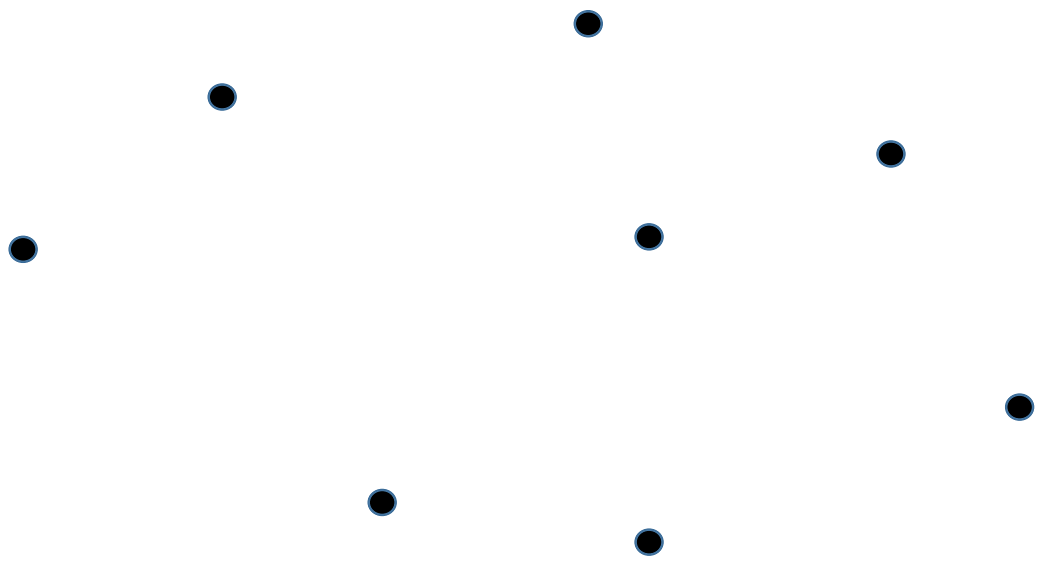
Three Edge-disjoint Plane Spanning Paths in a Point Set

Philipp Kindermann, Jan Kratochvil, Beppe Liotta, Pavel Valtr

Graph Drawing 2023

Isola delle Femmine, September 21, 2023

S is a set of *n* points in the plane, no three in a line (general position)



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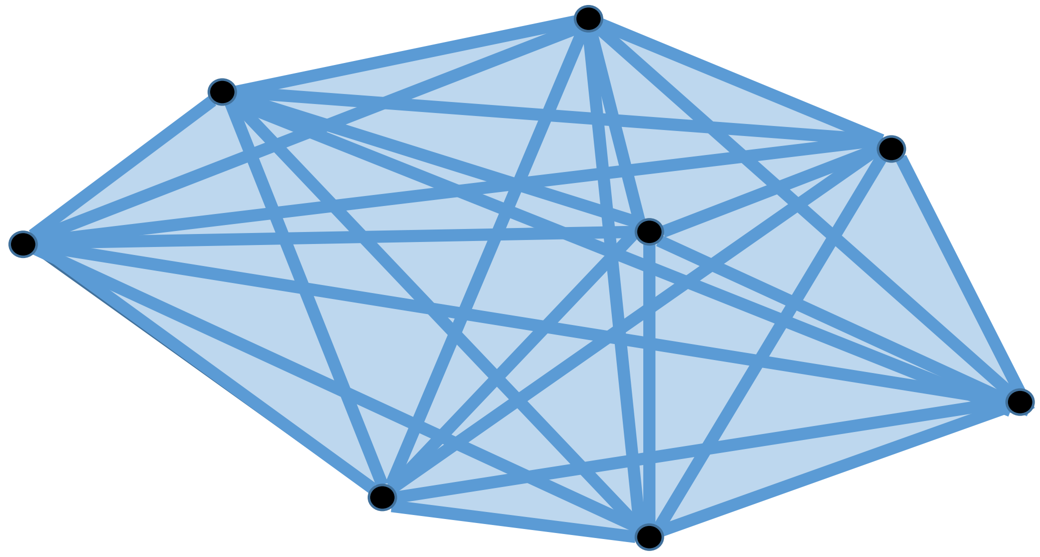
defines a complete geometric graph (edges are straight-line segments)



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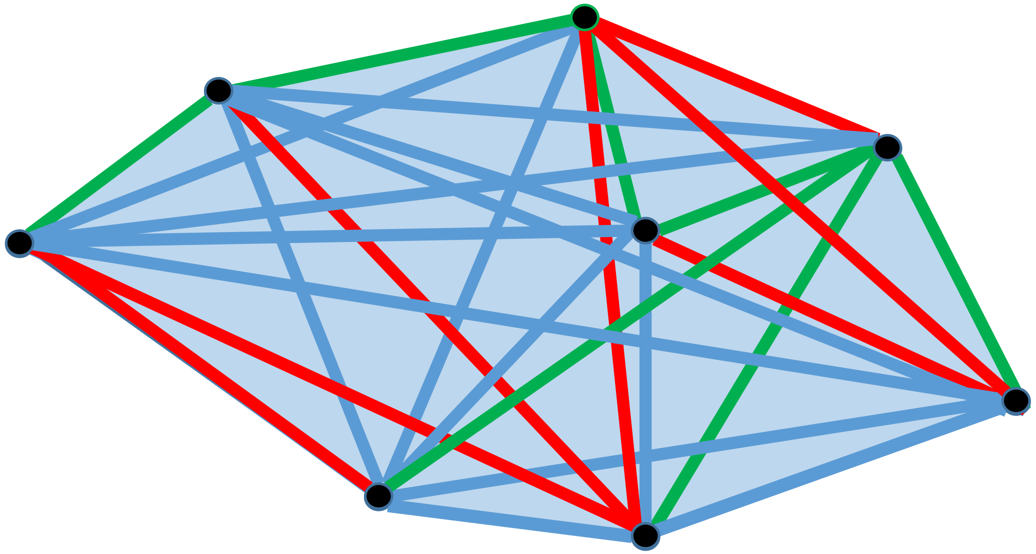
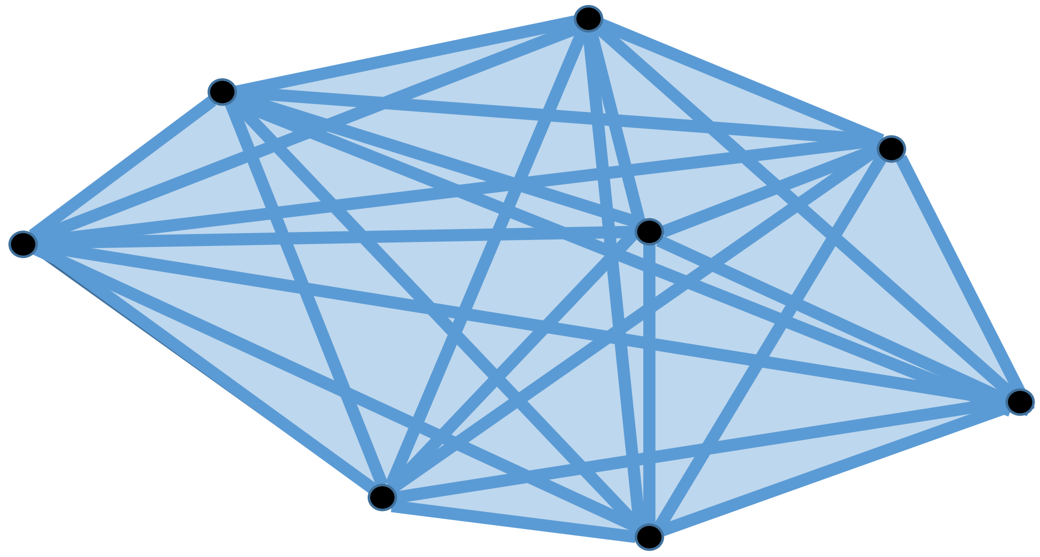


Goal: Packing edge-disjoint plane spanning subgraphs

Question: How many edge-disjoint plane spanning paths can we pack in a given complete geometric graph?



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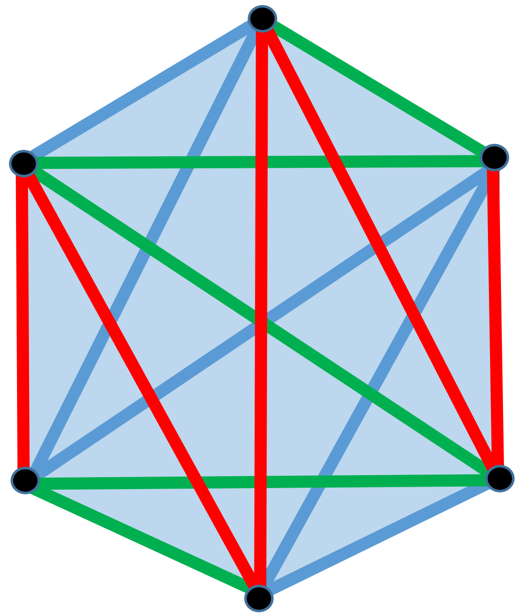

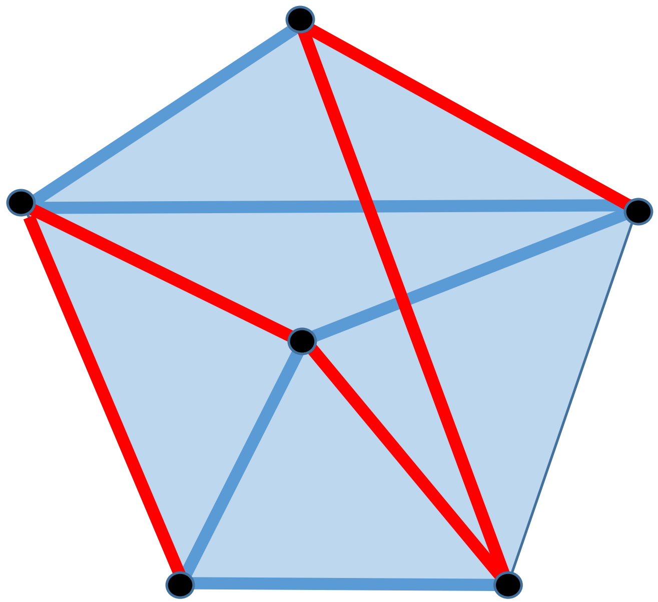

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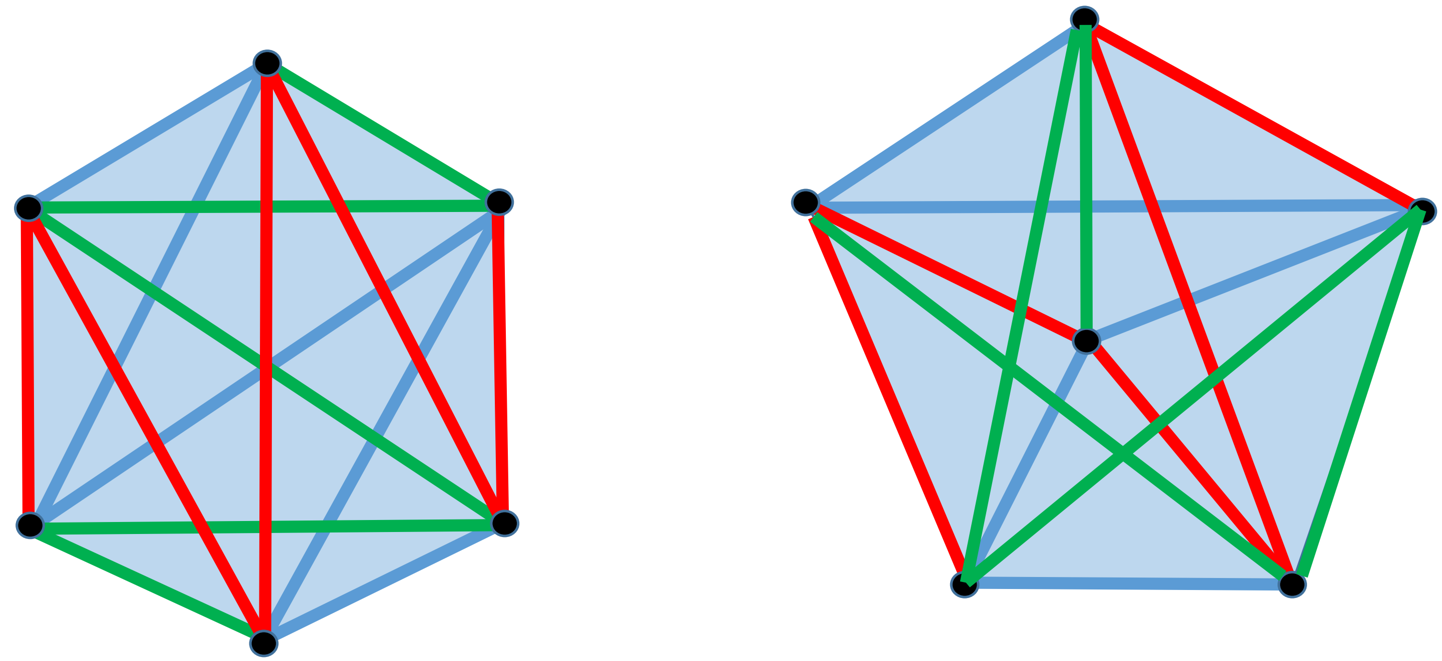


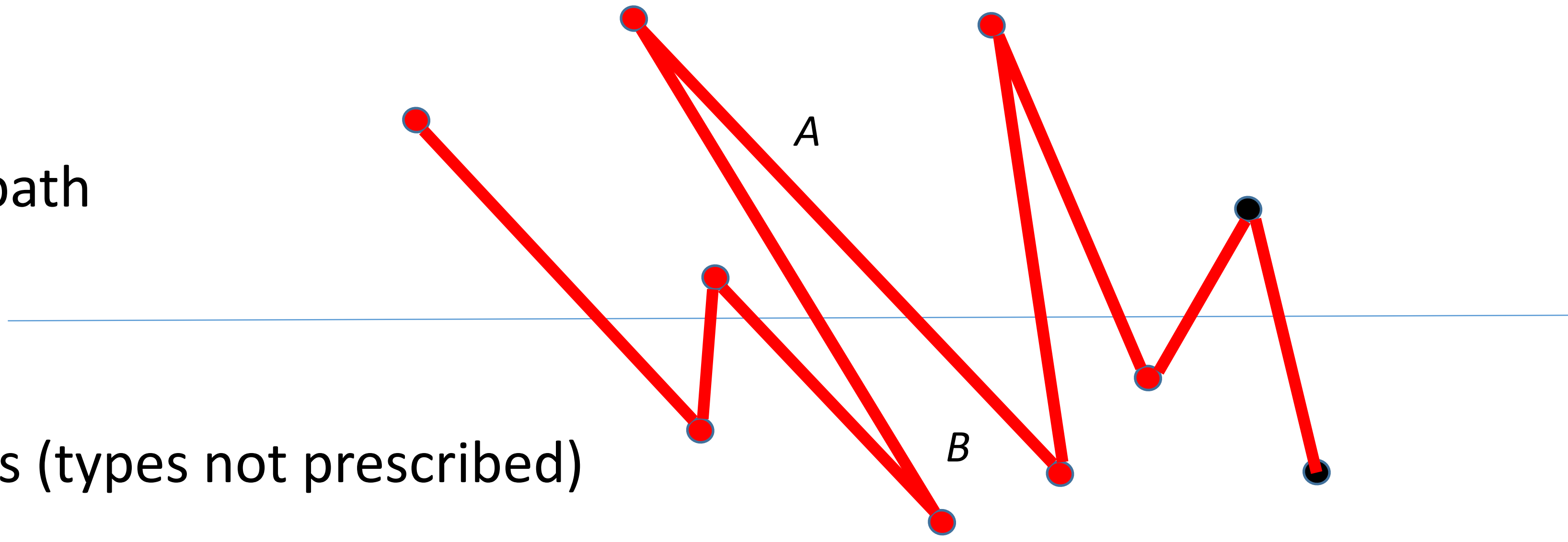
Our question: How many edge-disjoint plane spanning paths can we ALWAYS find in ANY given complete geometric graph with n points?

Packing edge-disjoint plane spanning subgraphs in complete geometric graphs

Known:

Folklore – 1 path

Abellanas et al. [1999] – zig-zag path



Aichholzer et al. [2017] – $\sqrt{n}$ trees (types not prescribed)

Aichholzer et al. [2017] – 2 paths

Packing edge-disjoint plane spanning subgraphs in complete geometric graphs

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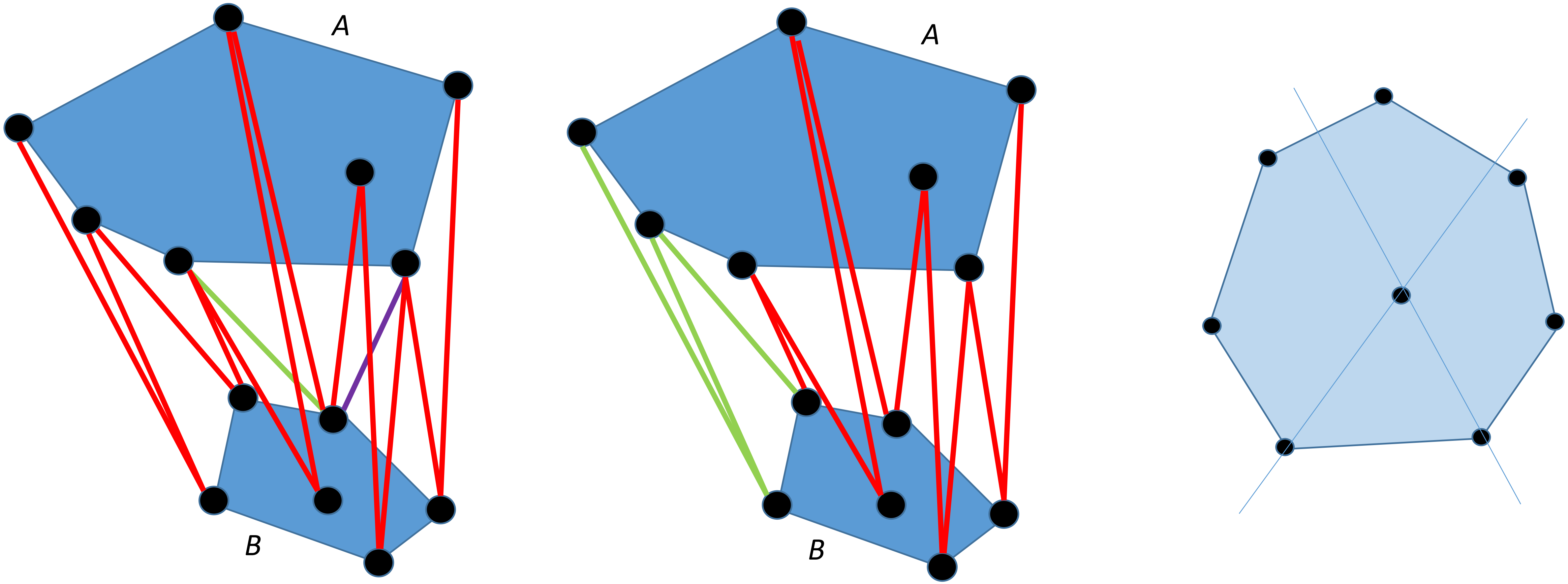
Our results:

- 2 paths with prescribed starting vertices (on the boundary of conv(S))
- 3 paths Theorem: Every set of $|S| \geq 10$ points admits 3 edge-disjoint plane spanning paths.

Theorem: Every set *S* of $\geq$ 10 points admits 3 edge-disjoint plane spanning paths.

Idea of the proof: Prove that every S is of at least one of the following three types



Theorem: Every set *S* of ≥ 10 points admits 3 edge-disjoint plane spanning paths.

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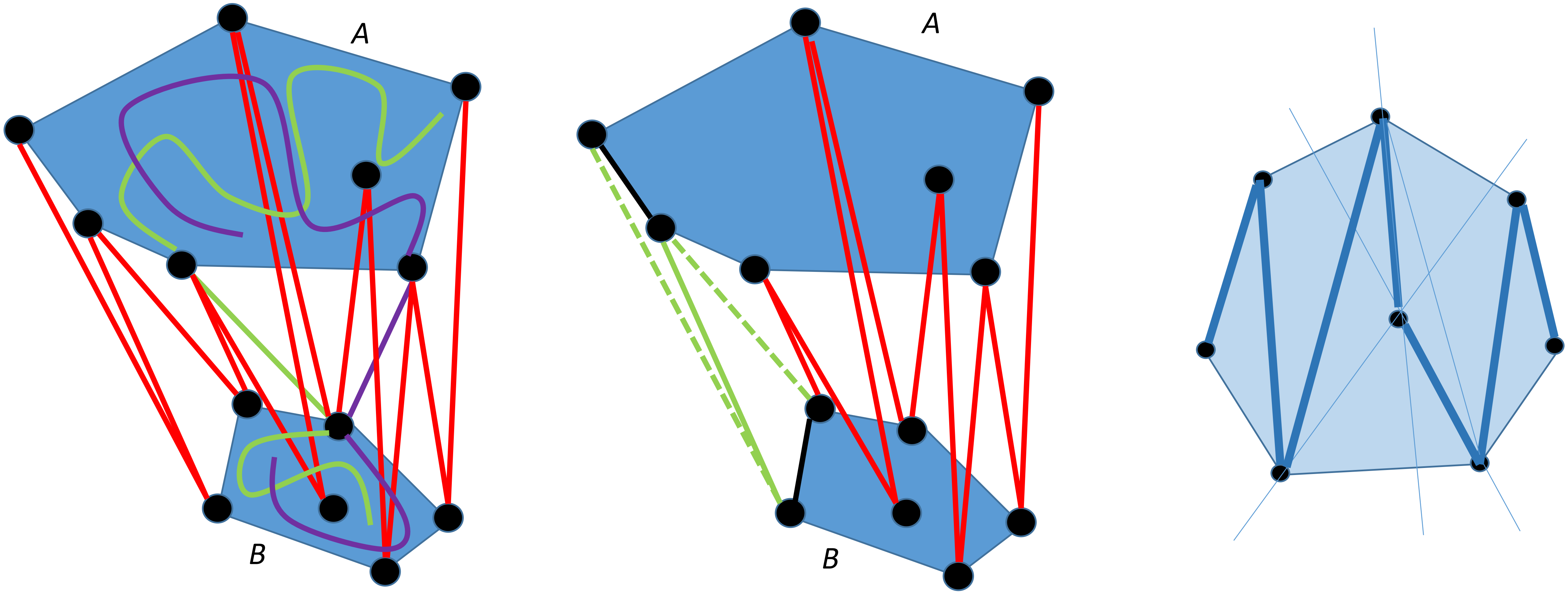

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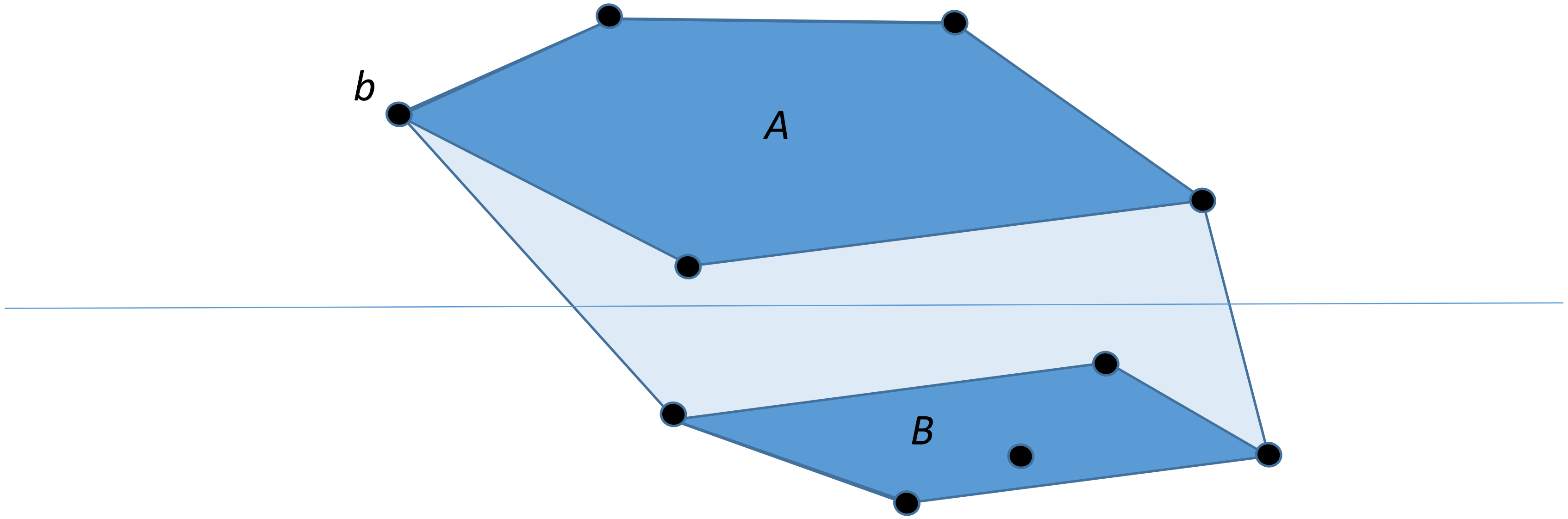
Abellanas [1999]: For every balanced separation (*A*,*B*) of *S*, there exists a zig-zag path starting in a bridged vertex of the larger part.



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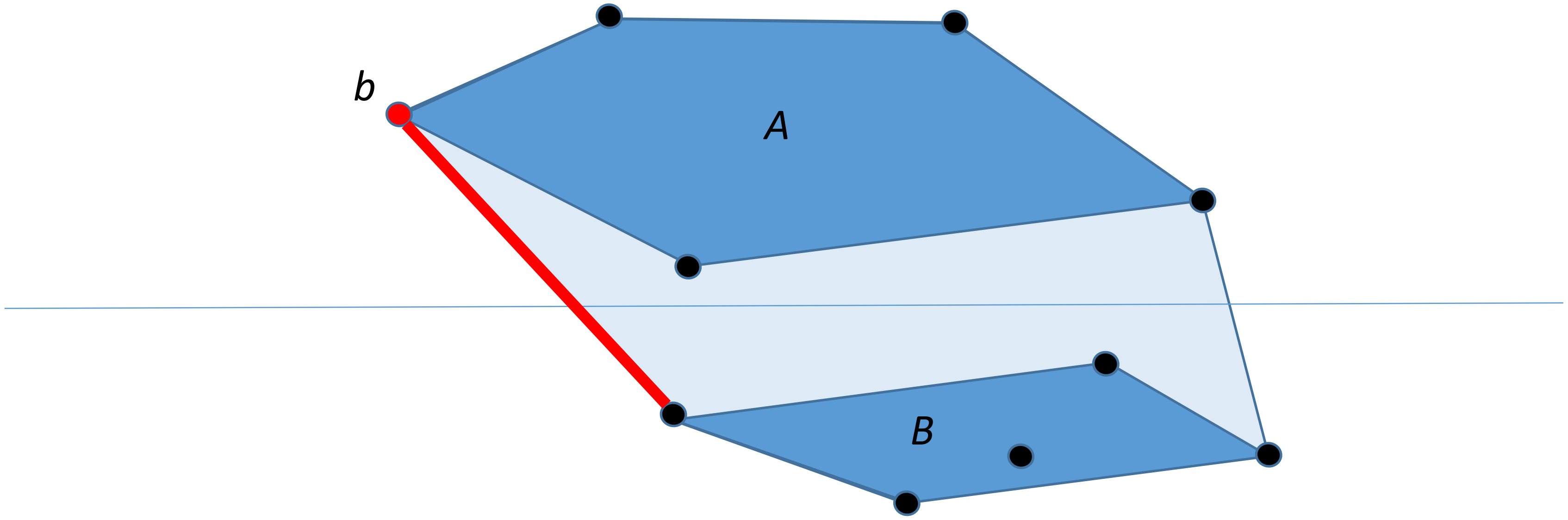

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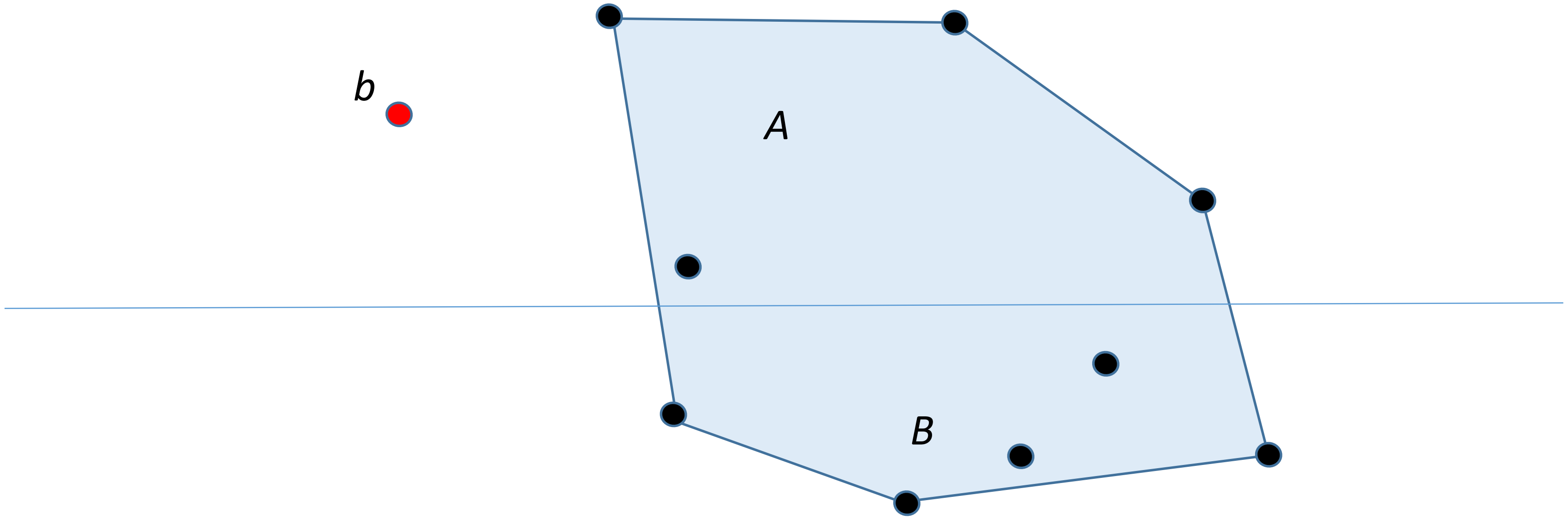

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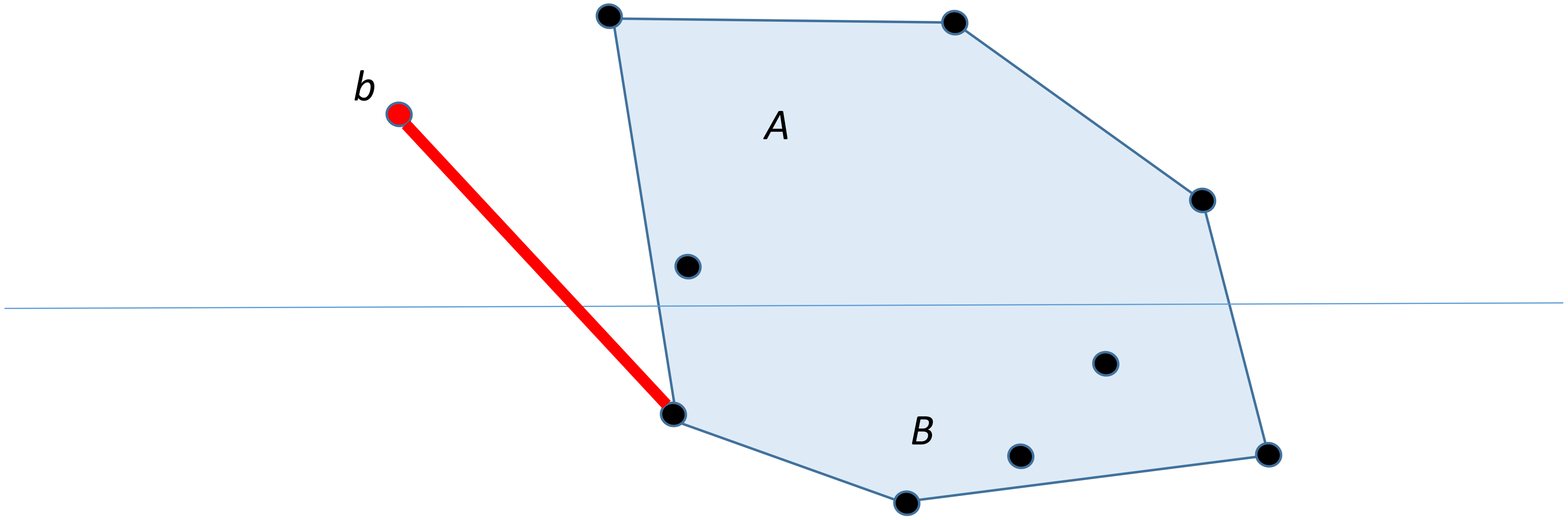

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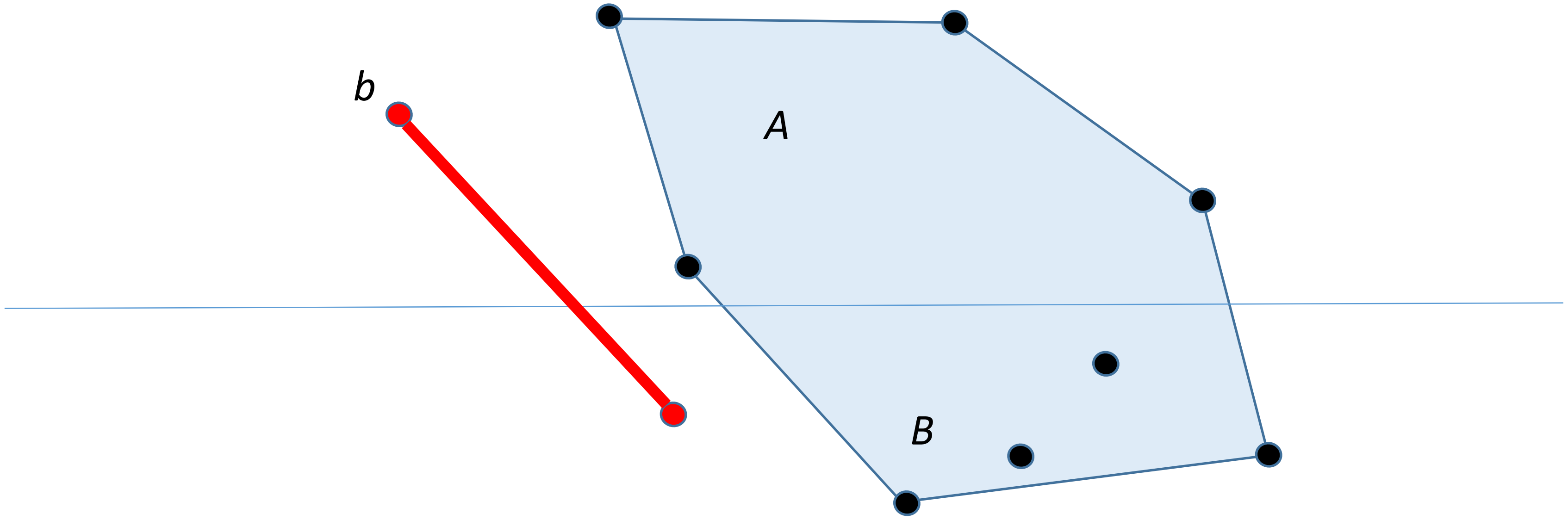

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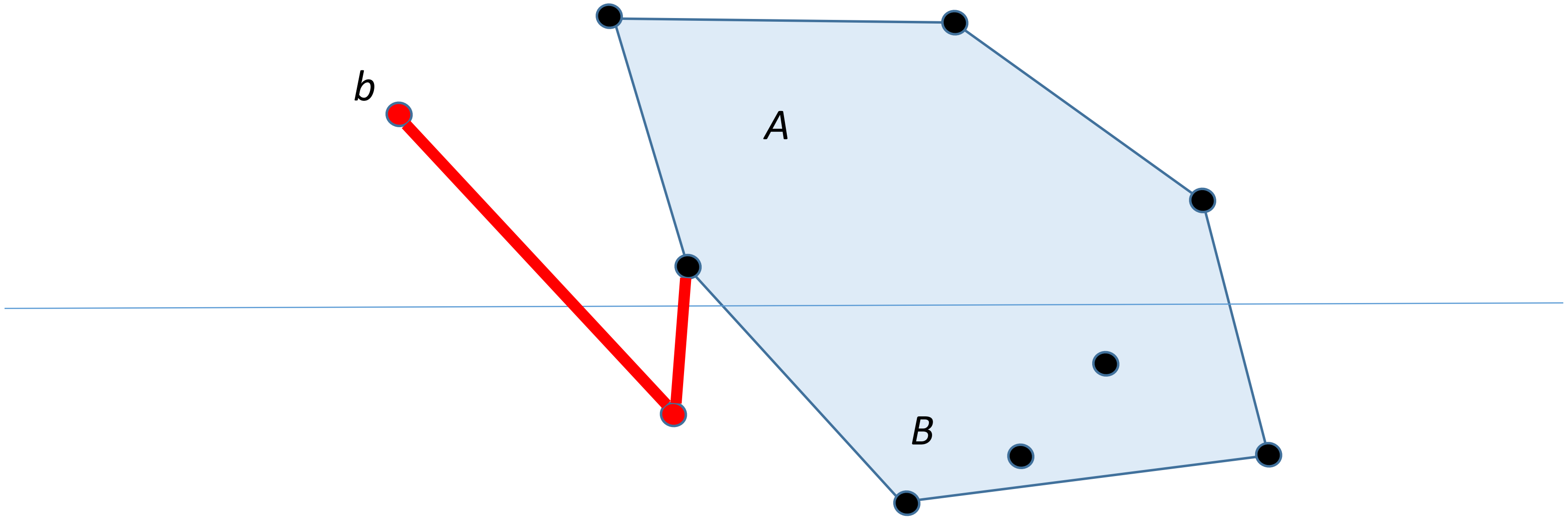

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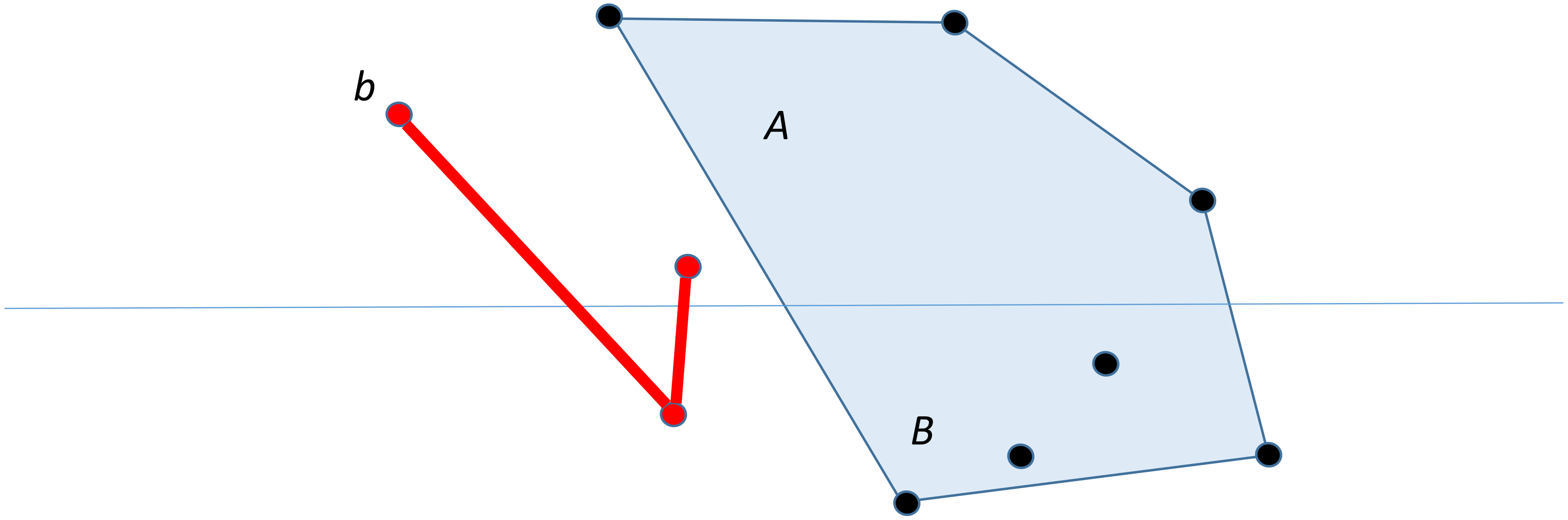

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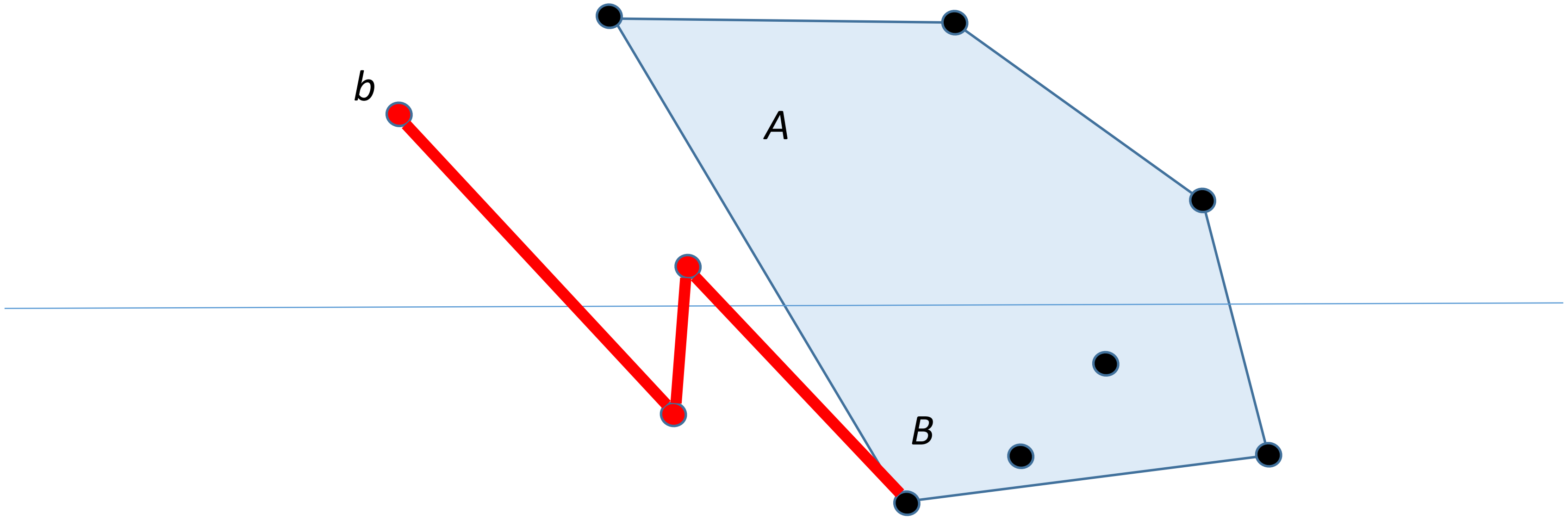

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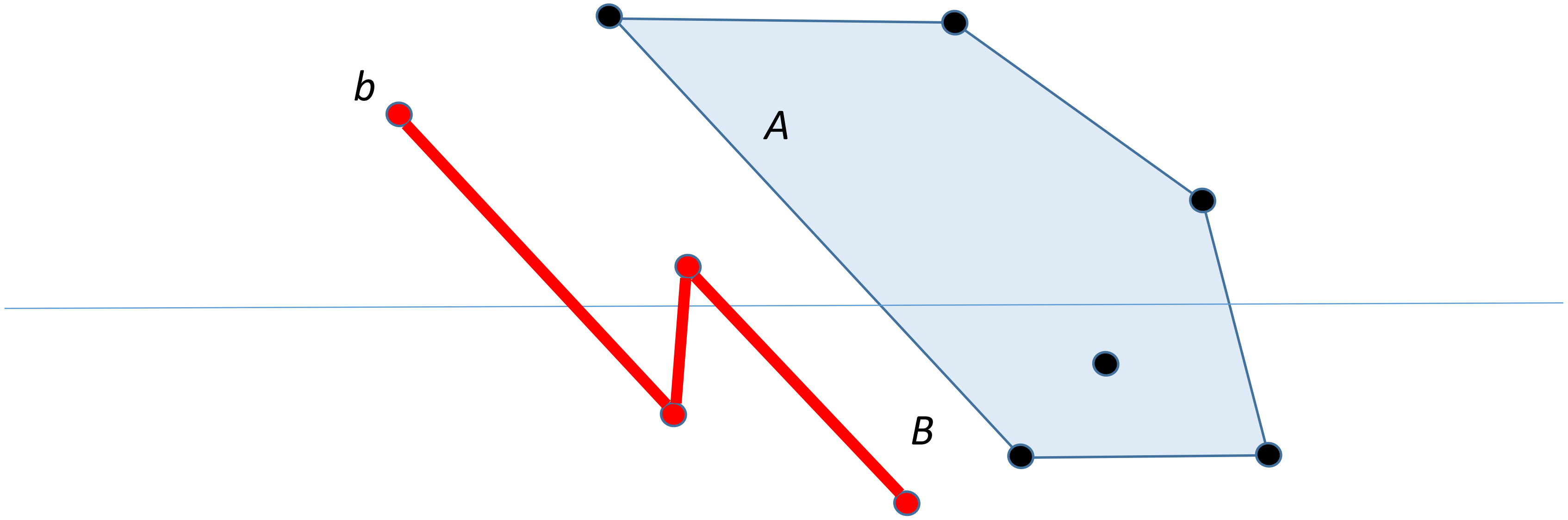

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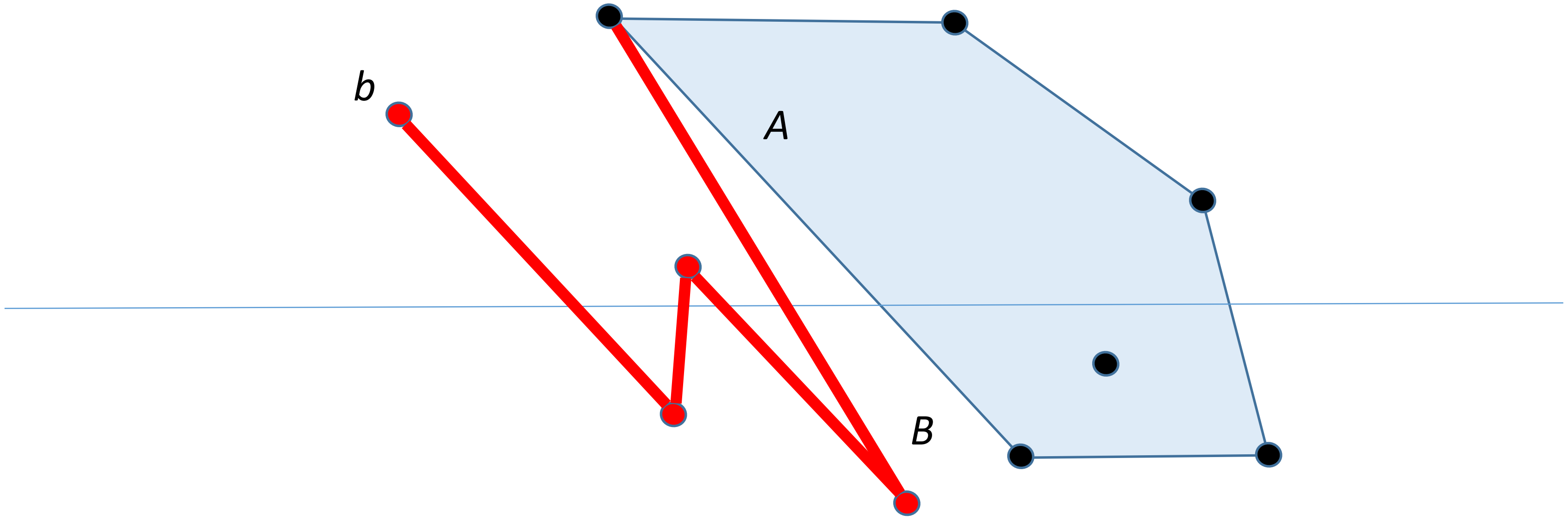

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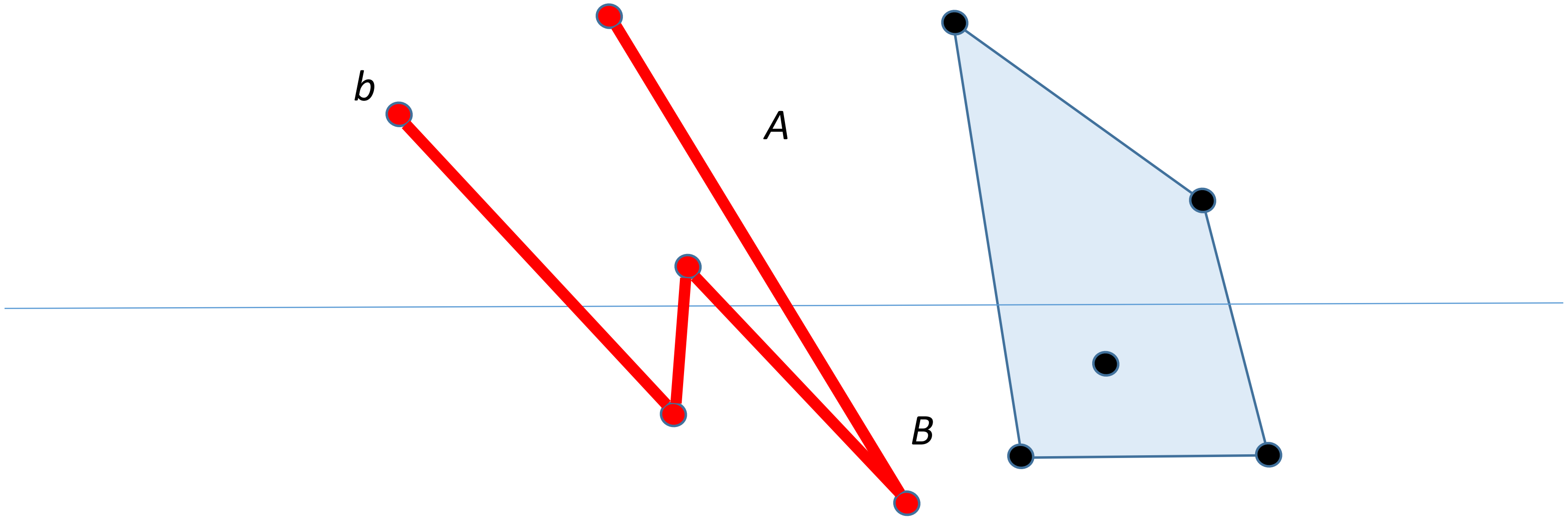

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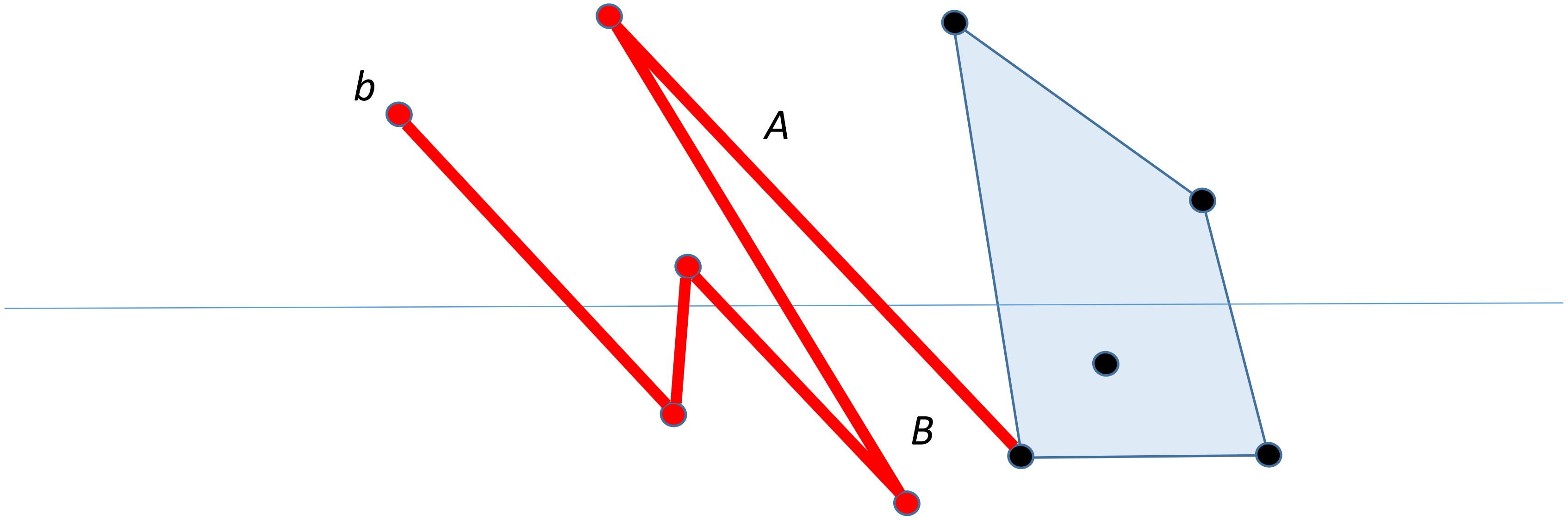

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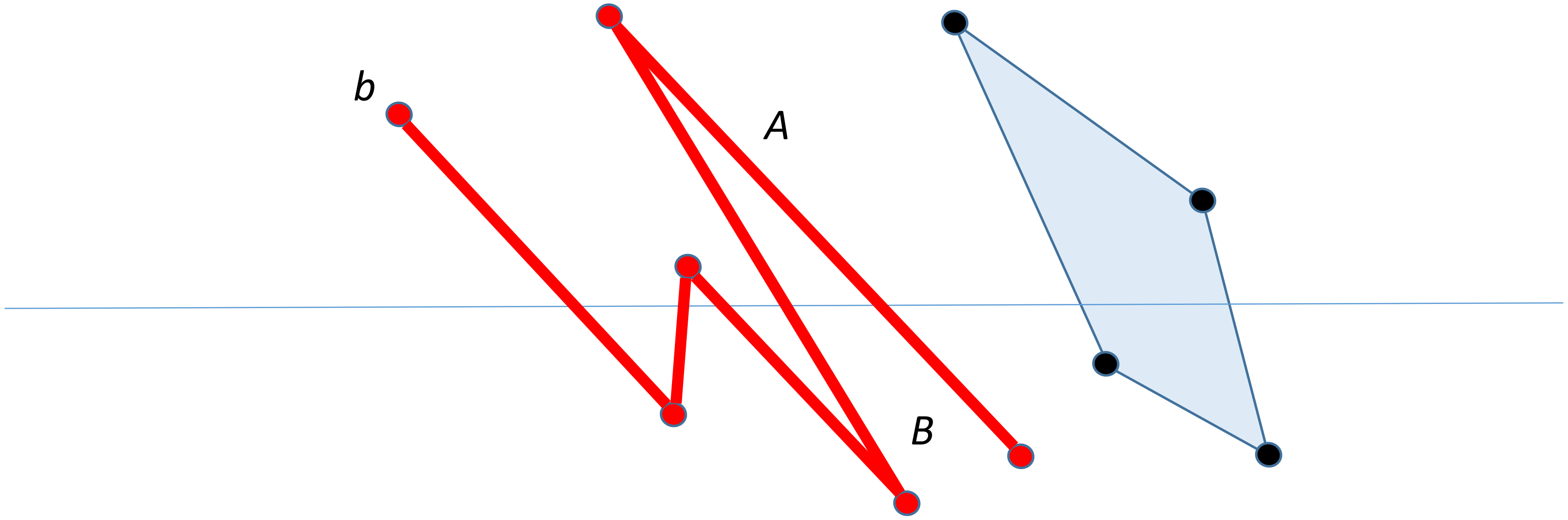

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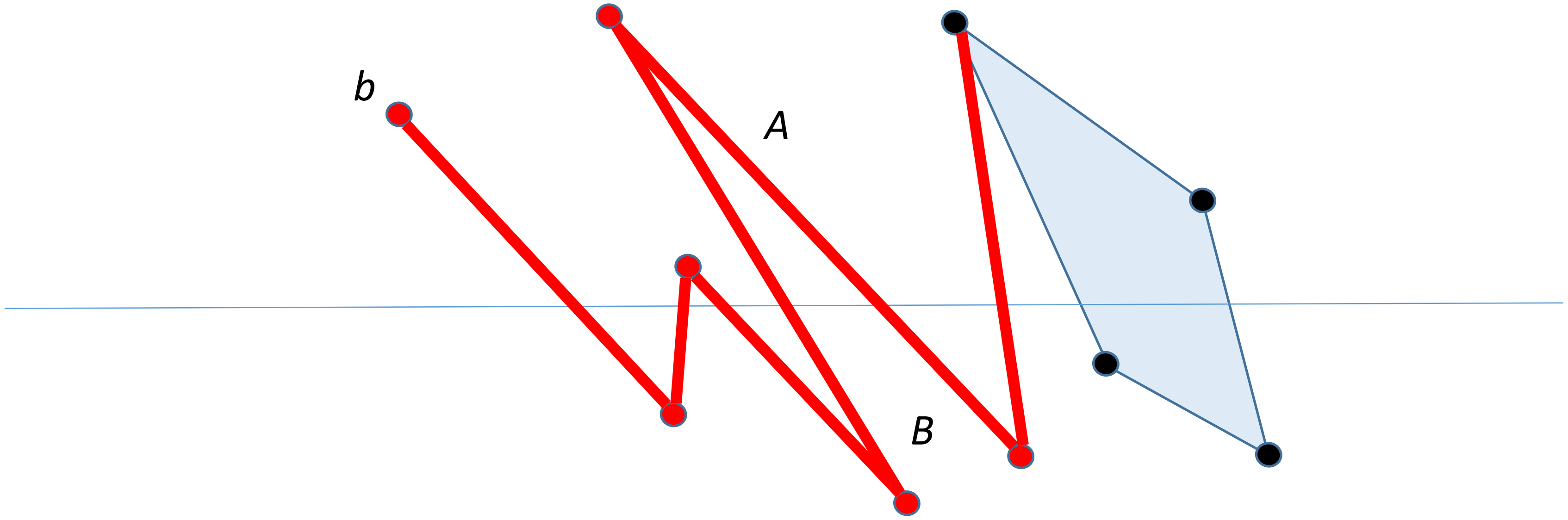

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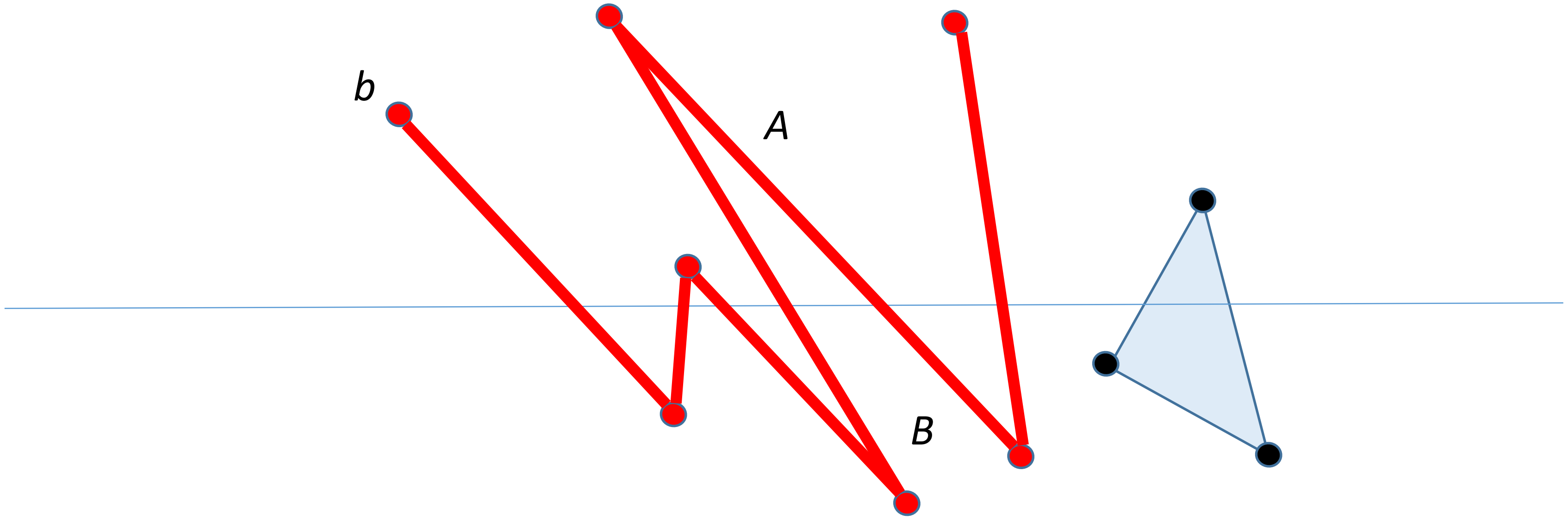

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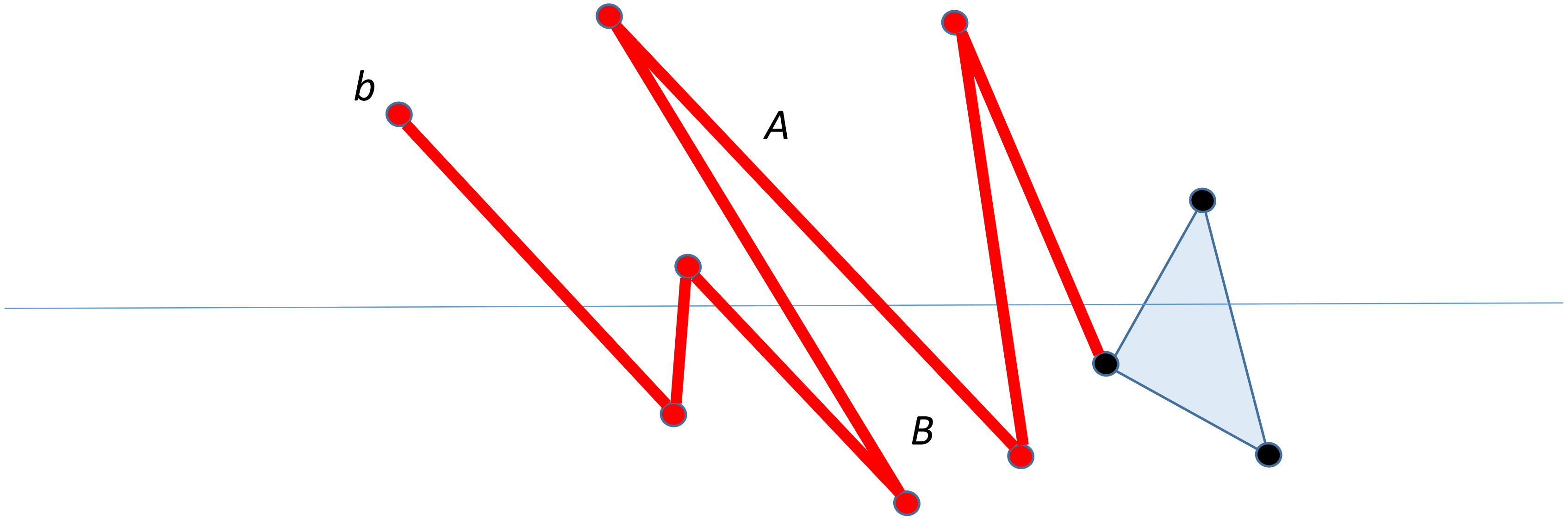

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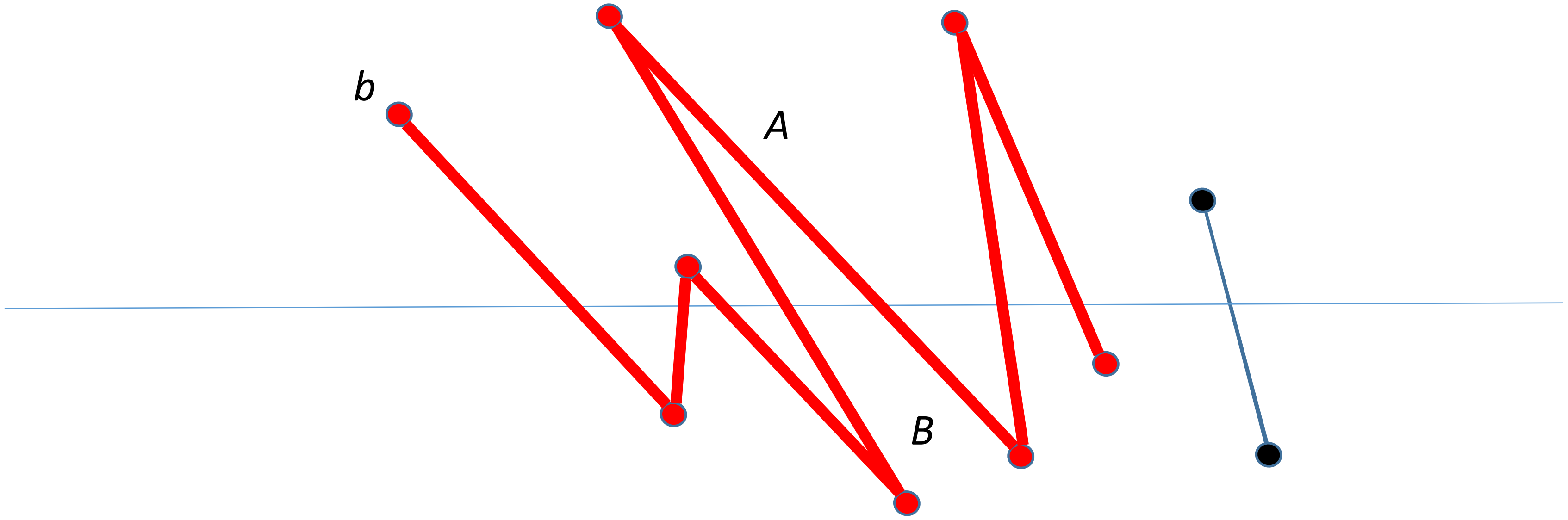

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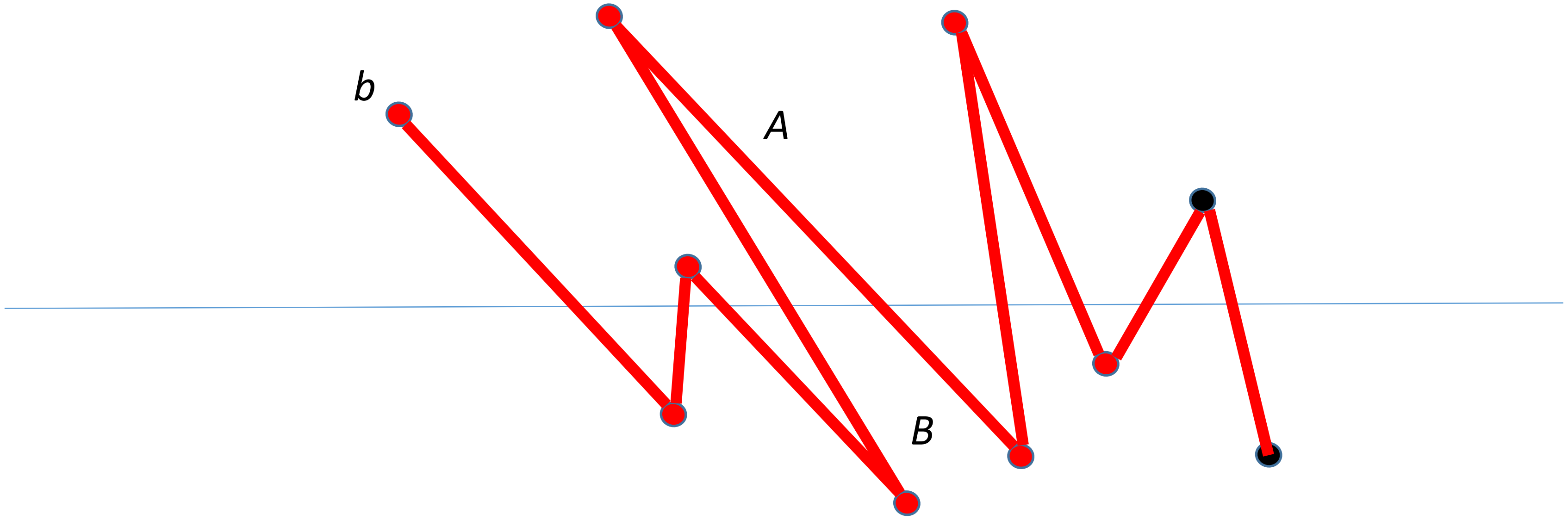


3 Paths

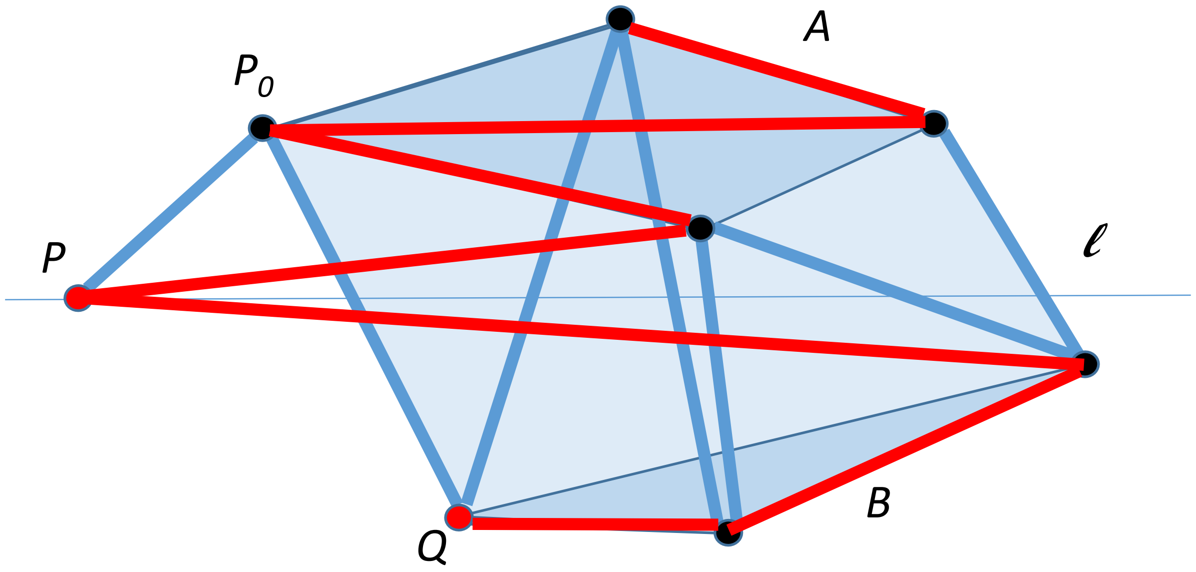
Remark: All steps of the proof were constructive. Thus given a set *S* of at least 10 points, we can construct 3 edge-disjoint plane spanning paths for *S* in polynomial time.

2 paths

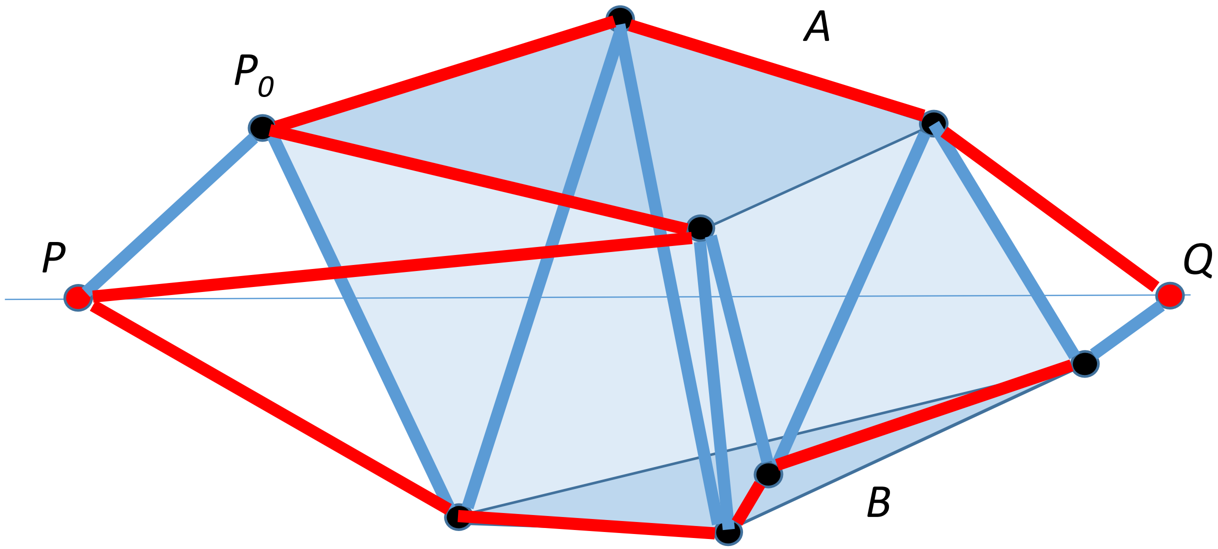
Theorem 2: Let P and Q be two (not necessarily distinct) points of S, lying on the boundary of conv(S), and let $|S| \geq 5$. Then S admits 2 edge-disjoint plane spanning paths, one starting in P, the other one starting in Q, and none of them using the edge PQ (in case P and Q are distinct).

Proof: Case 1, $P \neq Q$.

a) Subcase $|S|$ odd or PQ not a halving line.



b) Subcase $|S|$ even and PQ is a halving line and $|A(P) \cup A(Q)| \geq 3$.



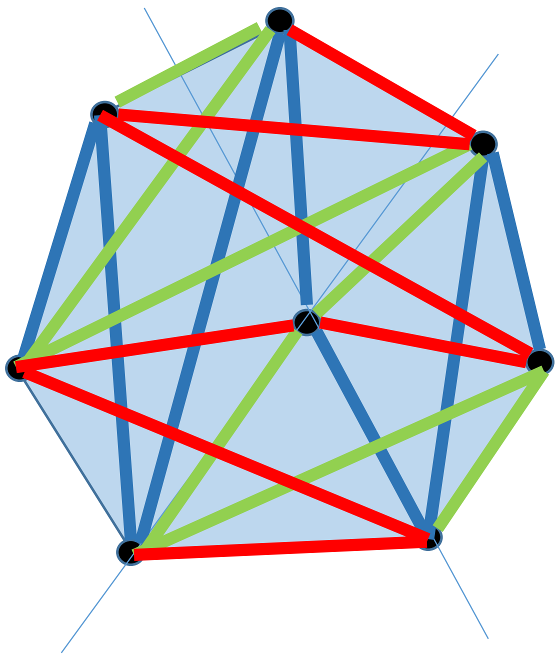
3 Paths

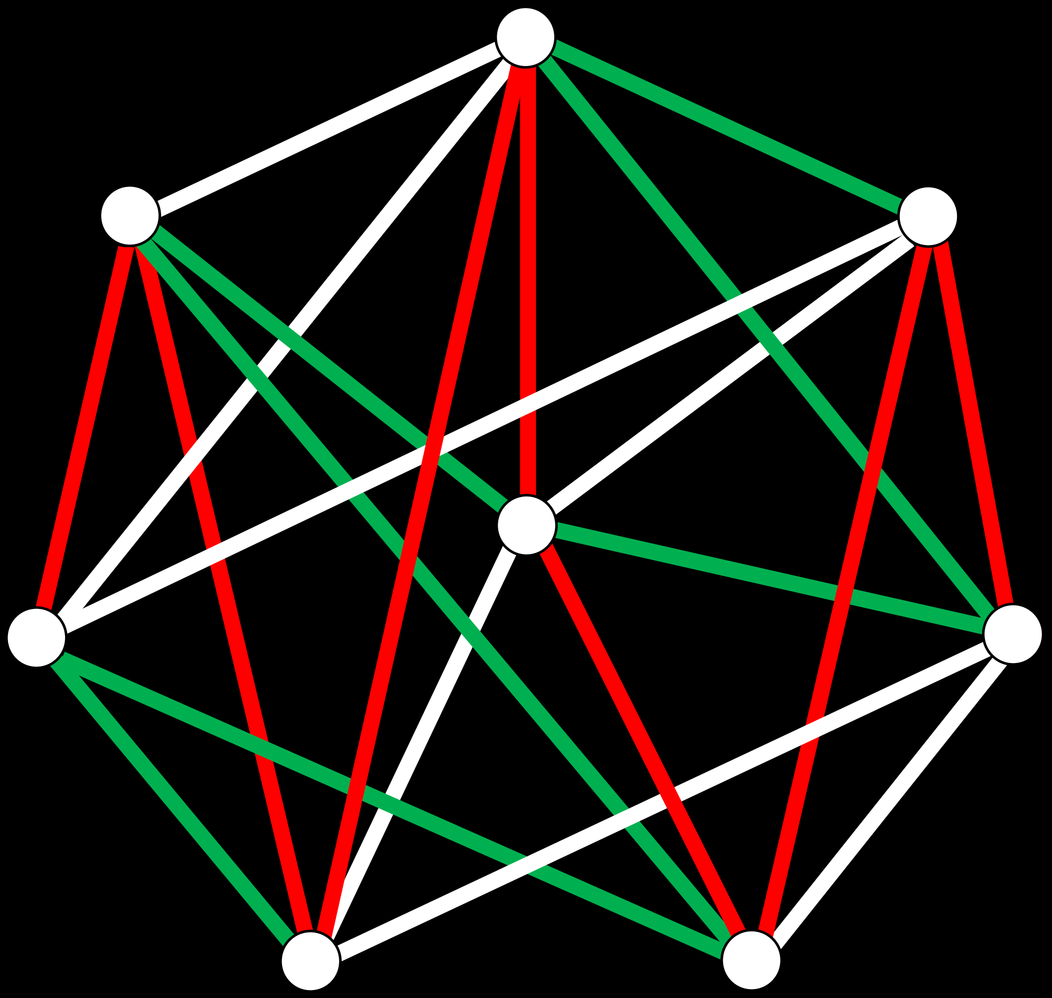
Theorem: Every set of $|S| \geq 10$ points admits 3 edge-disjoint plane spanning paths.

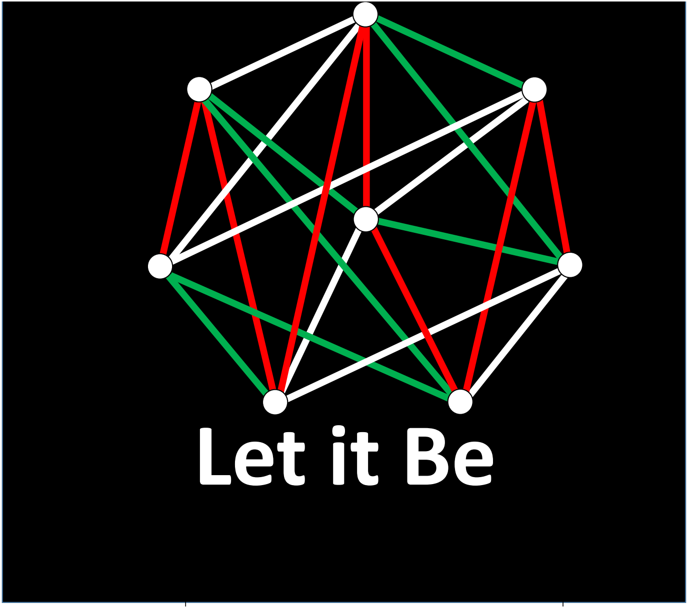
Proof:

Case C. S is in the wheel position.

An ad hoc construction shows that *S* has $(n\text{-}2)/2 \geq 3$ edge-disjoint plane spanning paths.



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3 PATHS !
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NOT FOR SALE (Sorry)!

ONLY AS A GIFT!

ONLY AS A GIFT!

UPOZORNĚNÍ!

První praní je možné nejdříve po třech dnech po převzetí potištěných oděvů, tehdy je potisk dostatečně vytvrzen.
Trička perte na maximálně 40°C, žehlete dle pokynů na vinětě trička, zajistíte tak dlouhou životnost potisku.
Nedoporučujeme používat sušičku.

Děkujeme za Vaší přízeň a těšíme se na další spolupráci.

Tým tiskárny F&F