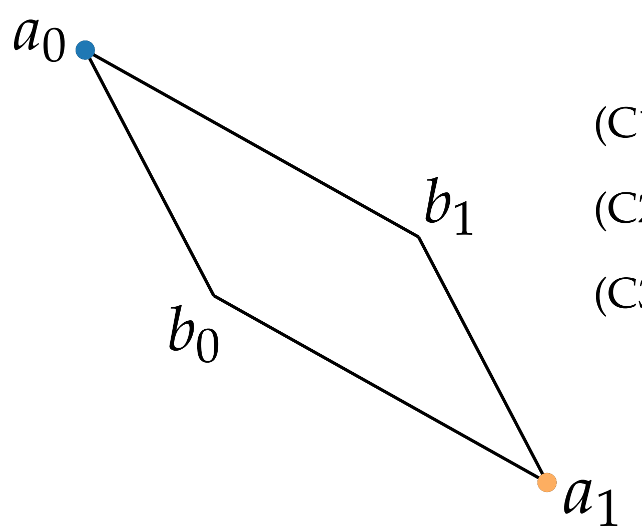

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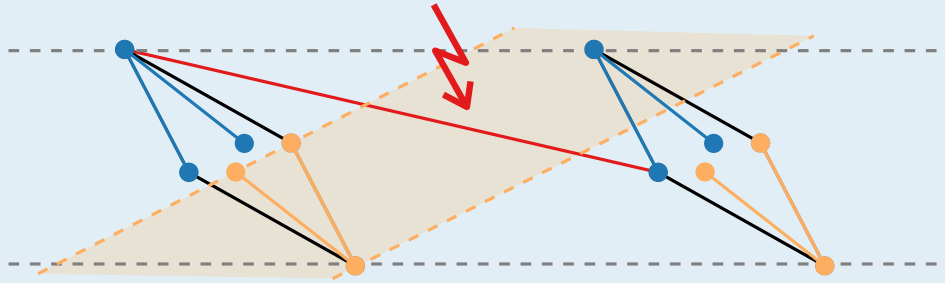

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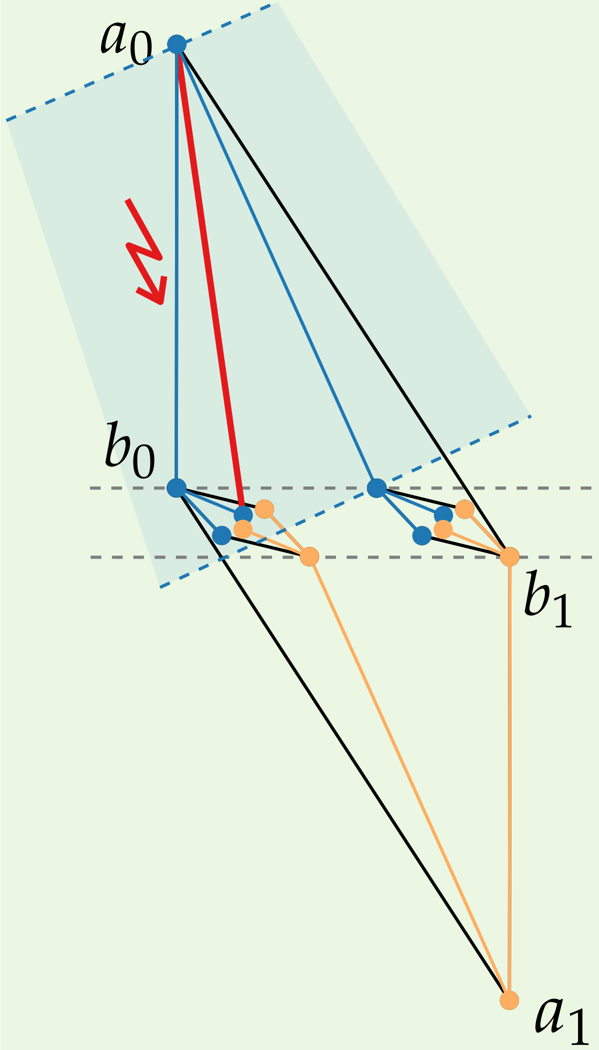

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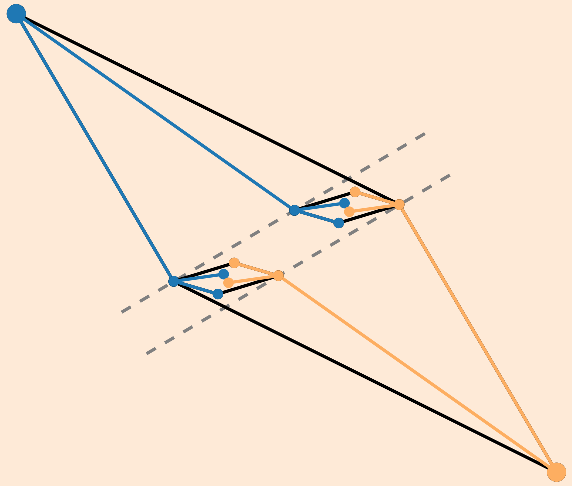
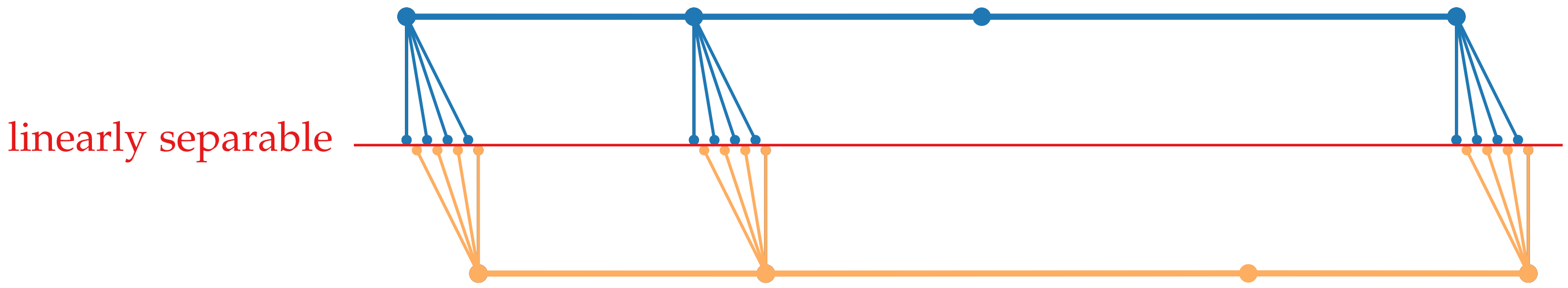


Our Contribution

$\beta = 1$

(1) Mutual Witness Gabriel (MWG) Drawings
of isomorphic caterpillars



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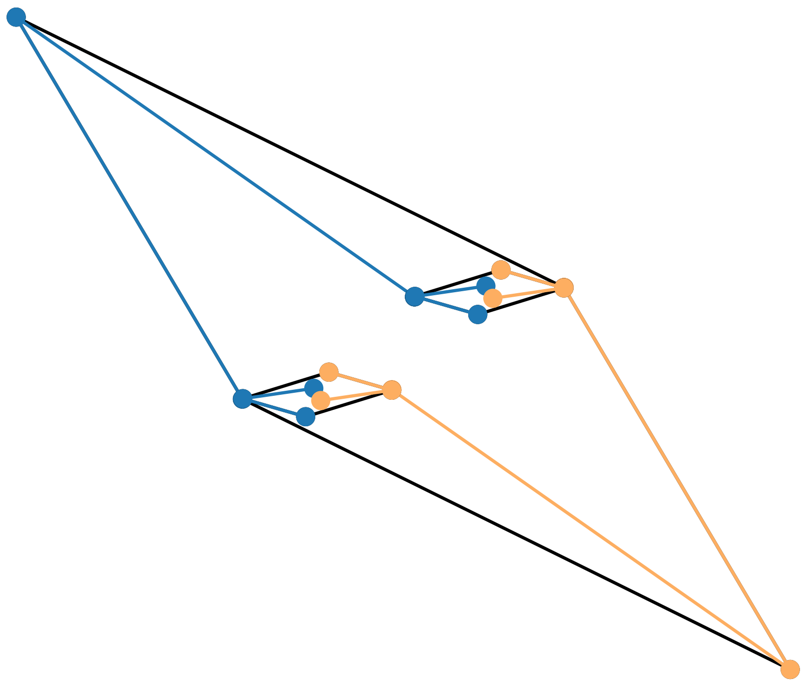

(3) Mutual Witness β Drawings
of almost isomorphic trees

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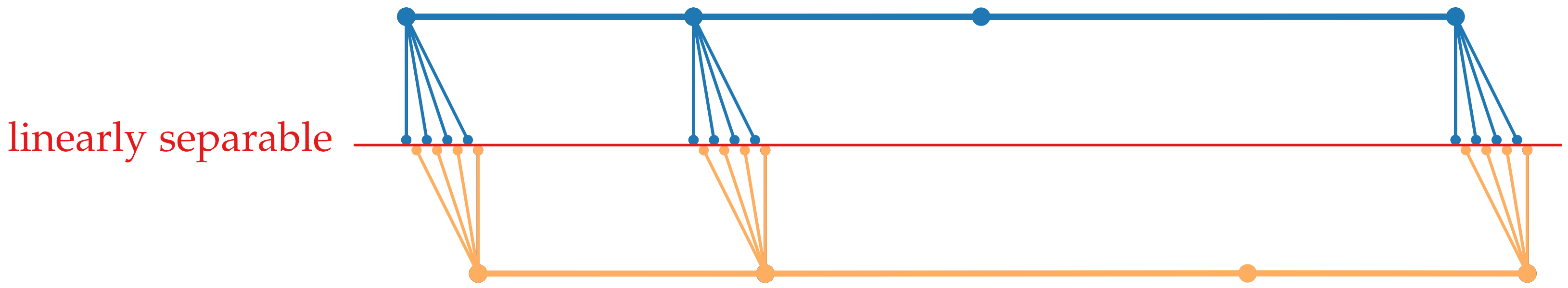

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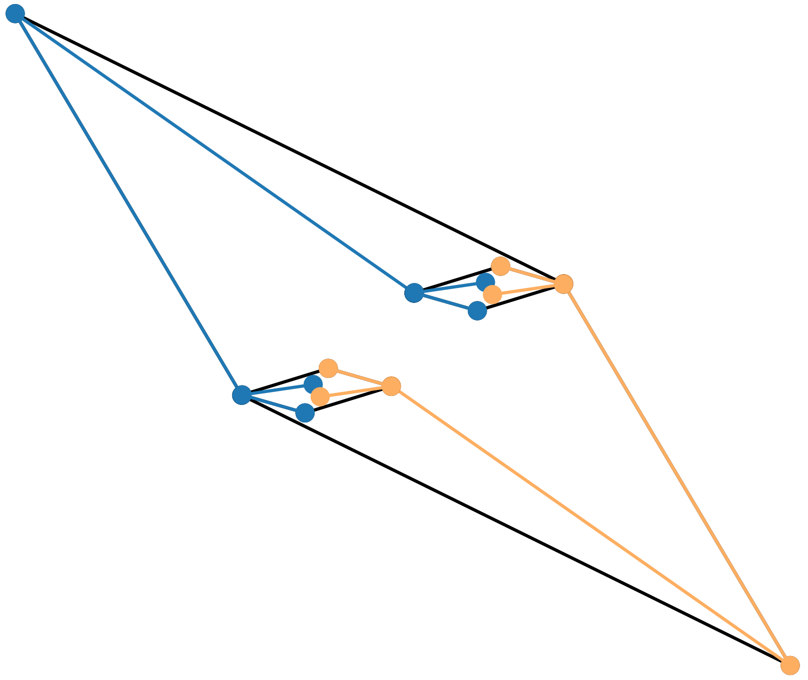

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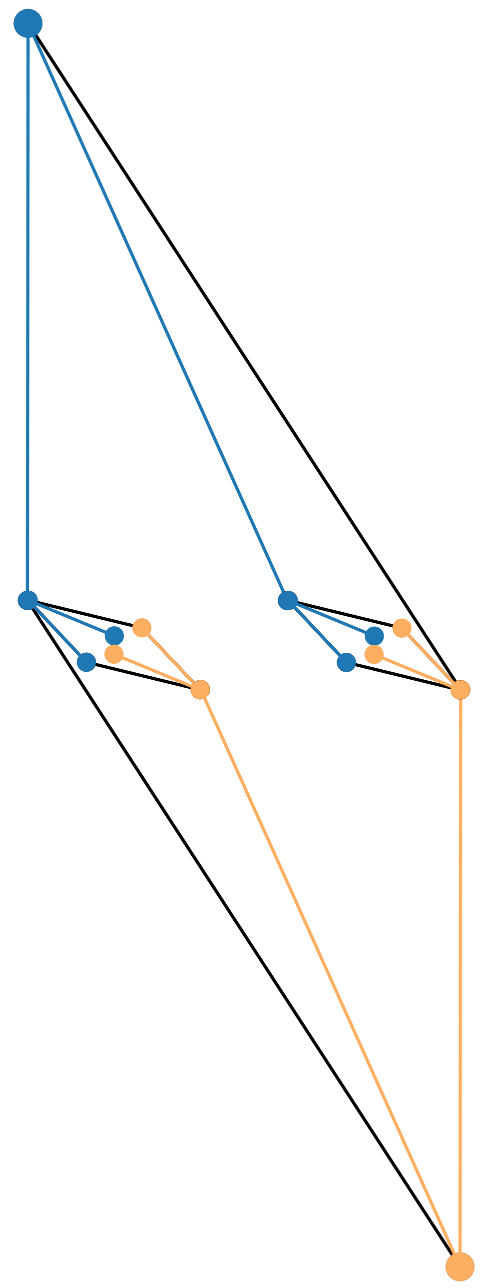

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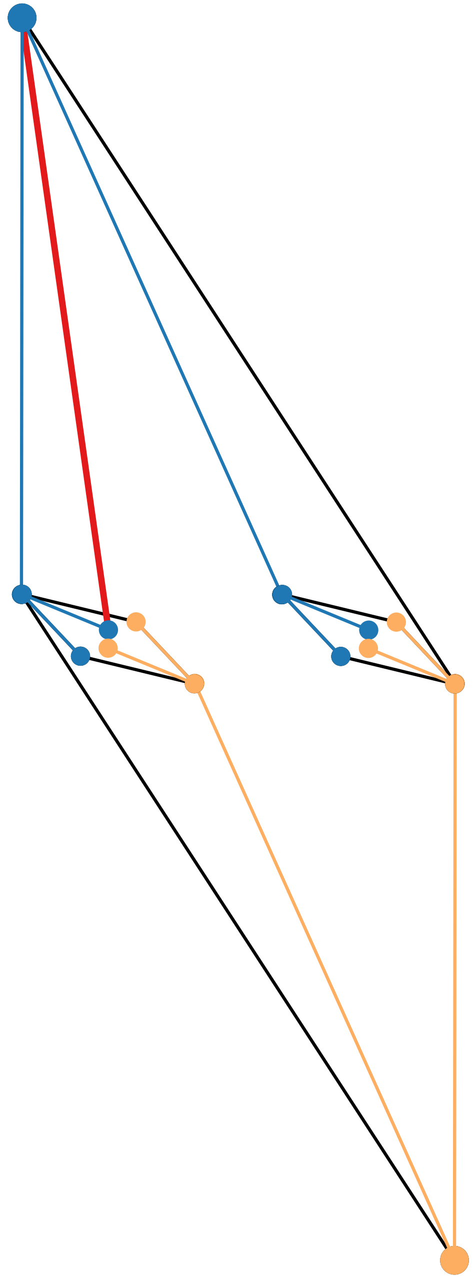

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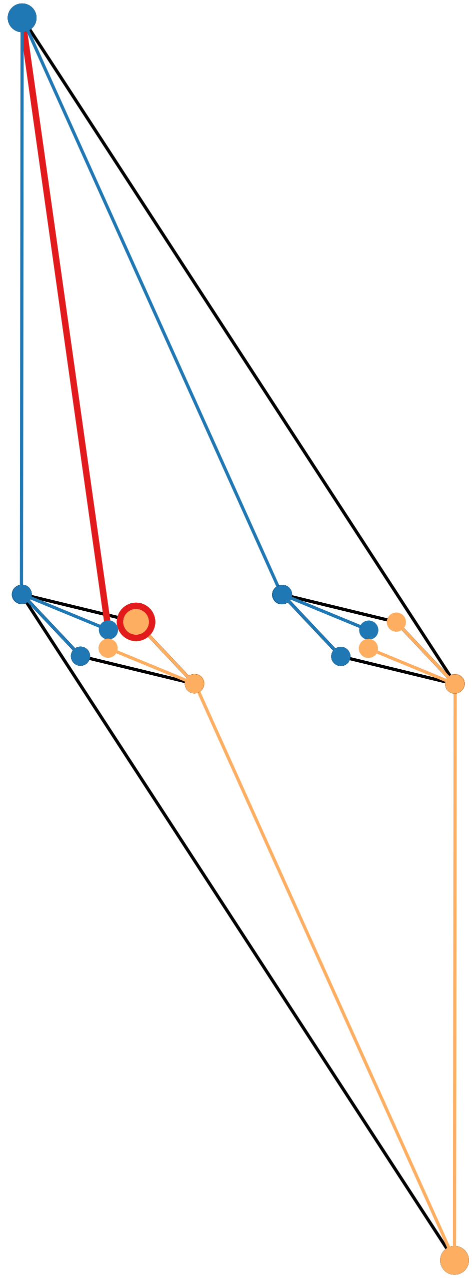

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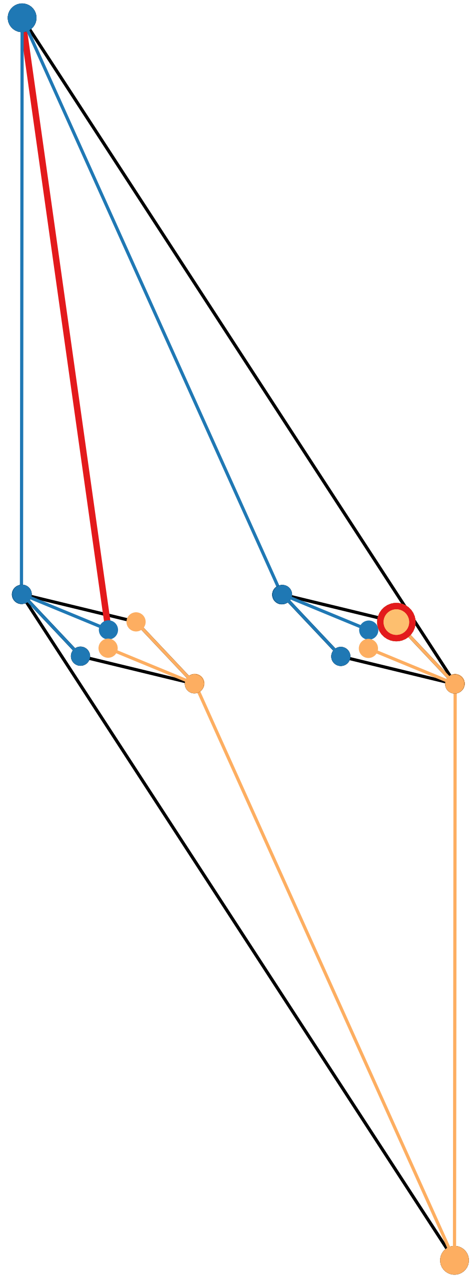

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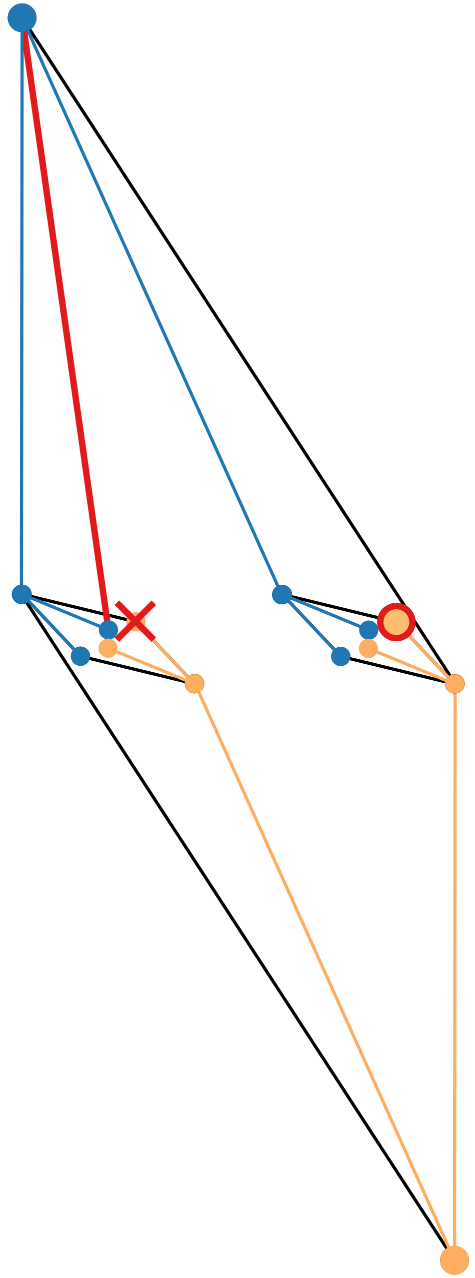

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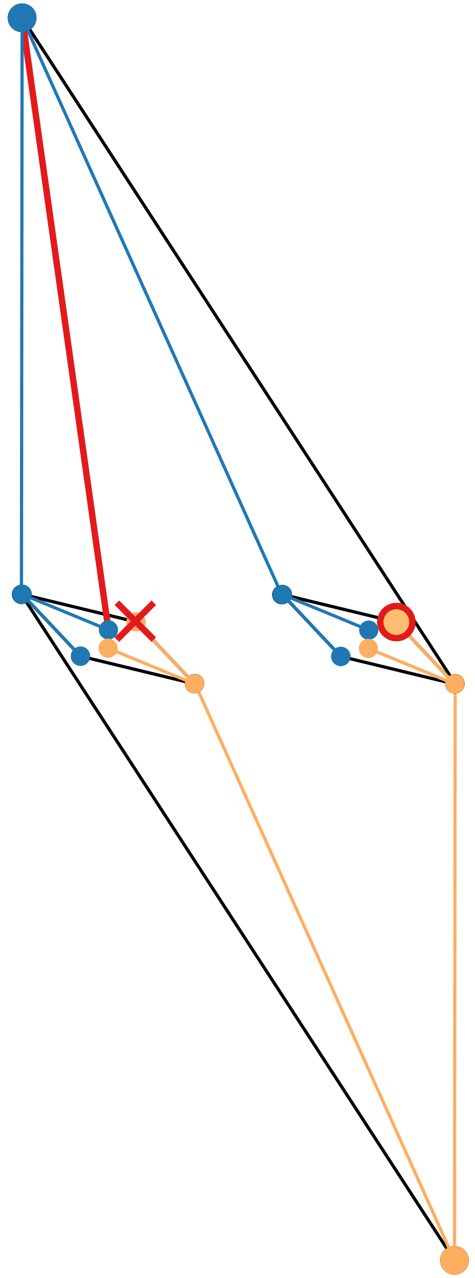
Theorem

Let (T, r) be a rooted tree and let $\mathcal{L}$ be a sparse set of leaves of T. Then the pair $\langle T, T \setminus \mathcal{L} \rangle$ of trees admits an MW-β drawing for all $\beta \in [1, \infty]$.



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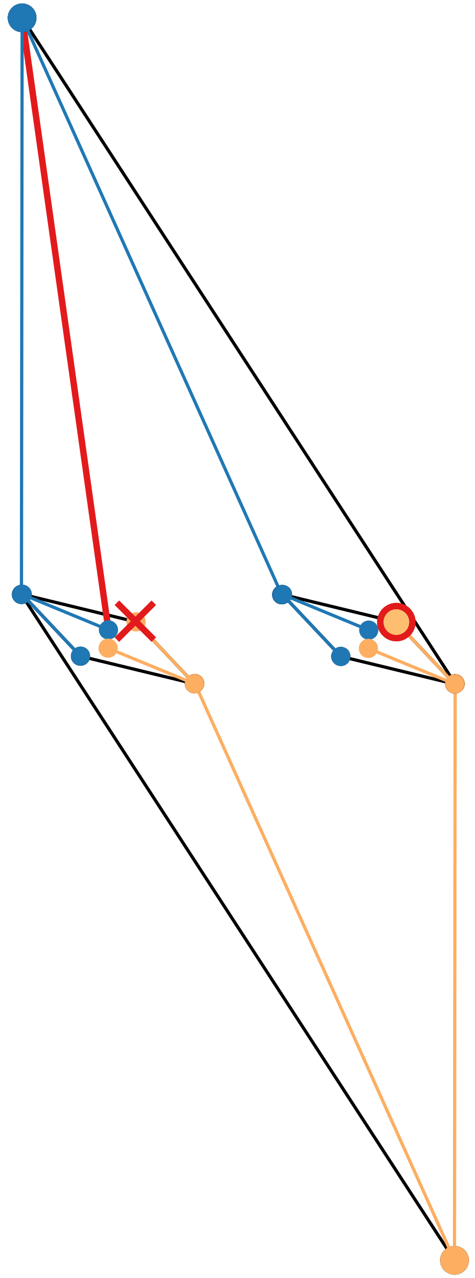

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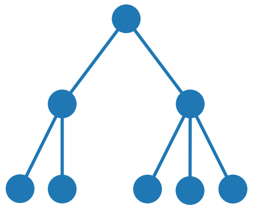

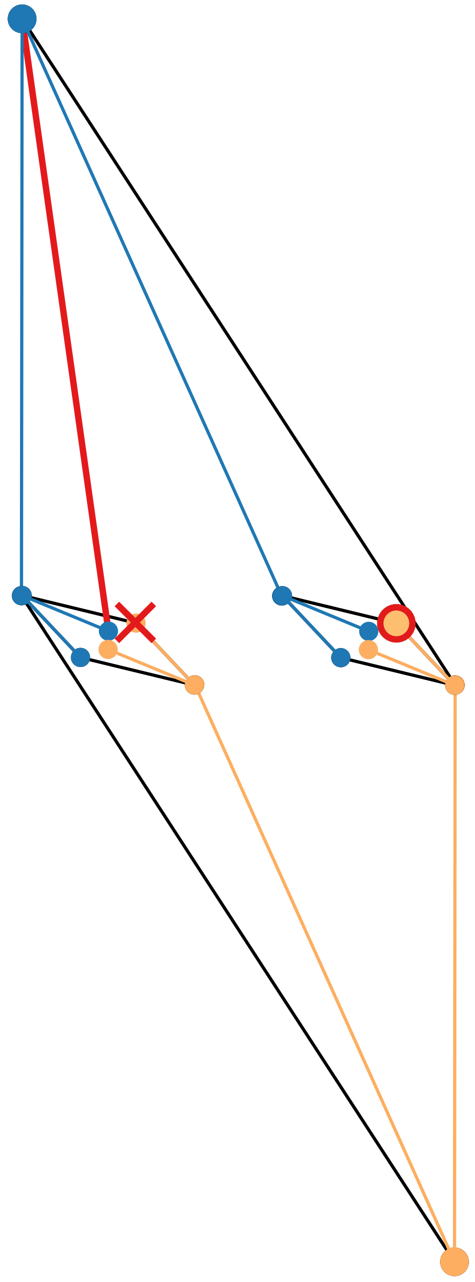

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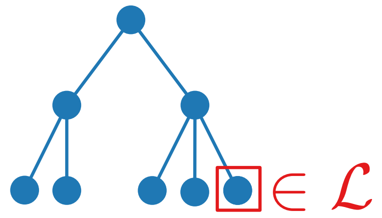

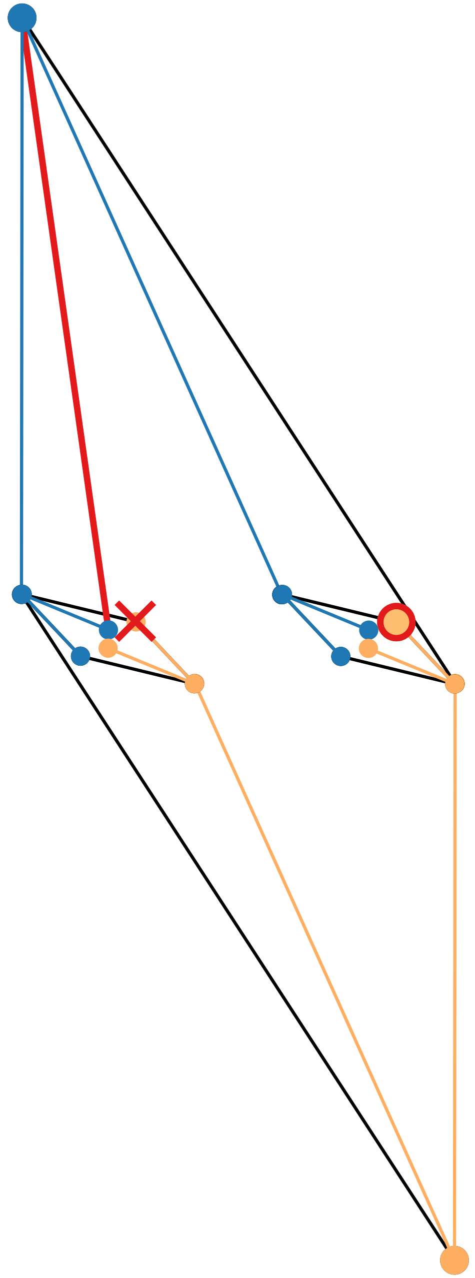

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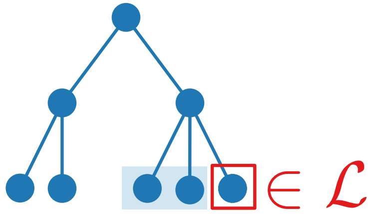

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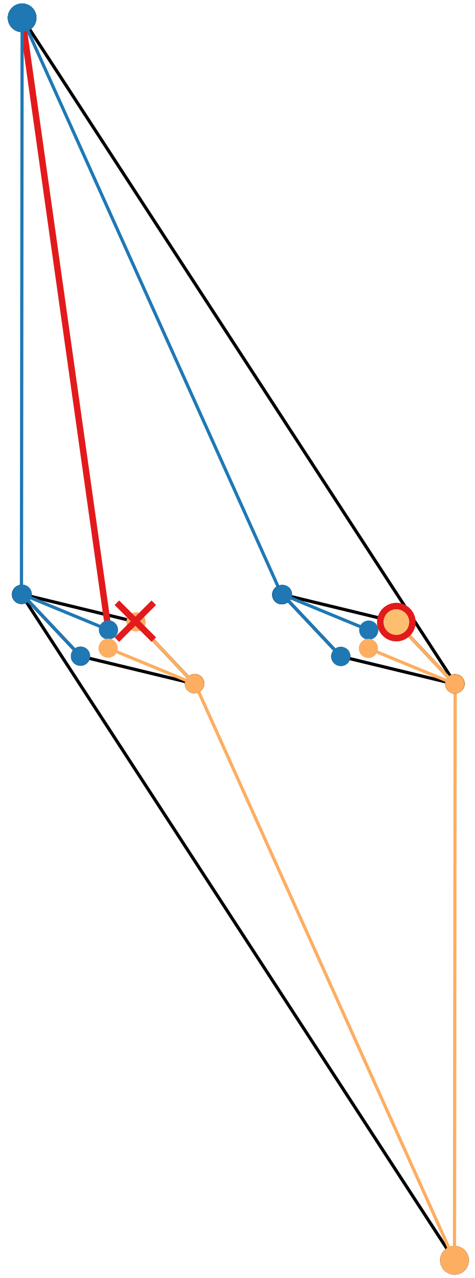

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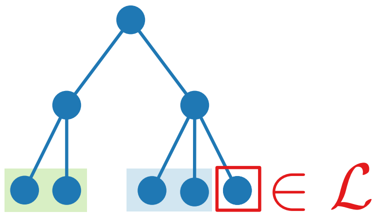

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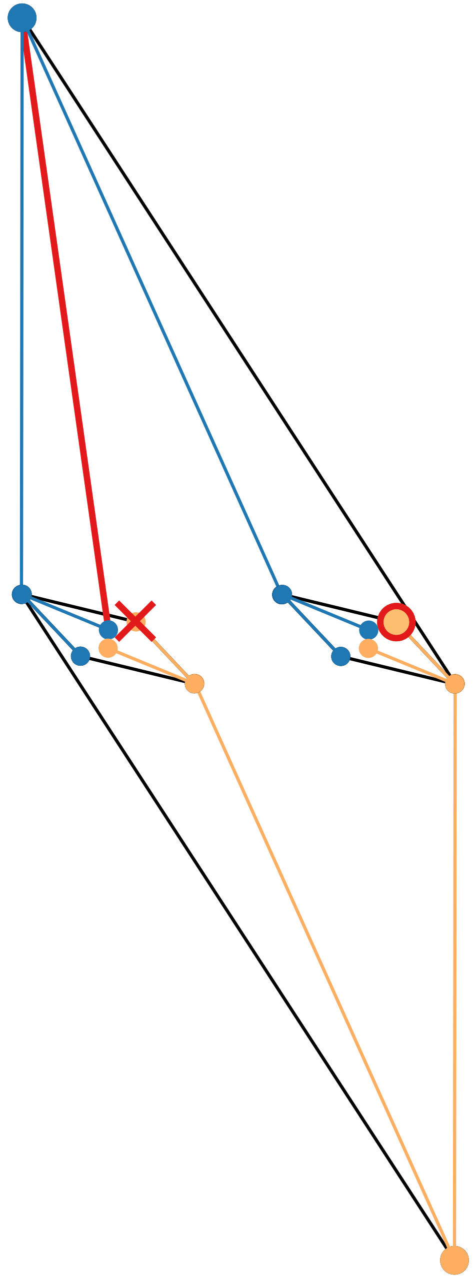

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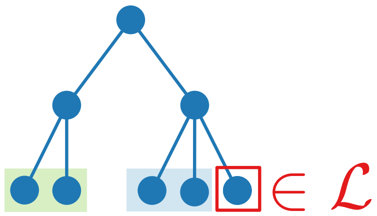

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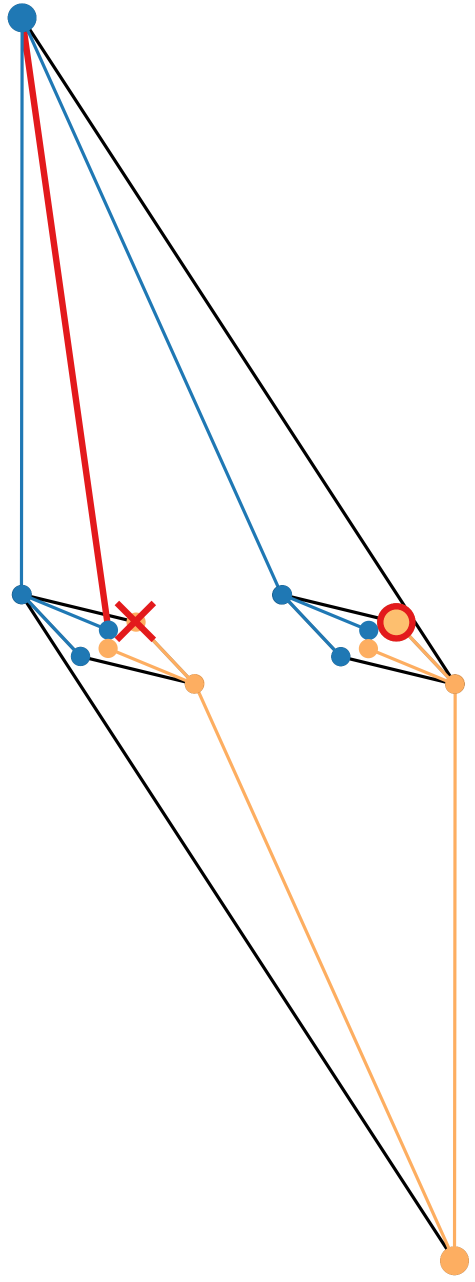
For any $m \geq 1$ and $n = 6m + 1$, there exist tree pairs $\langle T_0, T_1 \rangle$ with $|V(T_1)| \leq 1 + \frac{5}{6}(|V(T_0)| - 1)$ that admit an MW-β drawing for all $\beta \in [1, \infty]$.



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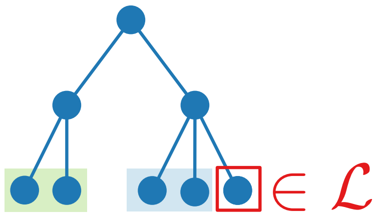

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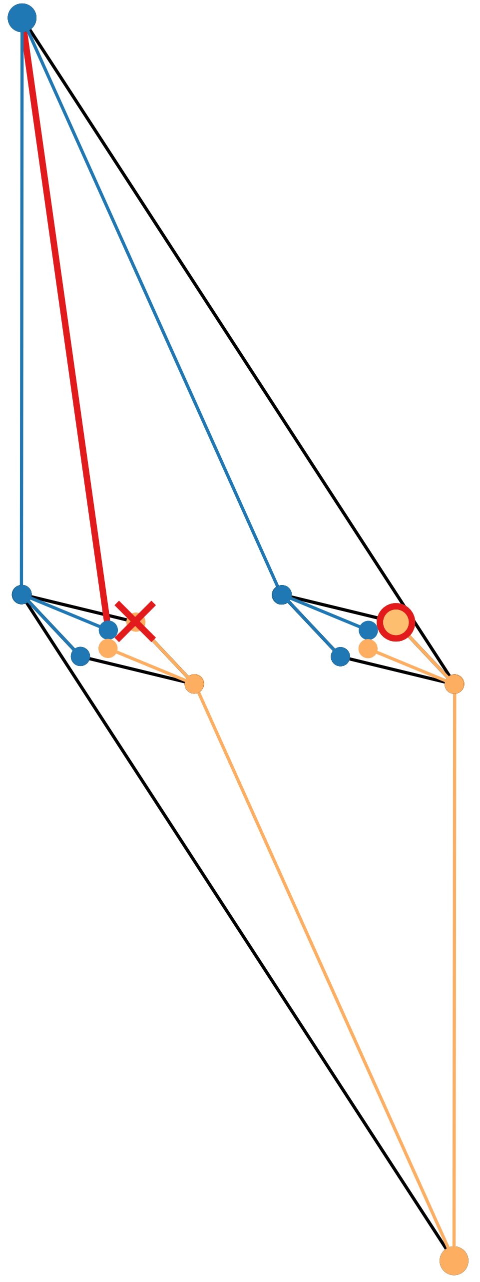

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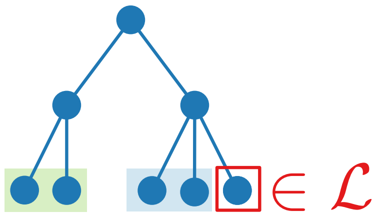

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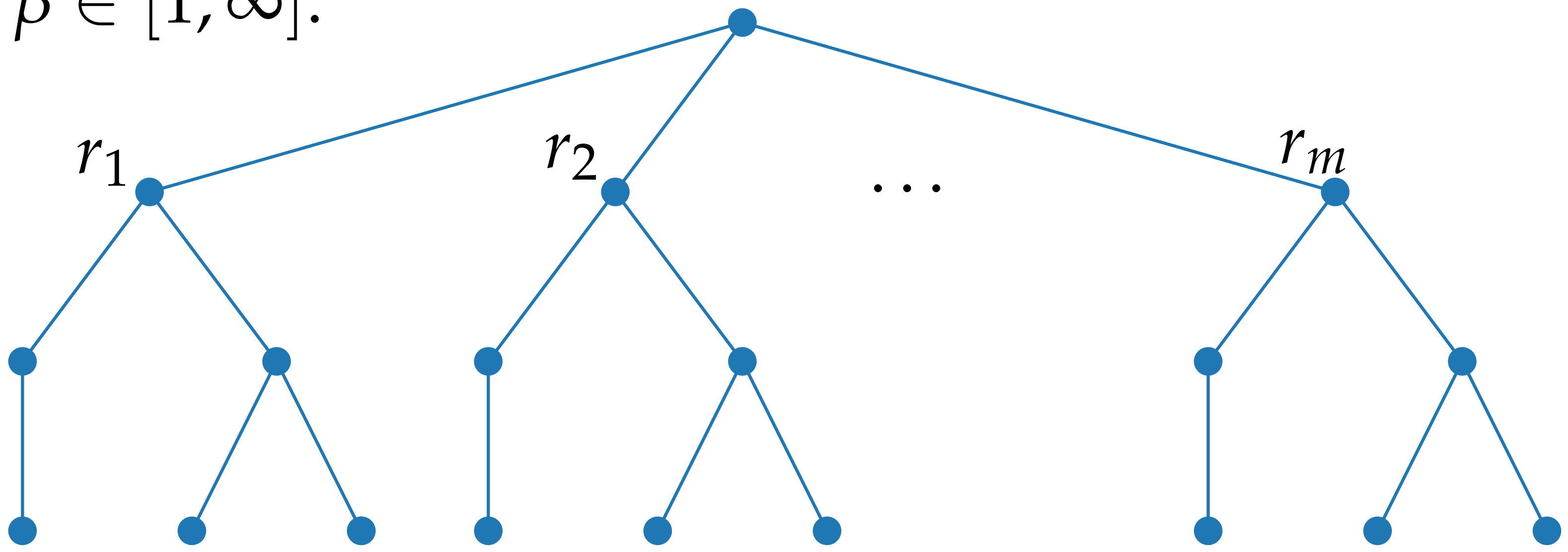

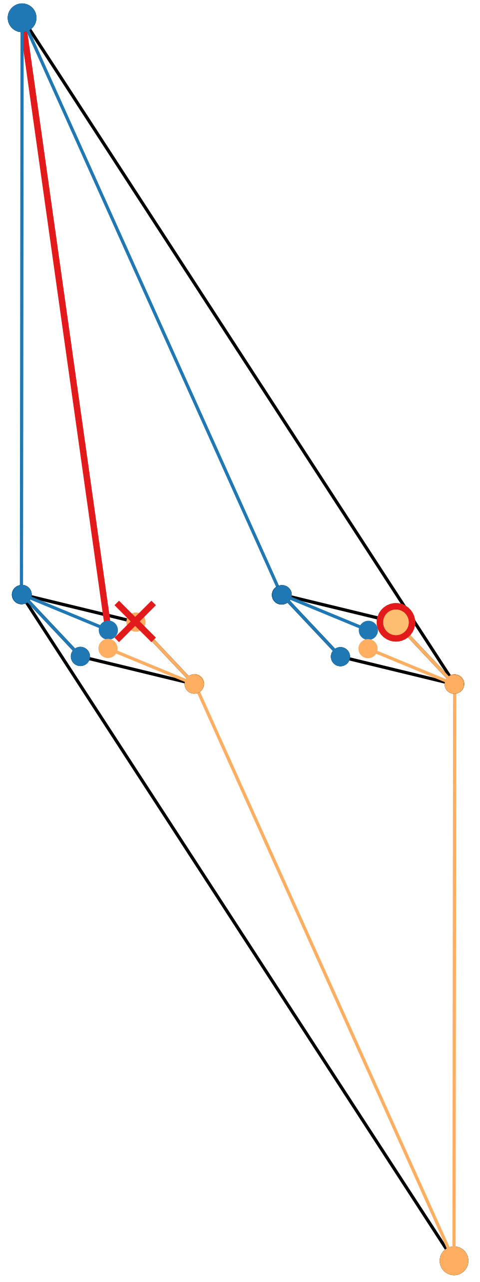

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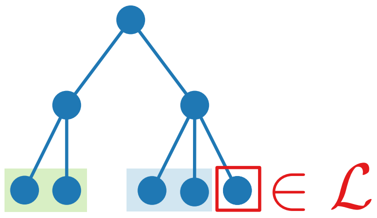

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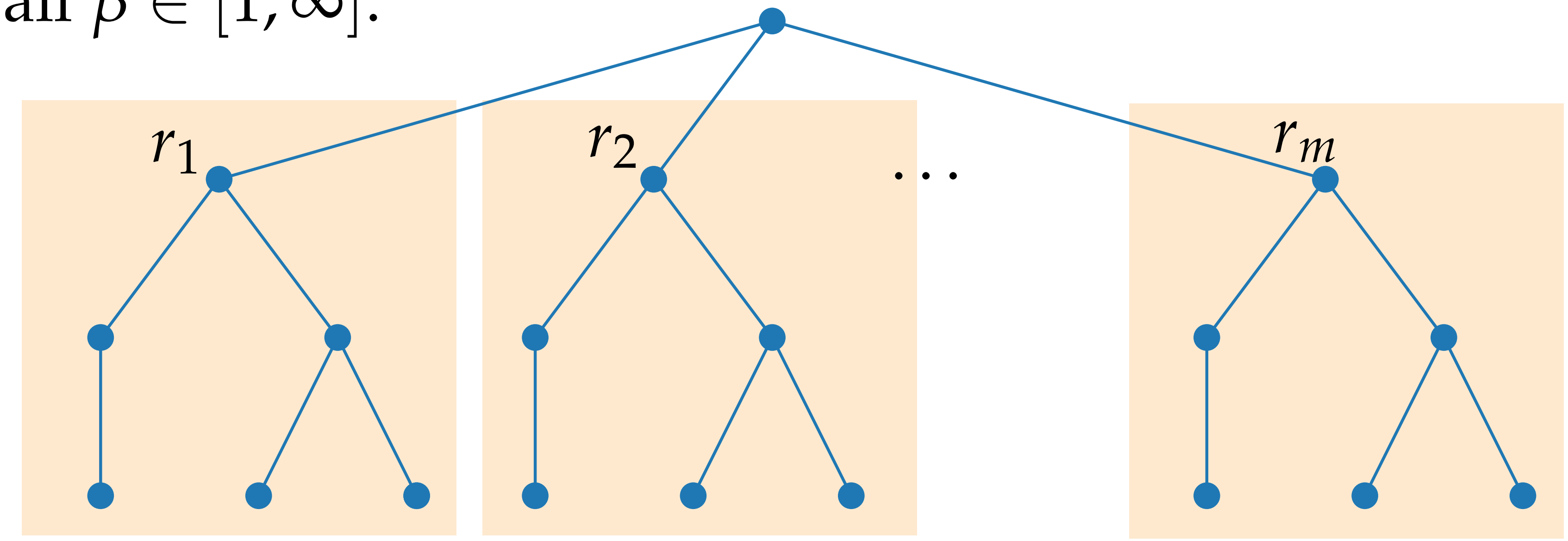

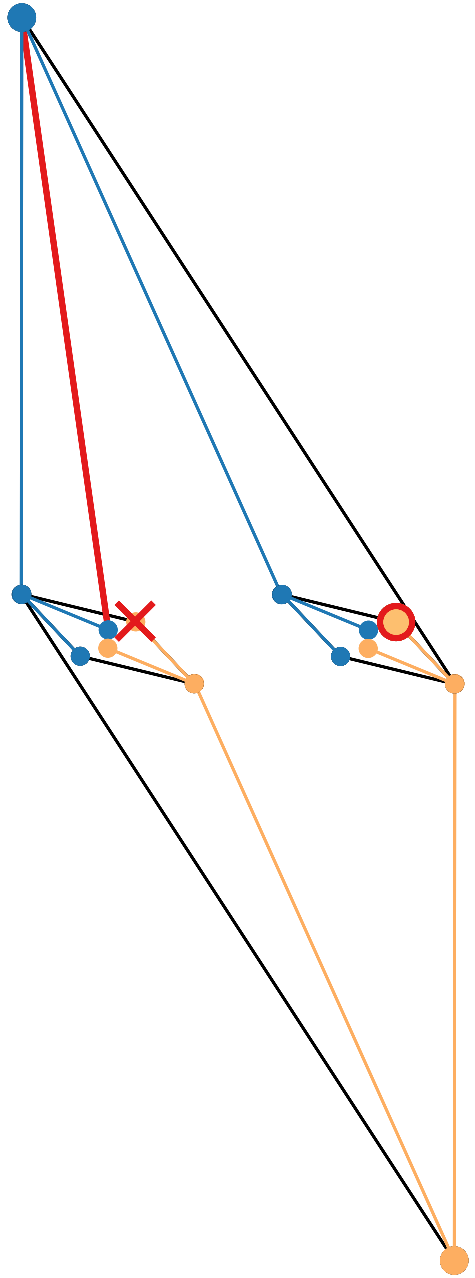

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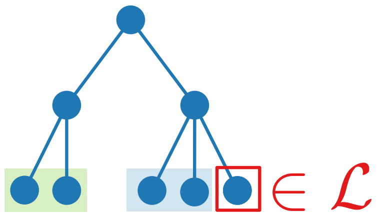

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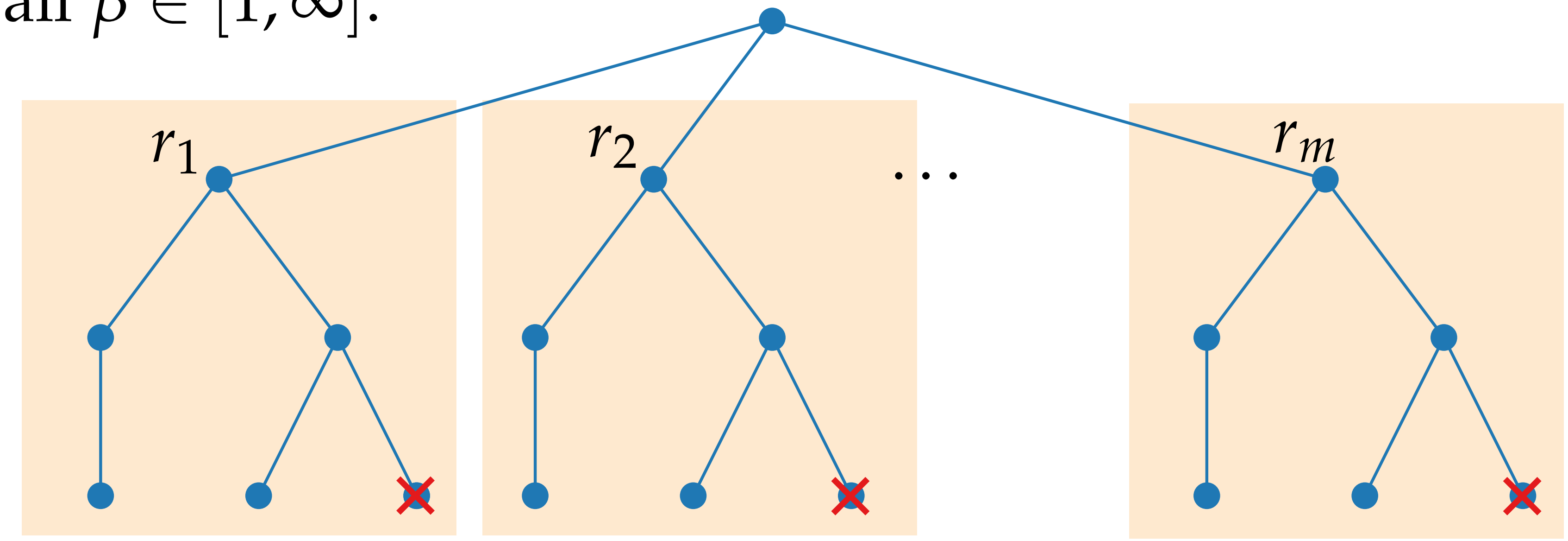

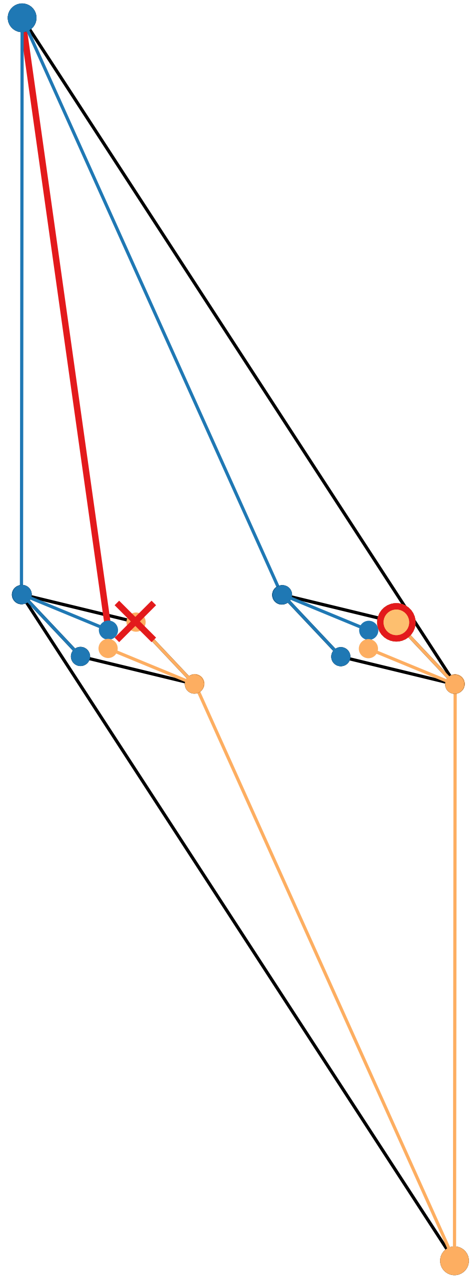

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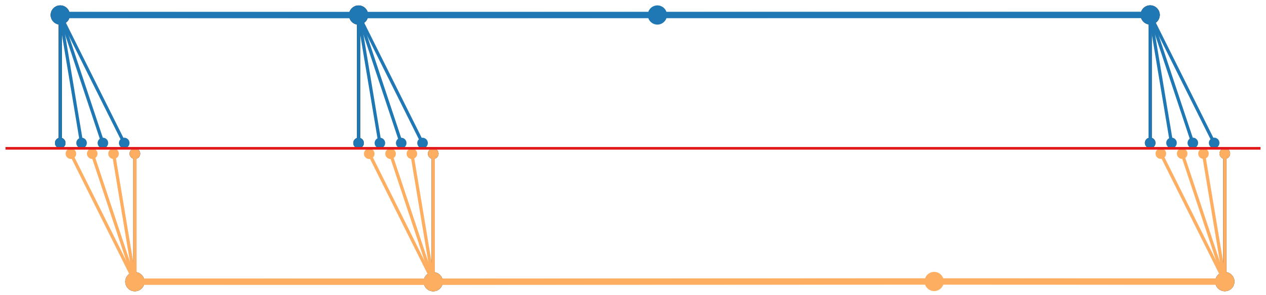

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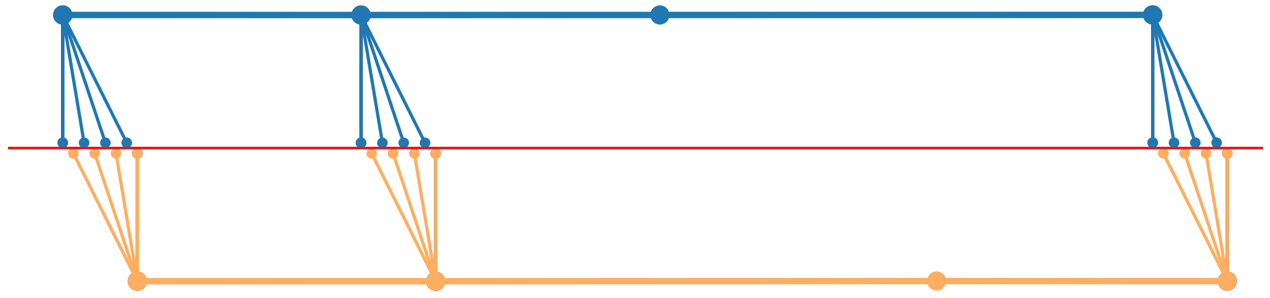

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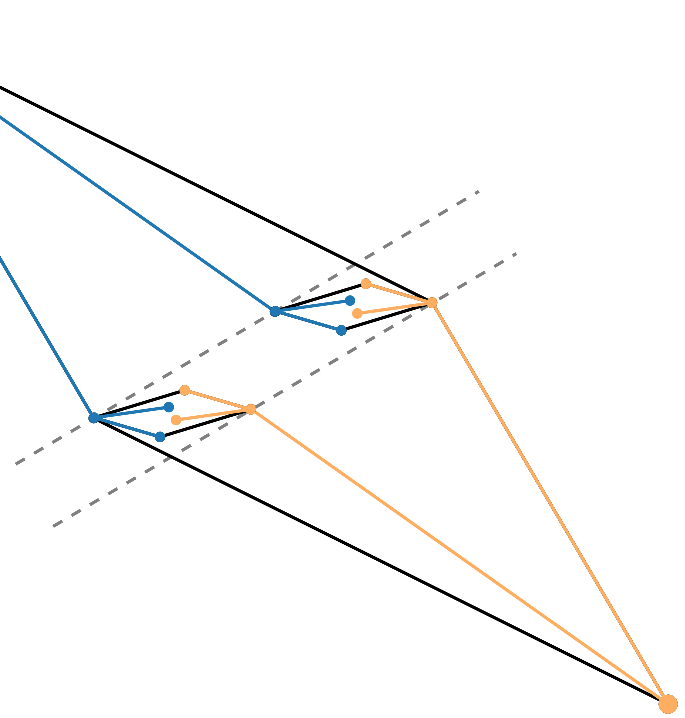

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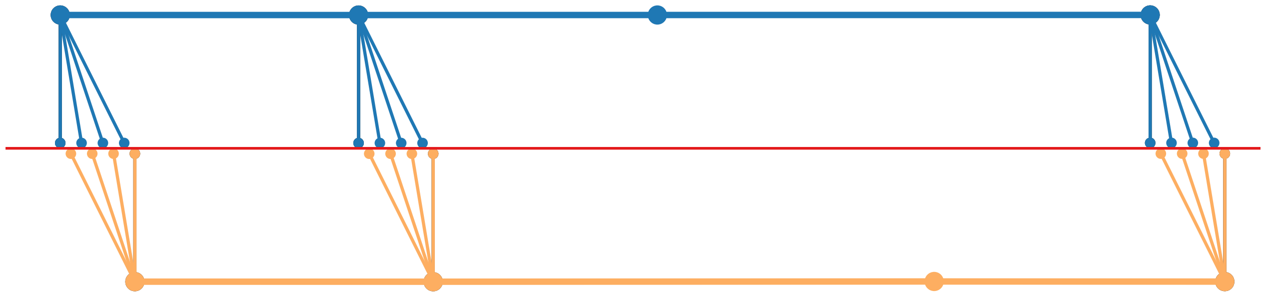

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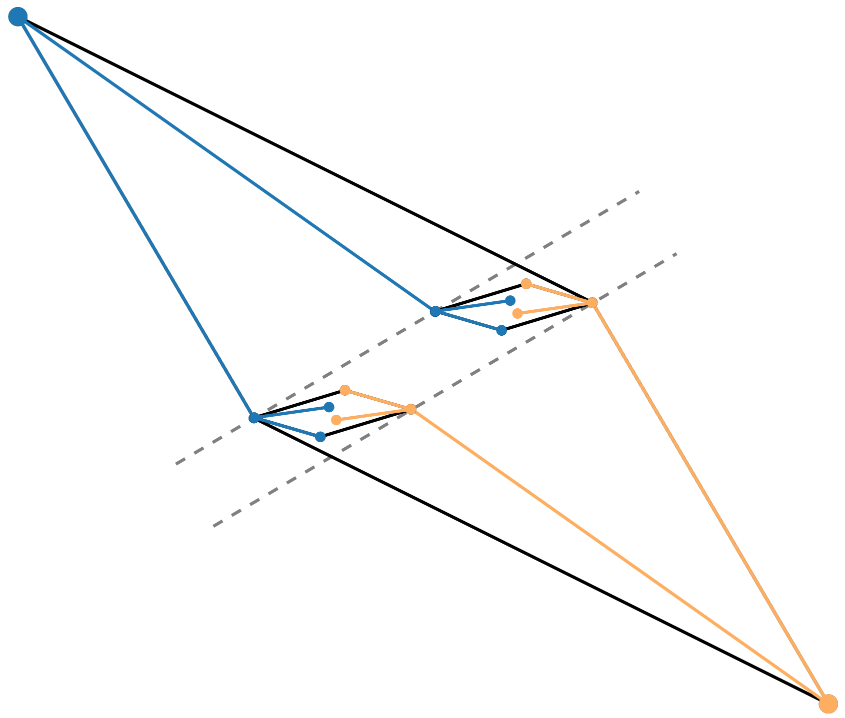

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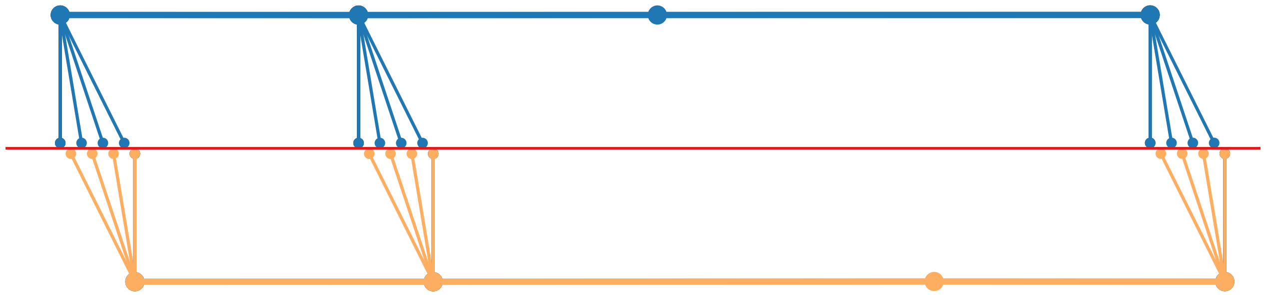

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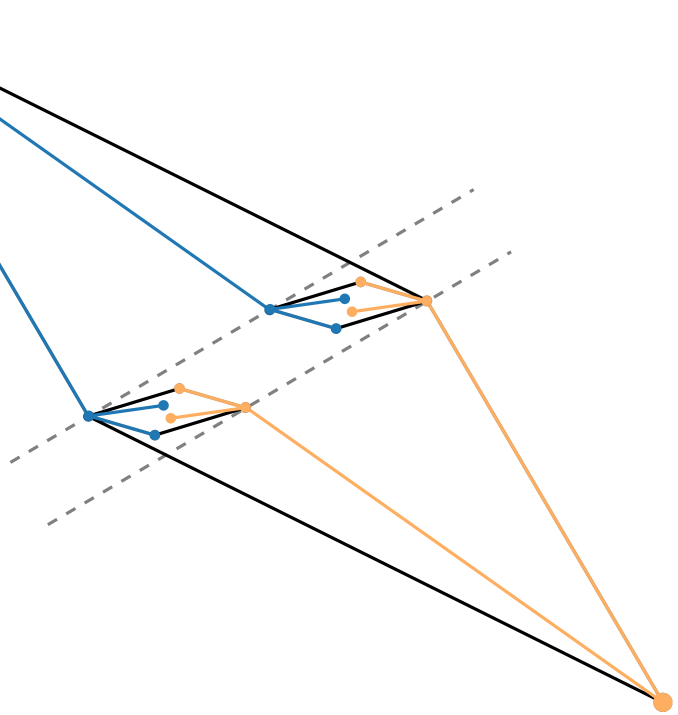
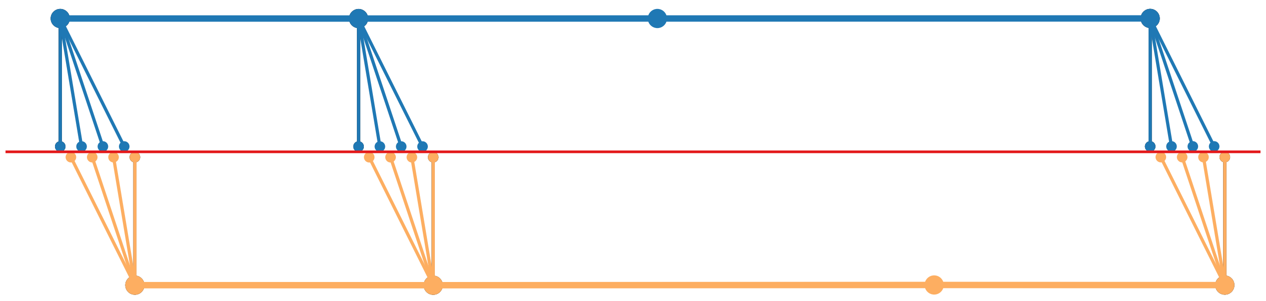
Let (T, r) be a rooted tree and let $\mathcal{L}$ be a sparse set of leaves of T. Then the pair $\langle T, T \setminus \mathcal{L} \rangle$ of trees admits an MW-β drawing for all $\beta \in [1, \infty]$.



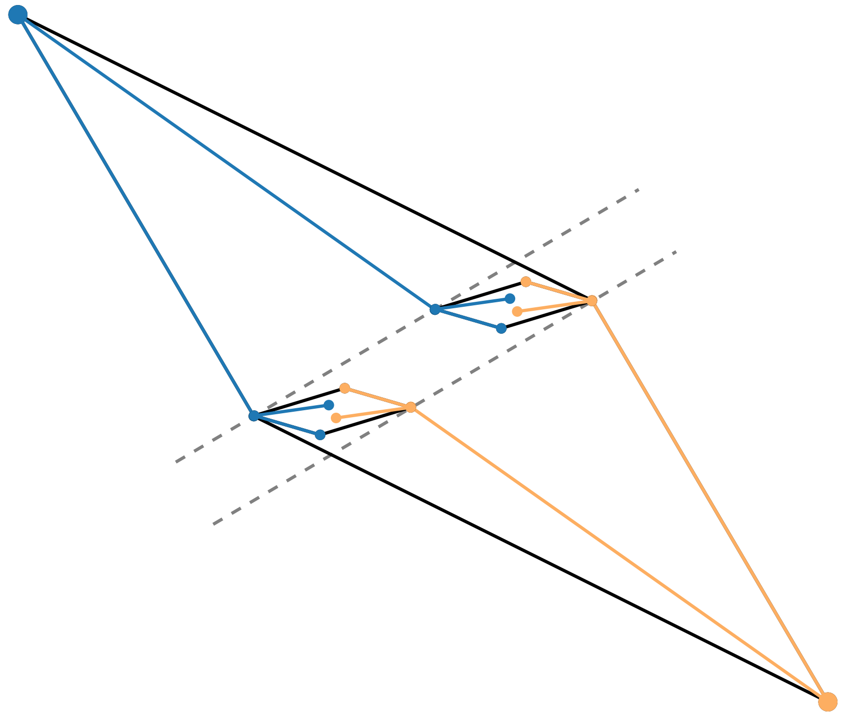
Open Questions

Summary



Theorem 1
Any pair $\langle T_0, T_1\rangle$ of isomorphic caterpillars admits a linearly separable MW-Gabriel drawing.

Theorem 2
Any two isomorphic trees $\langle T_0, T_1\rangle$ admit a MW-β-drawing for all $\beta \in [1, \infty]$.



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Open Questions
- linearly separable drawings for **any** pair of isomorphic trees?

Summary

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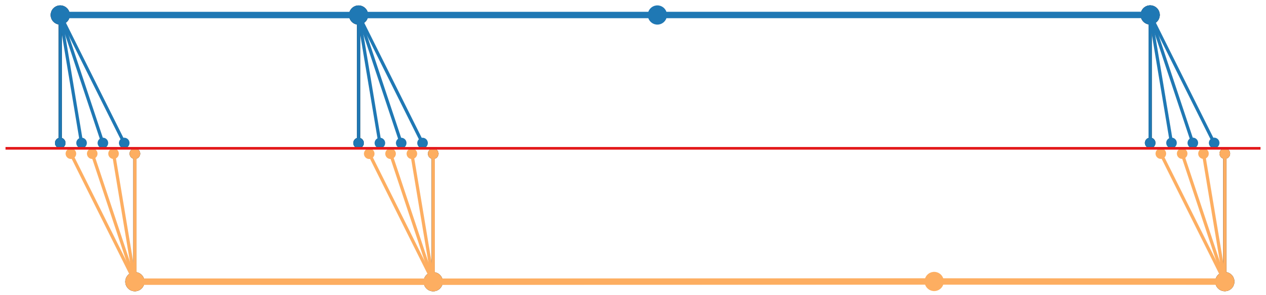
Any pair $\langle T_0, T_1 \rangle$ of isomorphic caterpillars admits a linearly separable MW-Gabriel drawing.



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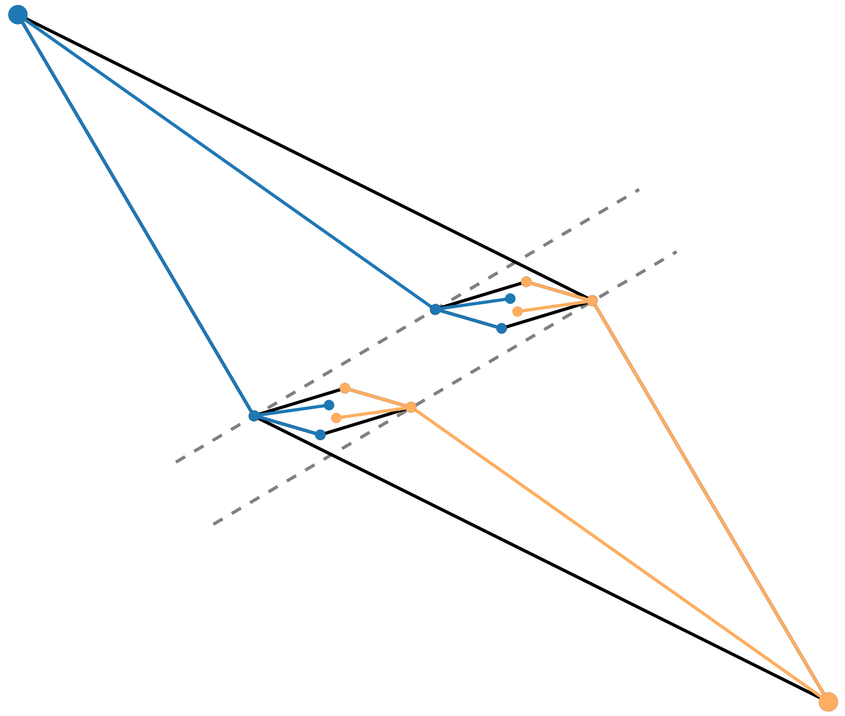

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Let (T, r) be a rooted tree and let $\mathcal{L}$ be a sparse set of leaves of T. Then the pair $\langle T, T \setminus \mathcal{L} \rangle$ of trees admits an MW-β drawing for all $\beta \in [1, \infty]$.

Open Questions

- linearly separable drawings for **any** pair of isomorphic trees?
- characterization of pairs of non-isomorphic trees that are drawable?