Cops and Robbers on 1-Planar Graphs

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September, 2023

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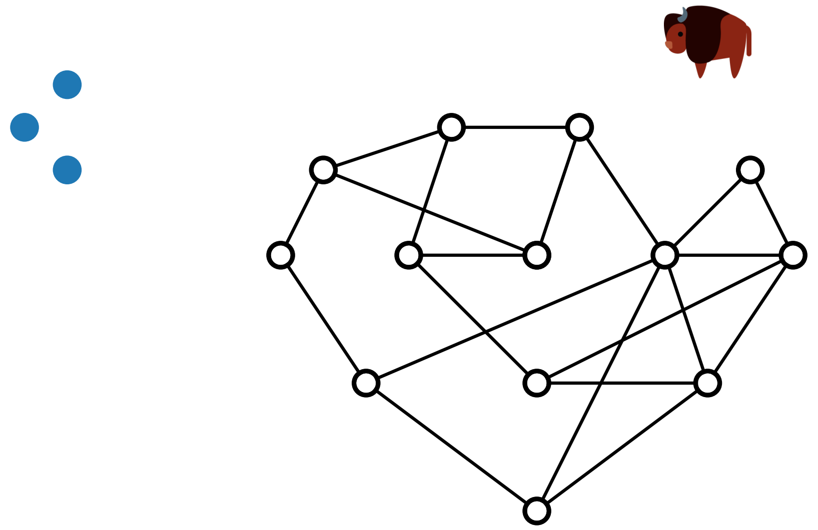

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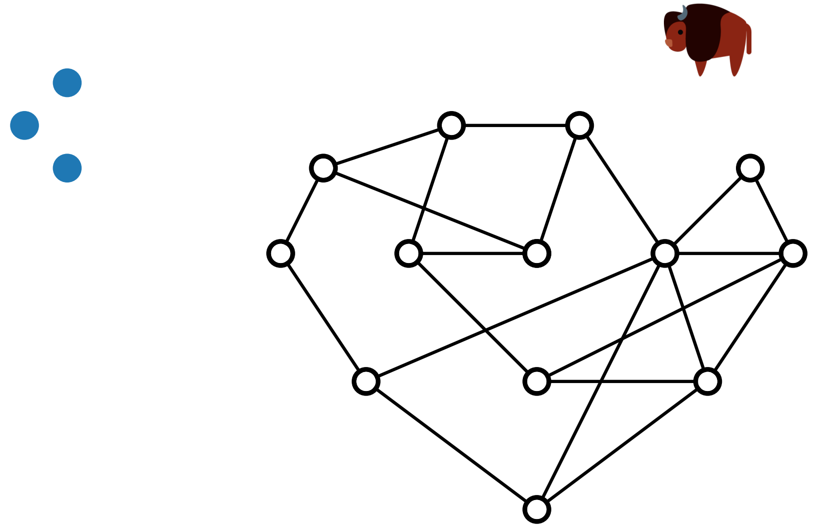

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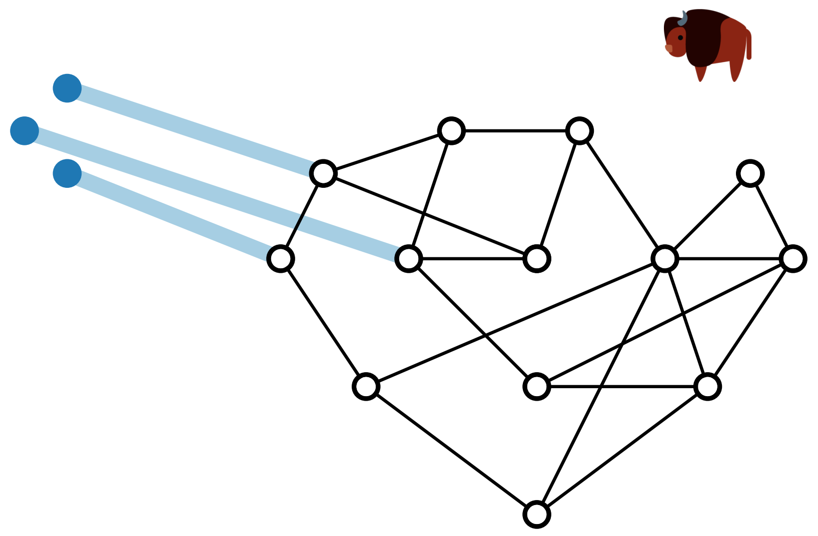

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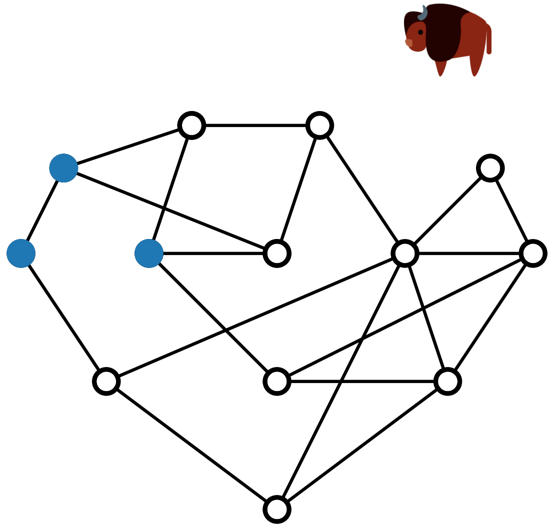

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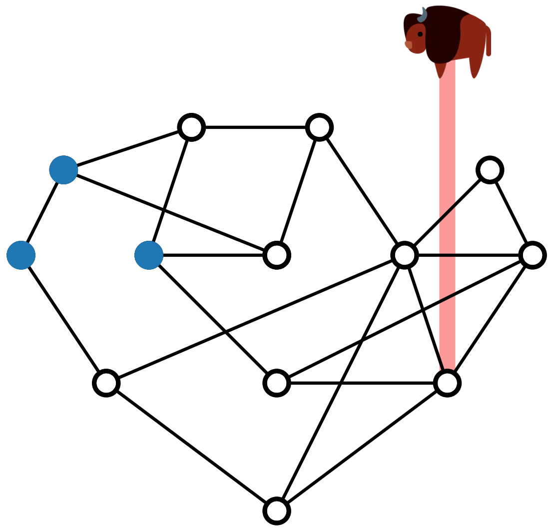

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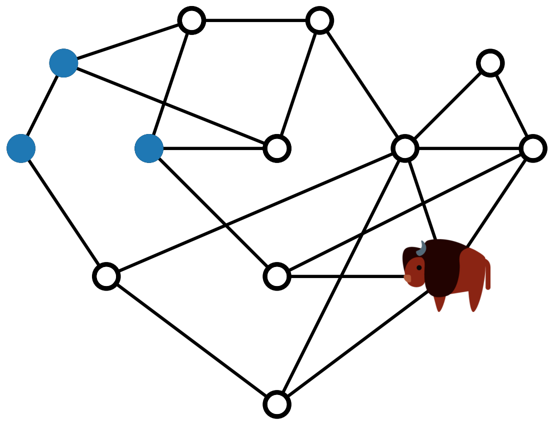

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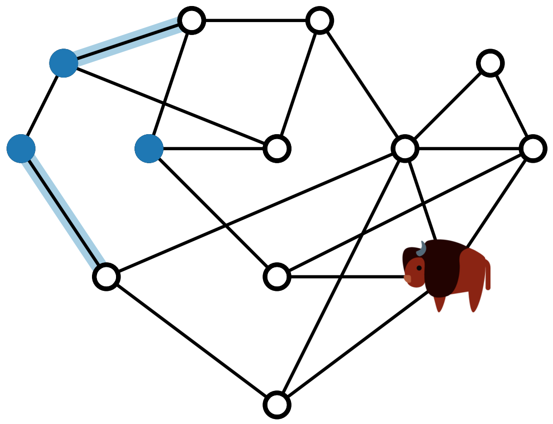

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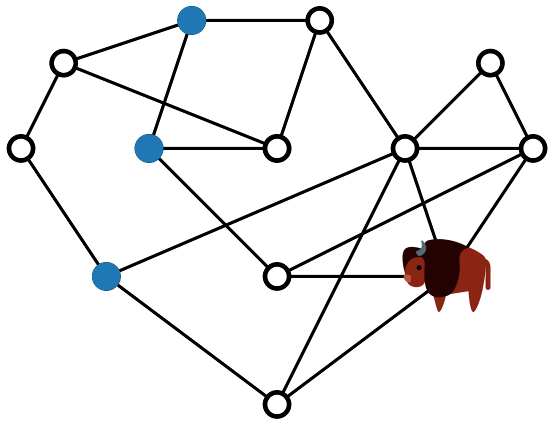

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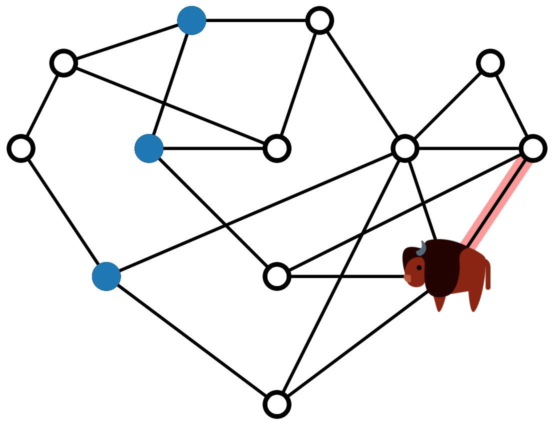

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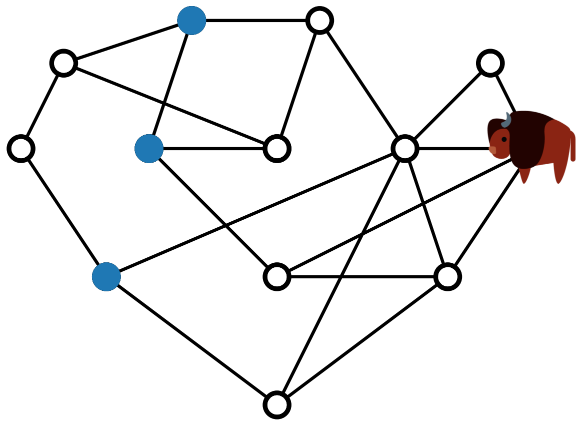

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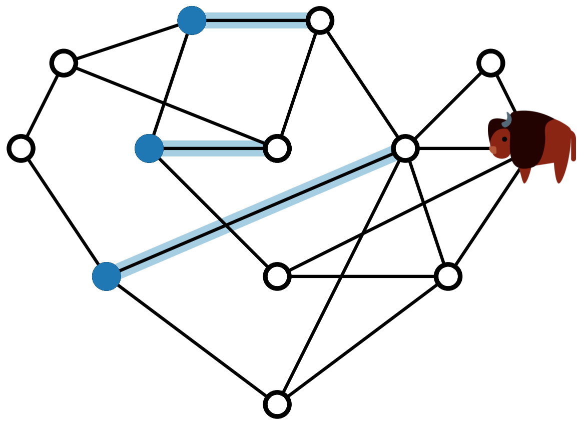

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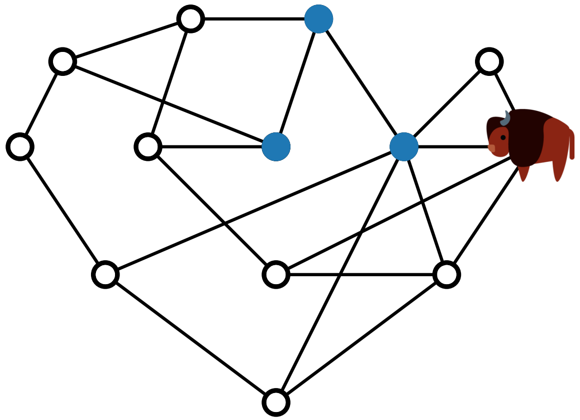

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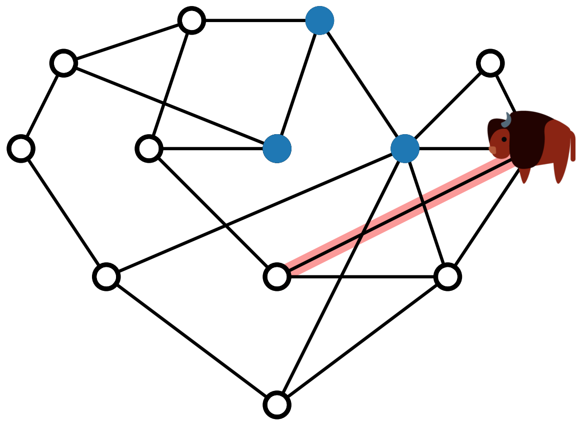

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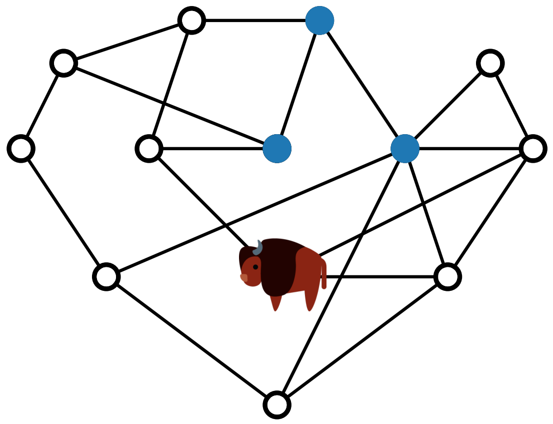

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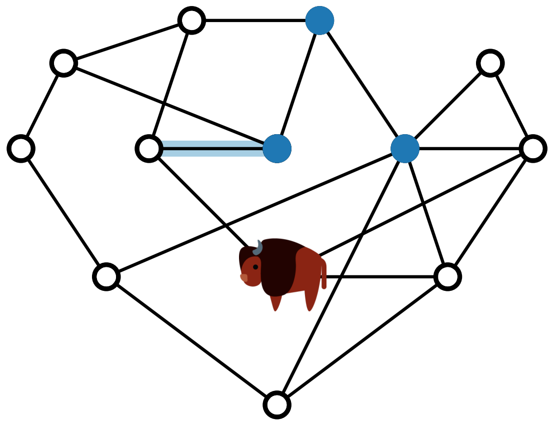

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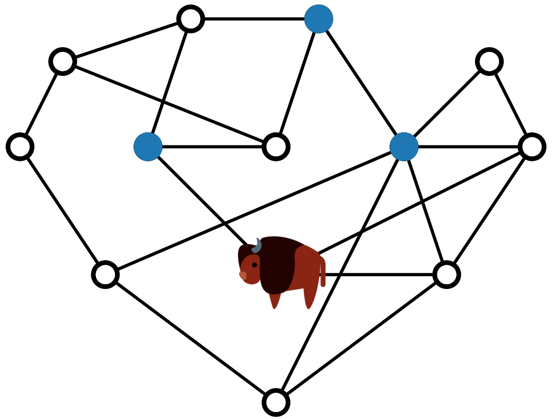

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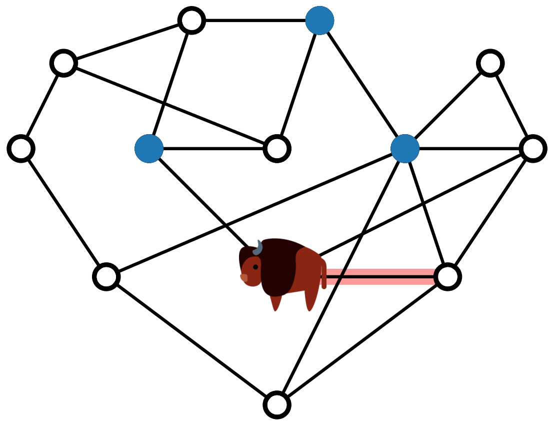

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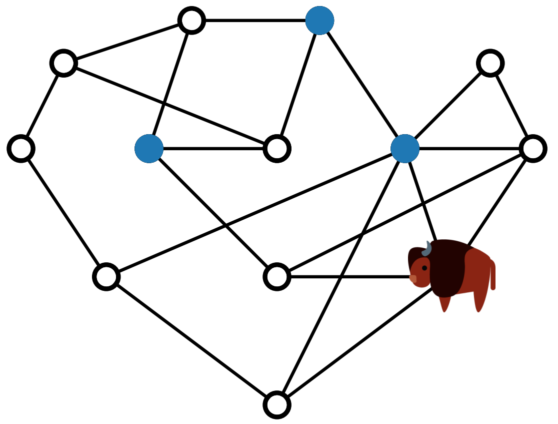

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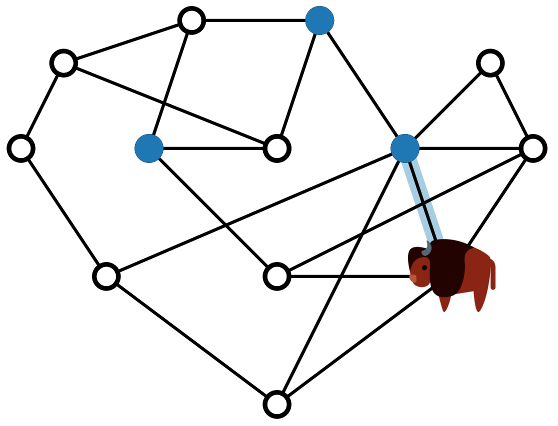

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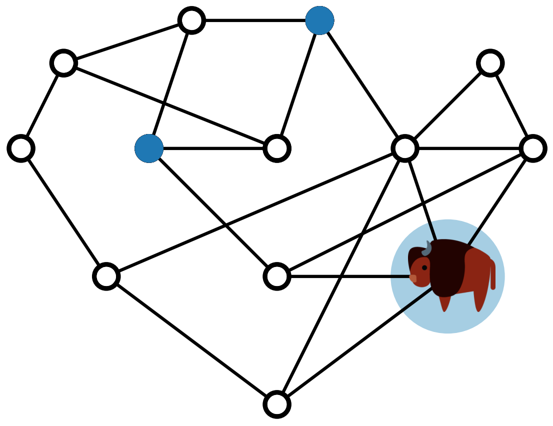

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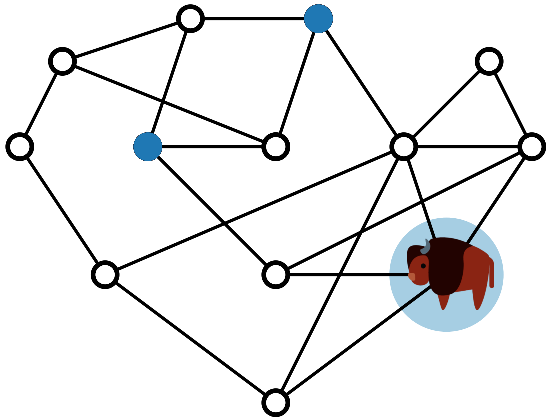

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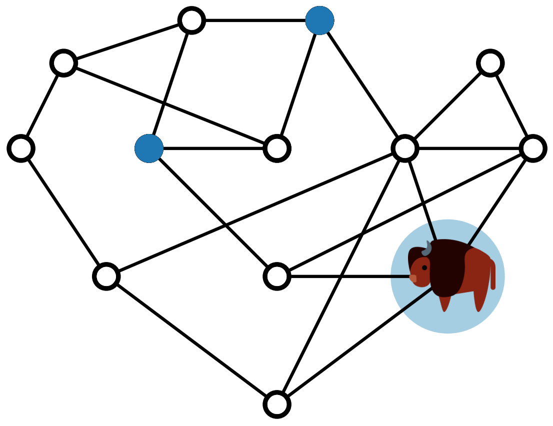

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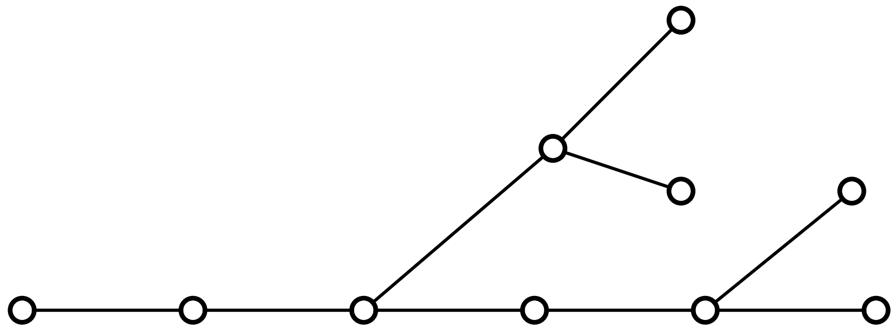

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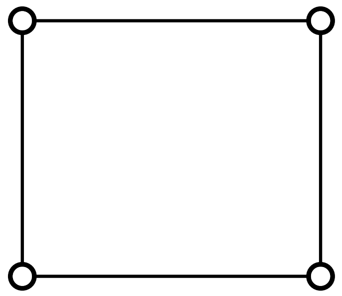

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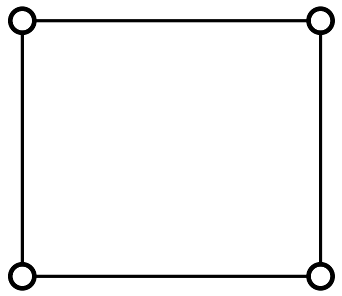

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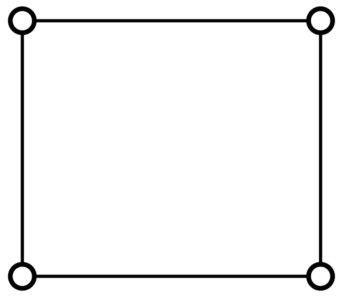

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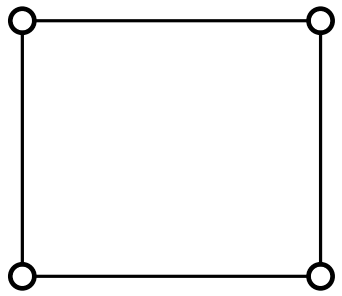

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- interesting from a GD point of view!

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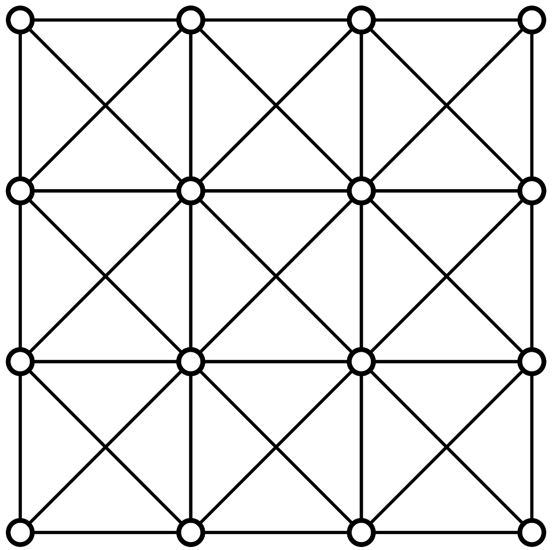

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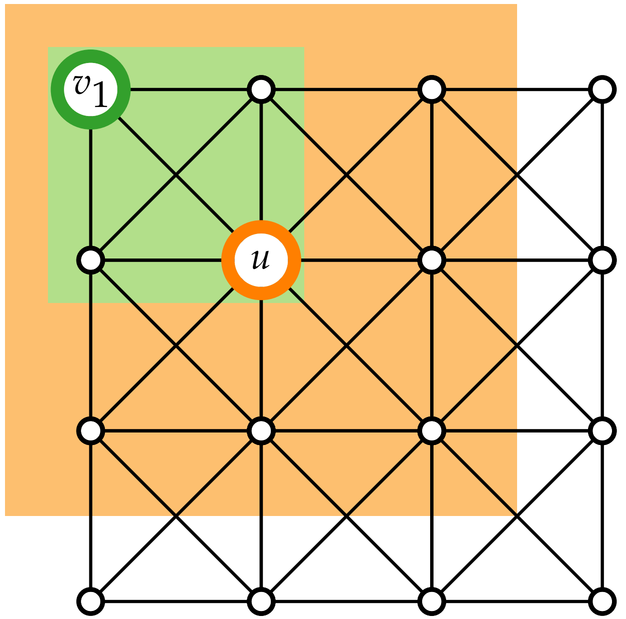

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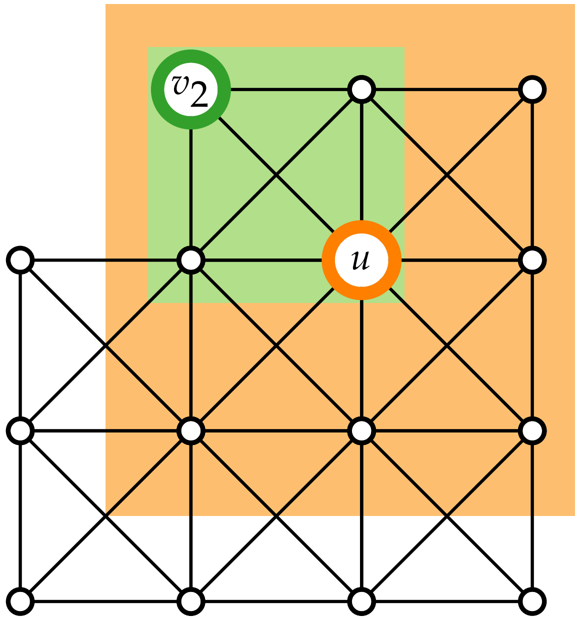

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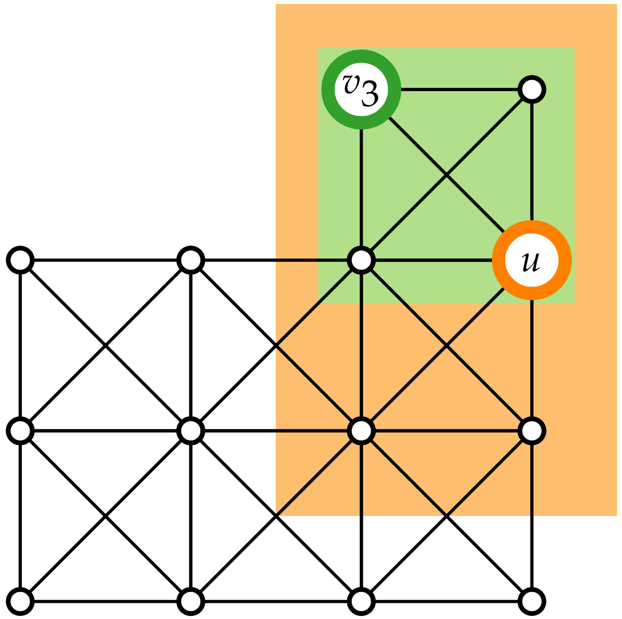

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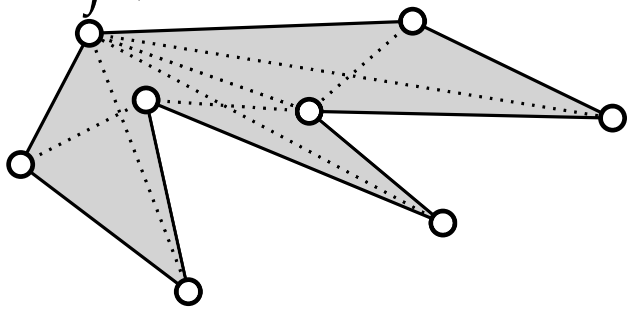

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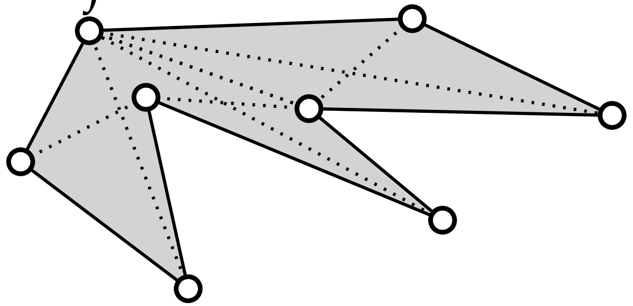

[Bonato and Nowakowski "Cops and Robbers: Covering by Cop-win Graphs", 2011]
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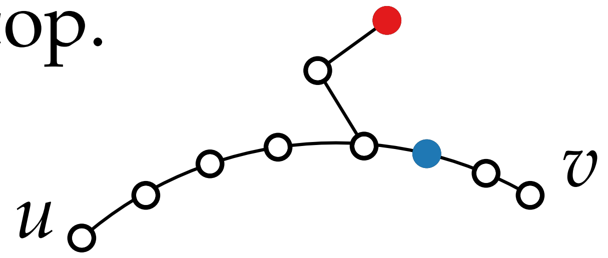

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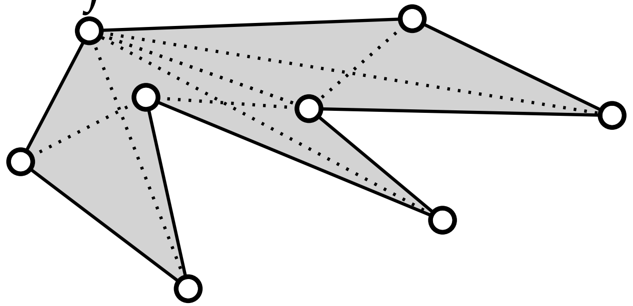

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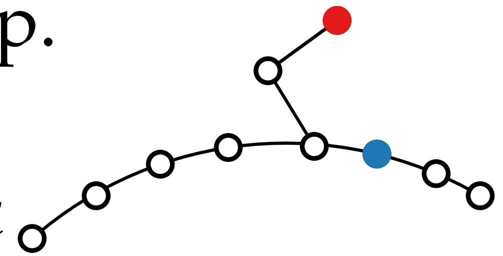

E.g.: - $c(\text{planar graph}) \leq 3$, [Aigner and Fromme, ’82]

- $c(\text{unit disk graph}) \leq 9$, [Berg, 2017]
- $c(\text{string graph}) \leq 13$, [Das and Gahlawat, 2022]
- $c(\text{genus} \leq g \text{ graph }) \leq 1.268g$, [Erde and Lehner, 2021]

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Means to bound the cop number from above:

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$v_1, v_2, \ldots, v_n$ s.t. $N[v_i] \subseteq N[v_j]$ in $G[v_i, \ldots, v_n]$, for some $j > i$.
E.g.: - chordal graphs,
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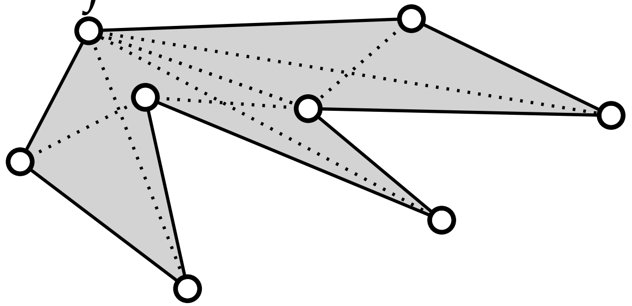

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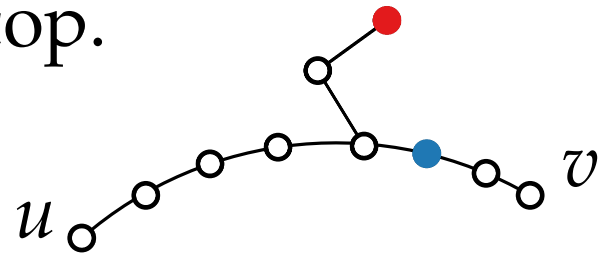

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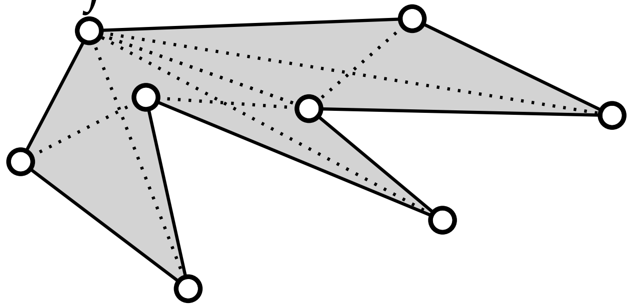

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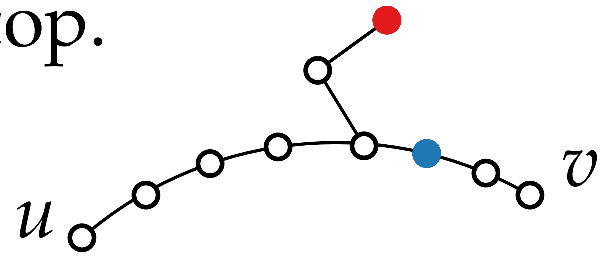

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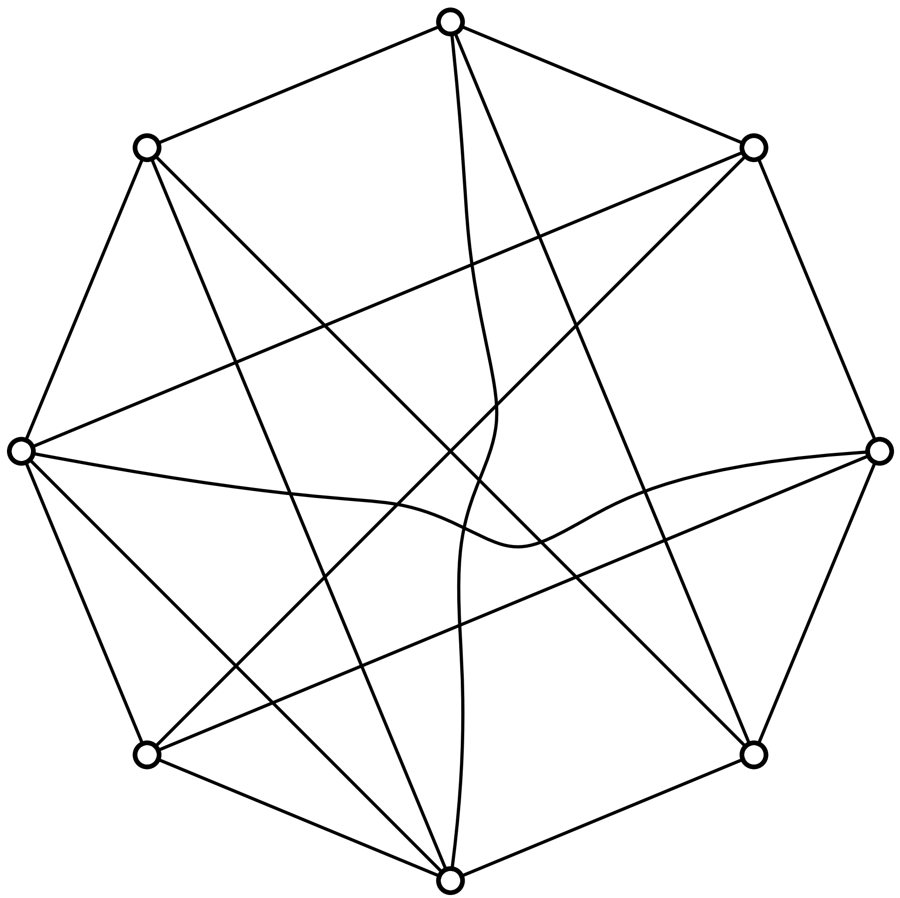

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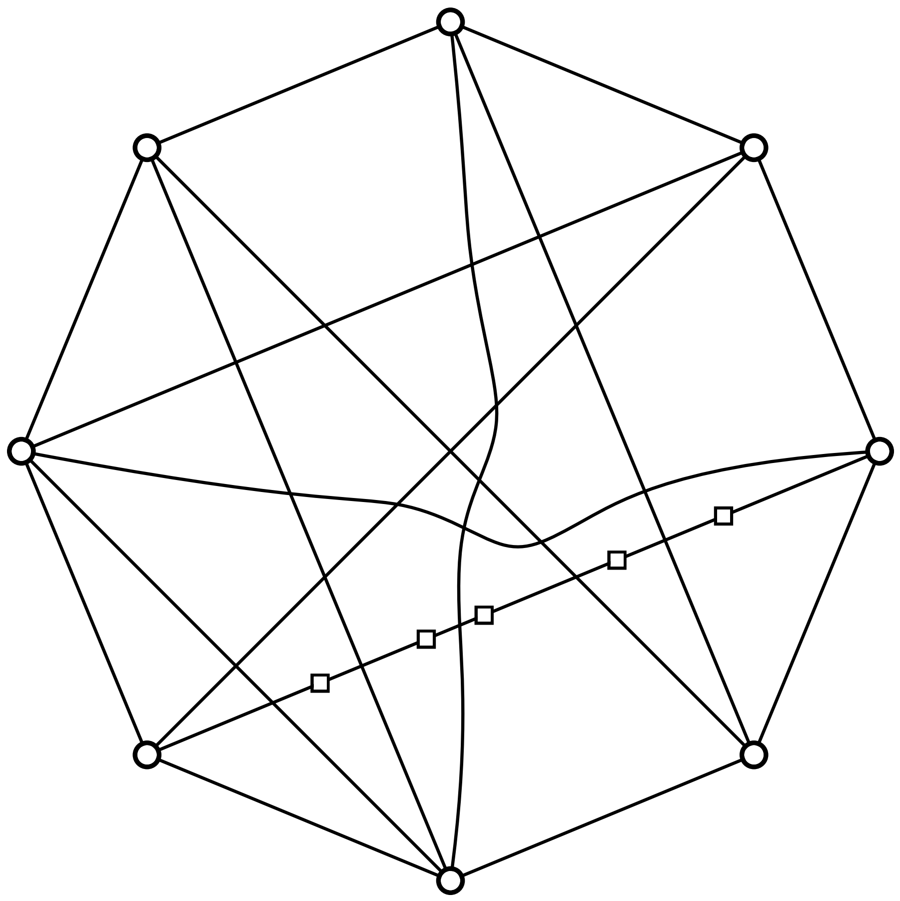

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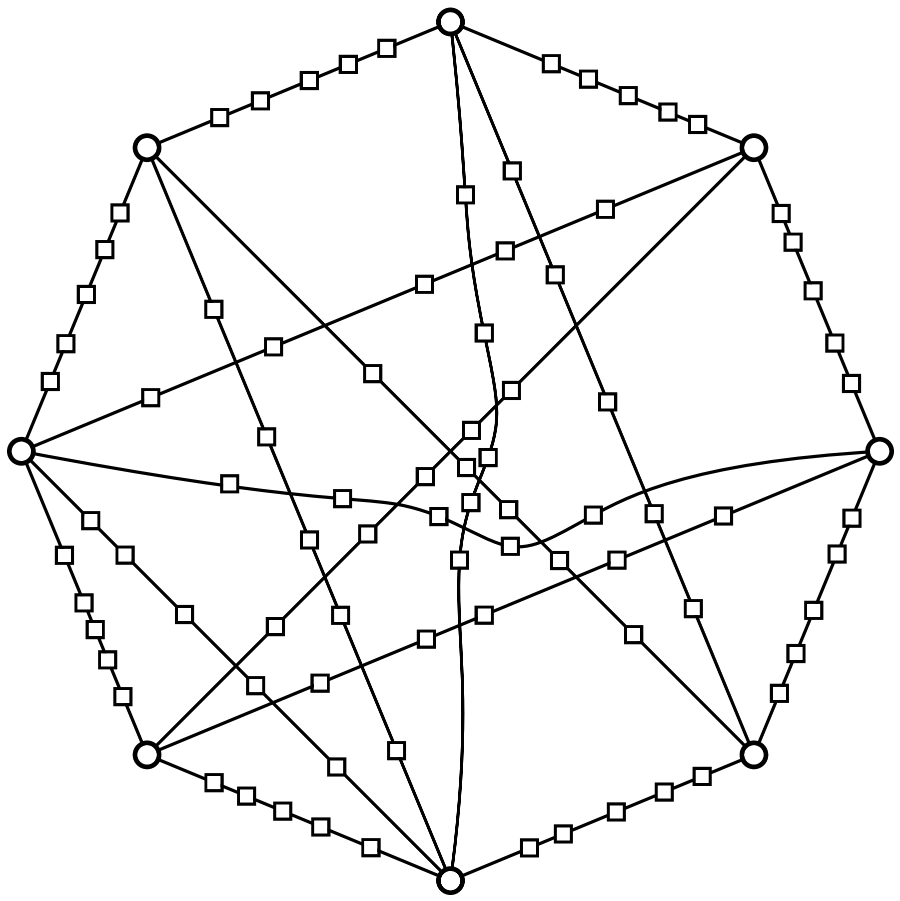

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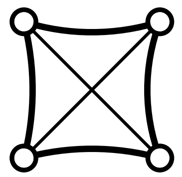

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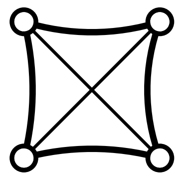

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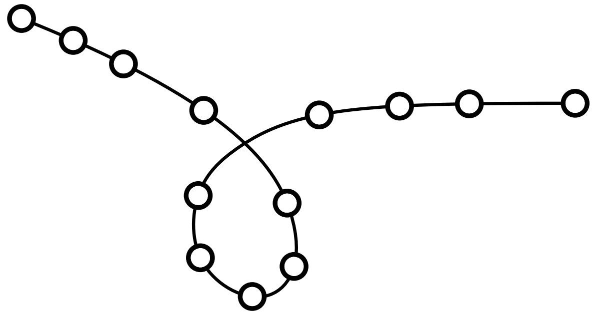

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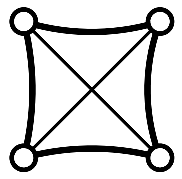

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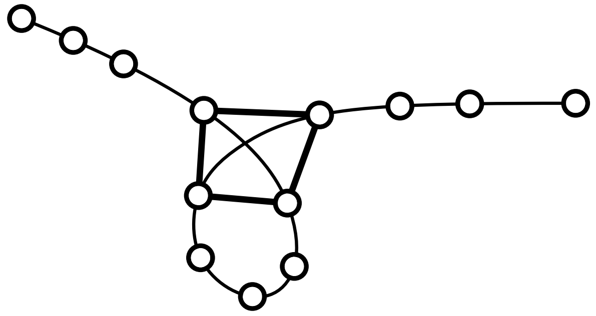

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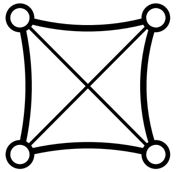

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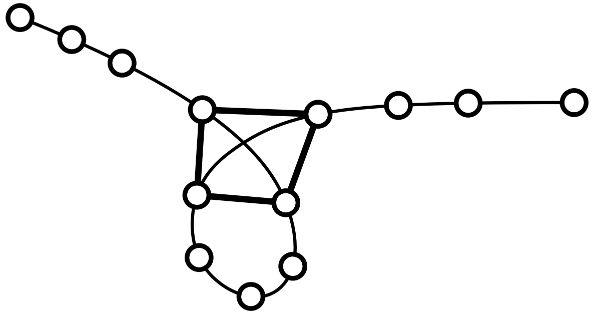

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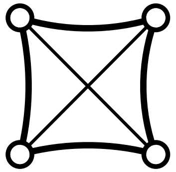

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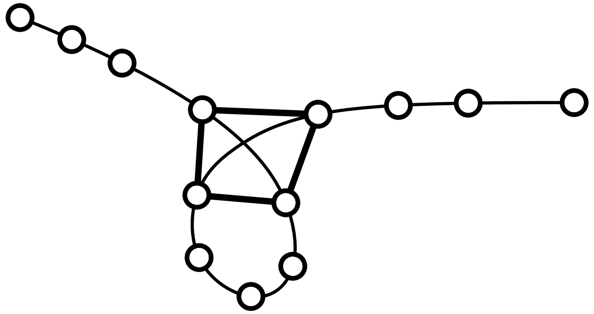

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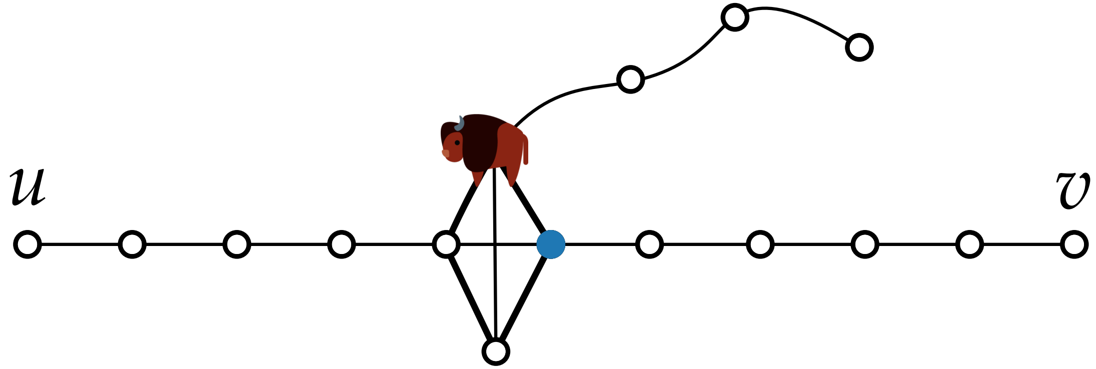

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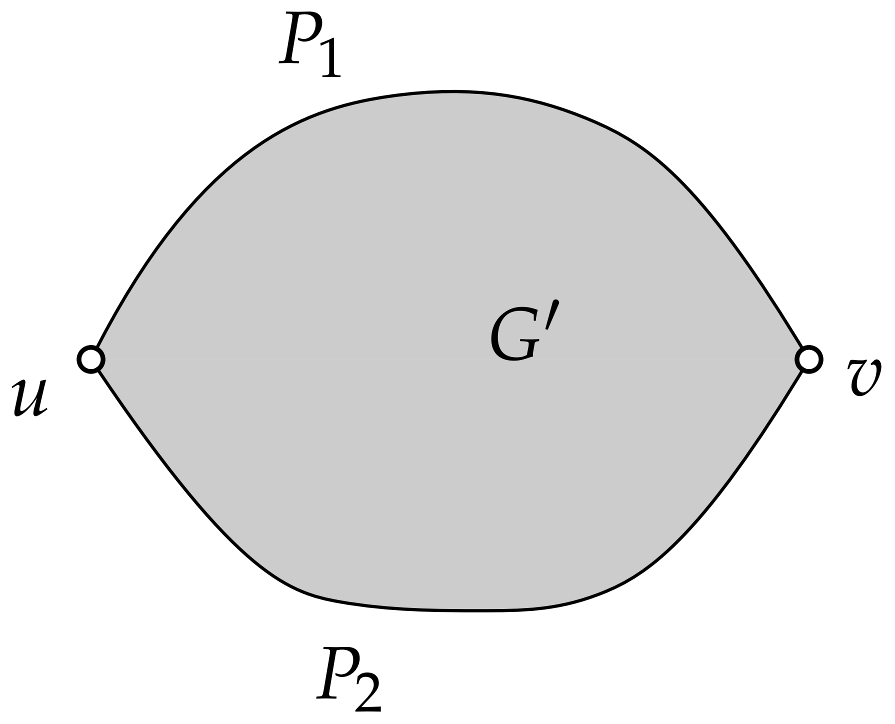

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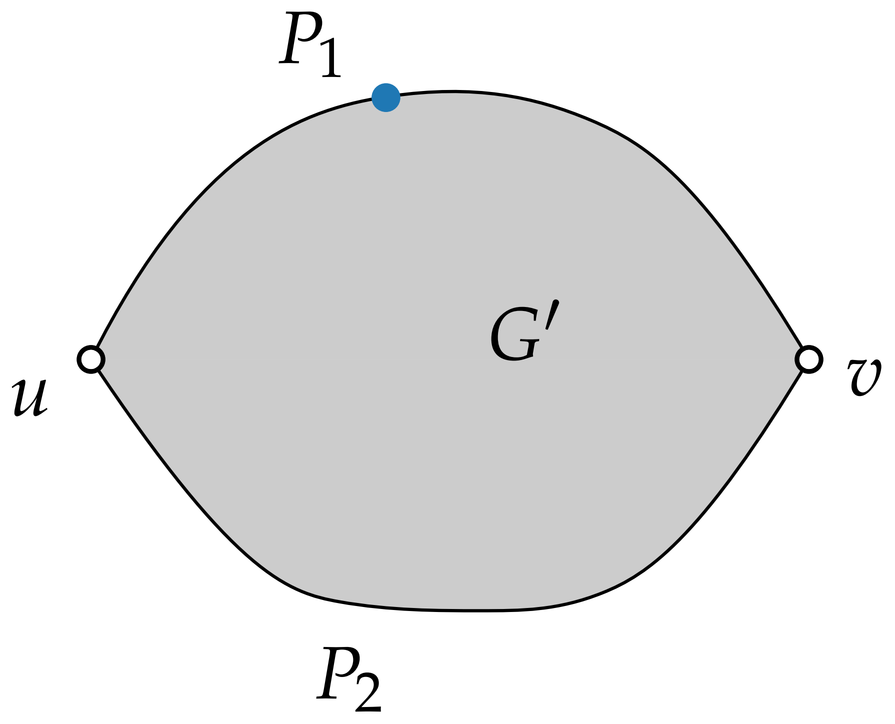

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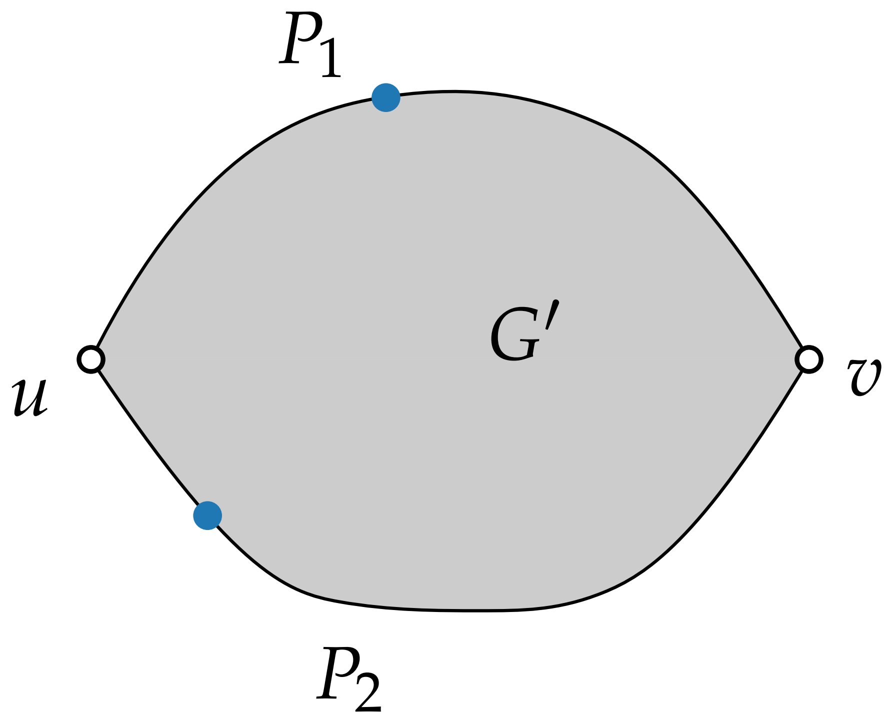

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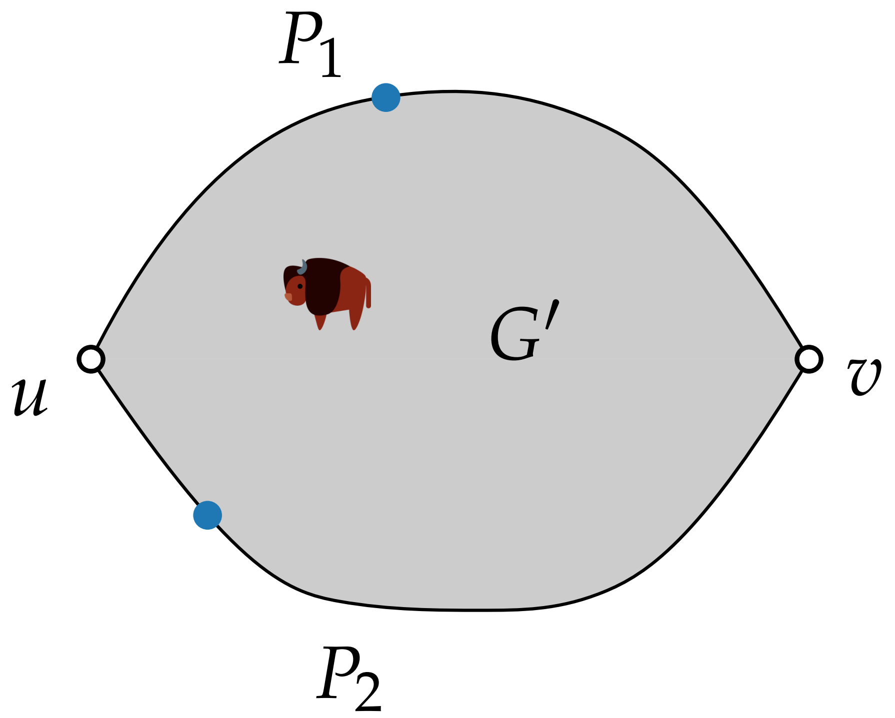

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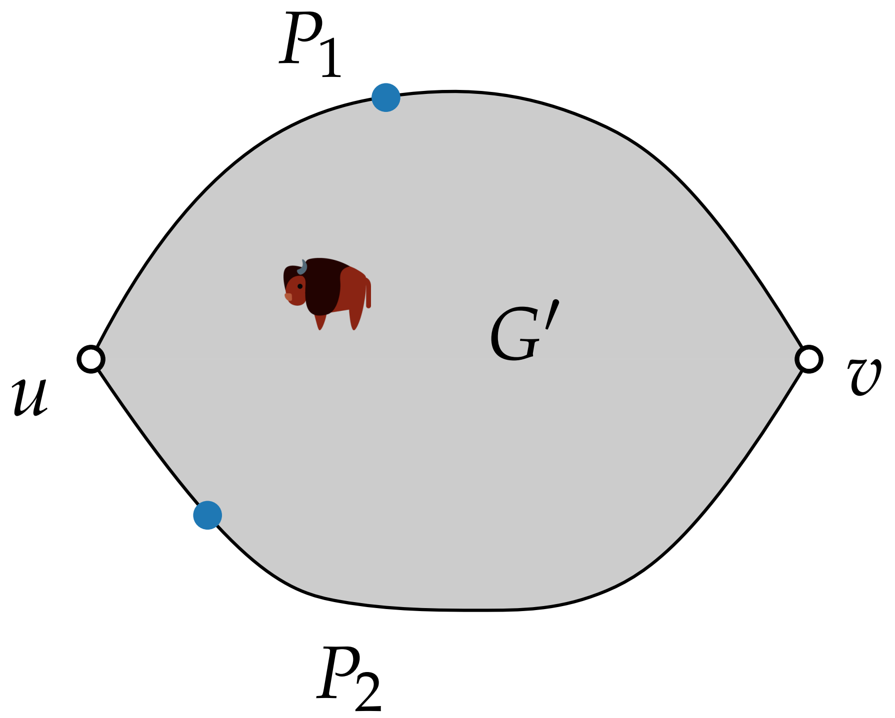

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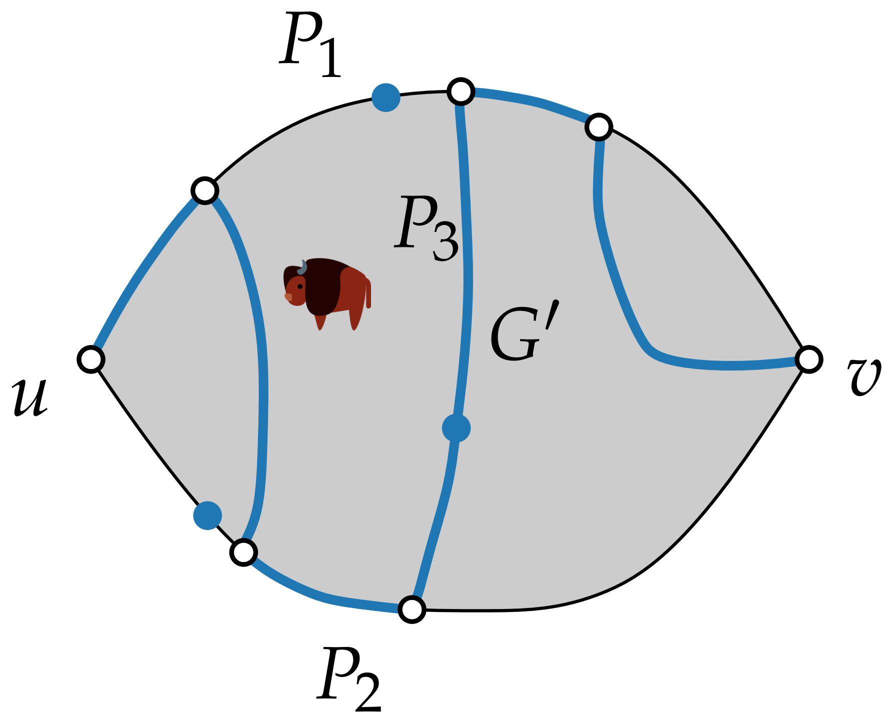

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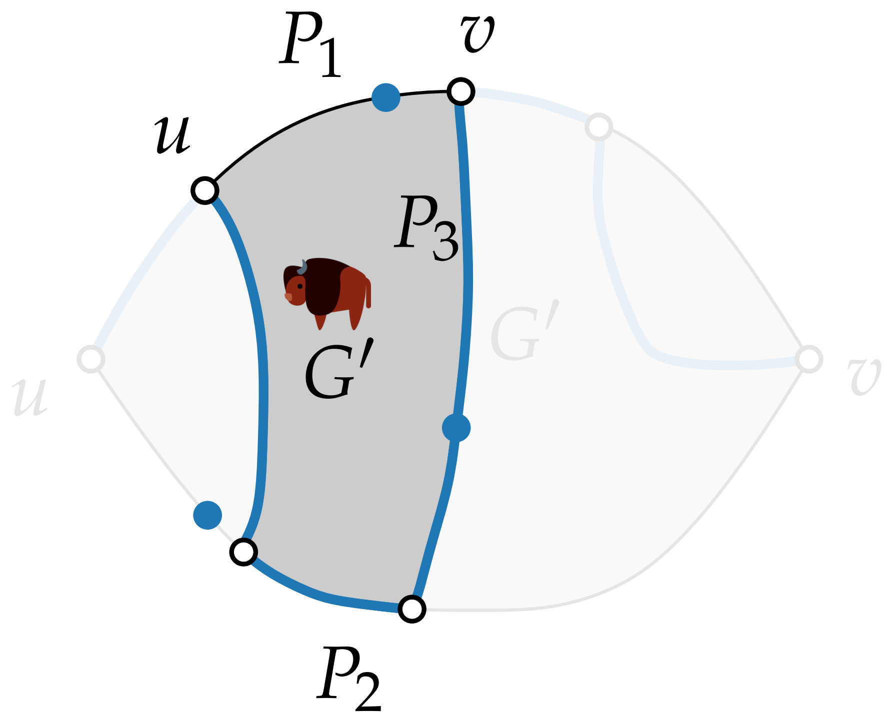

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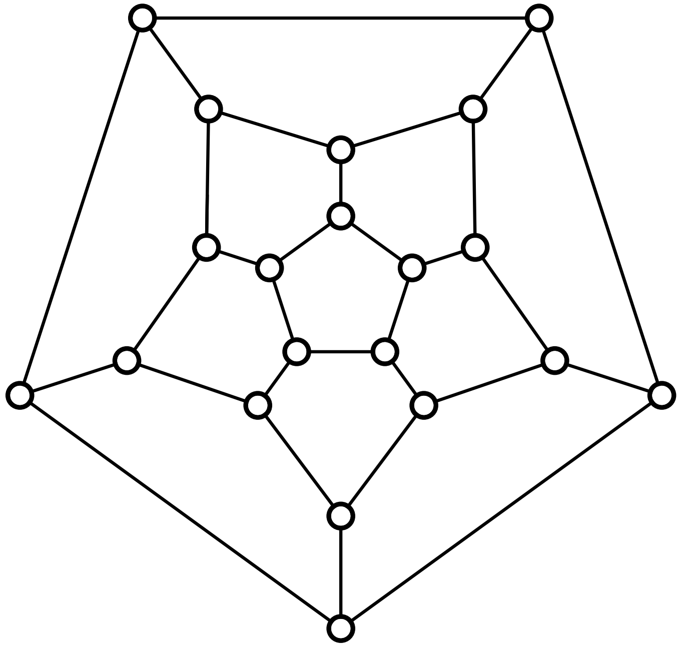

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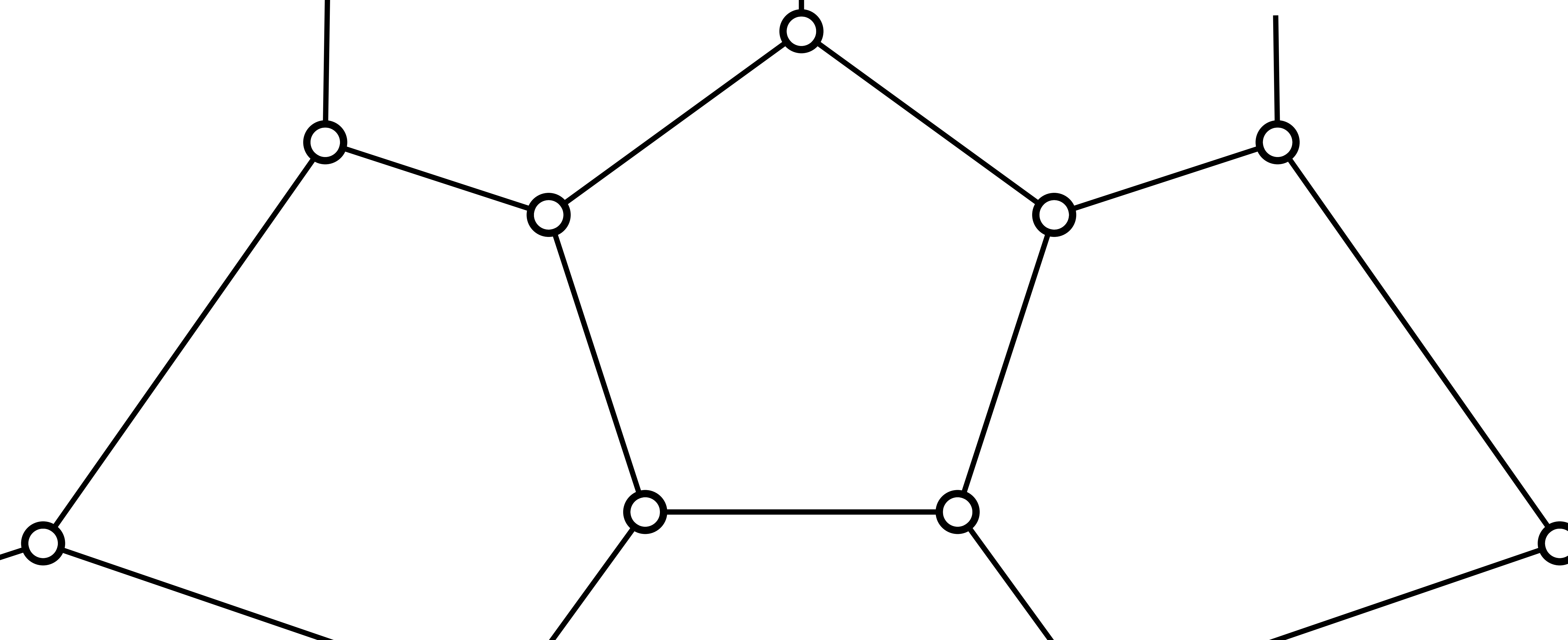

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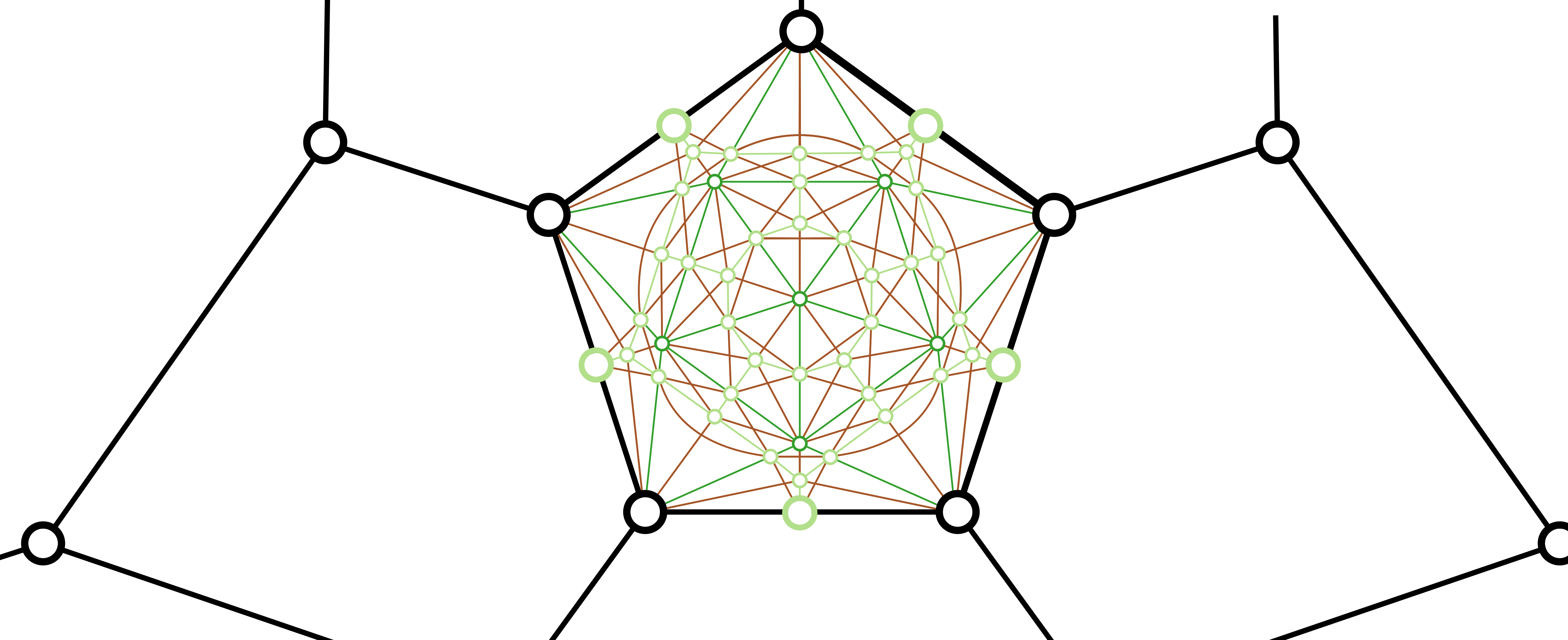

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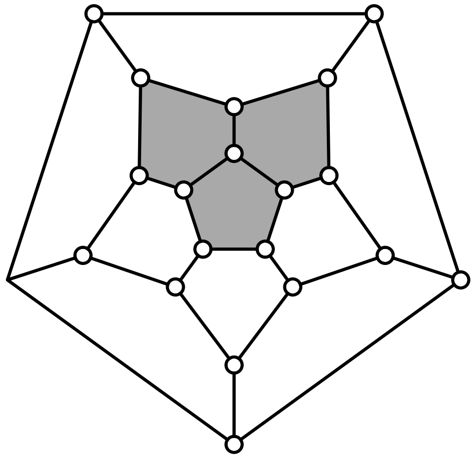

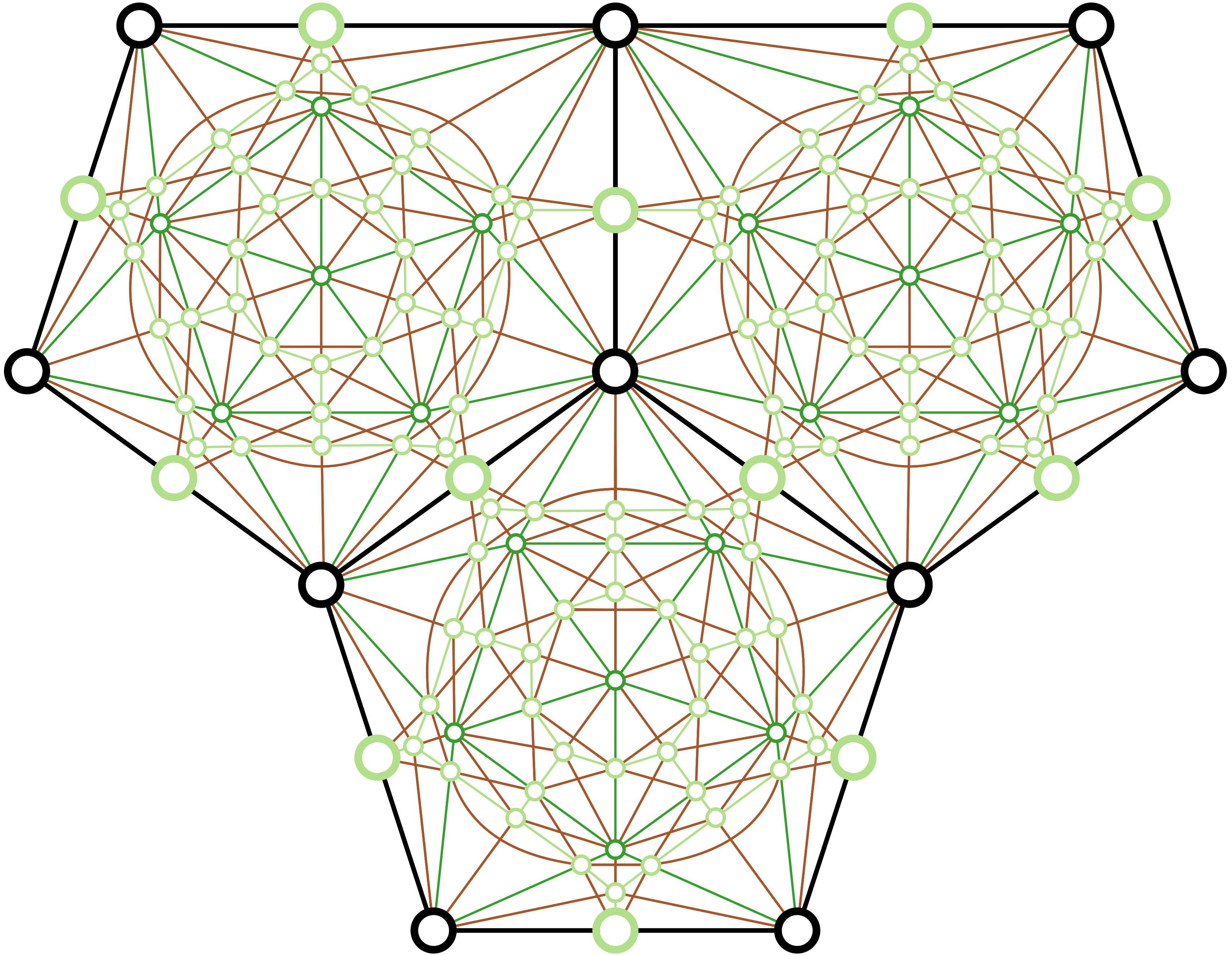

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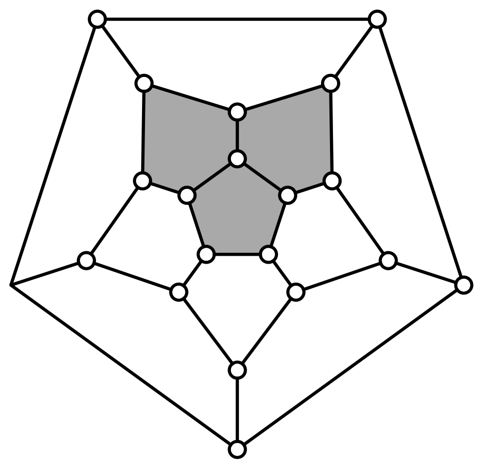

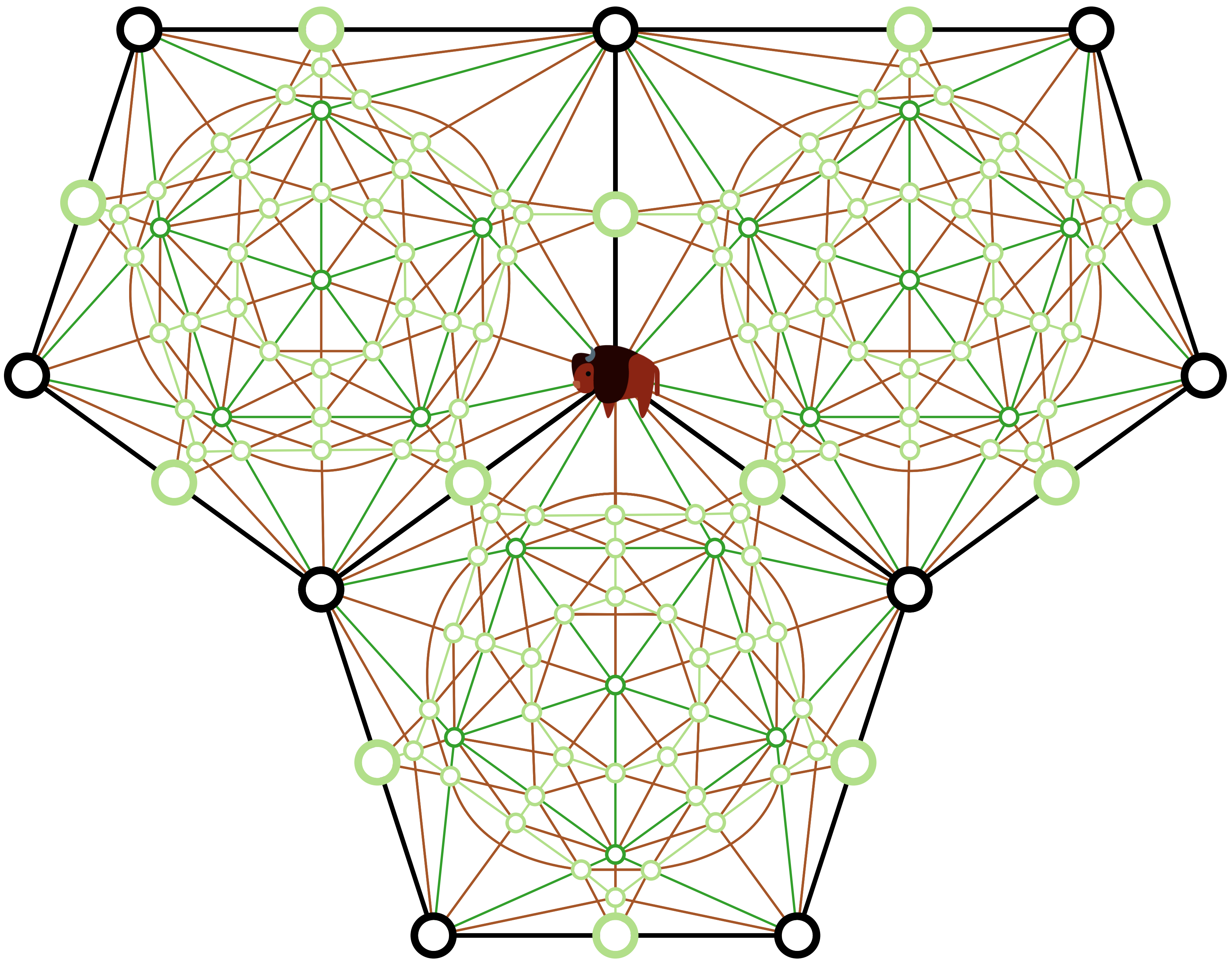

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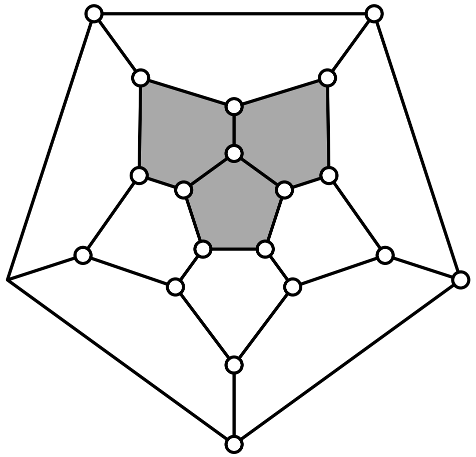

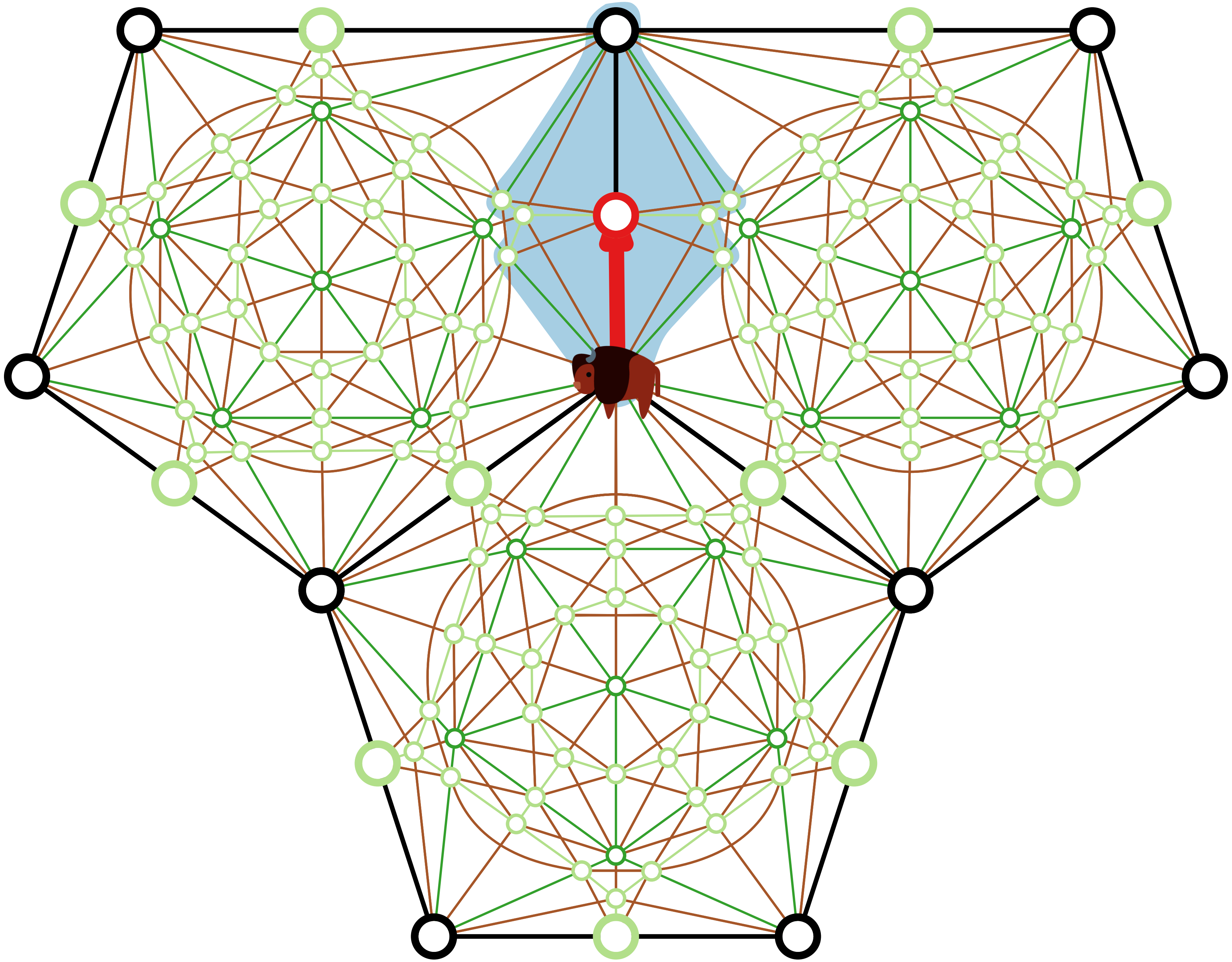

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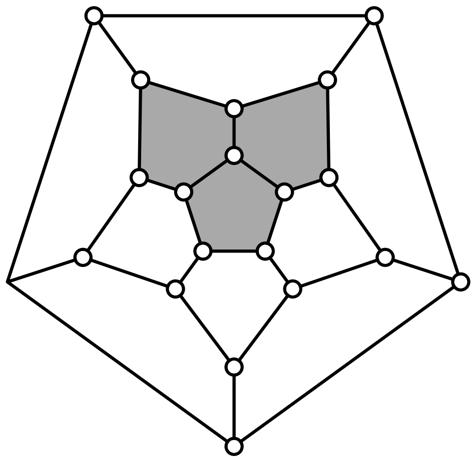

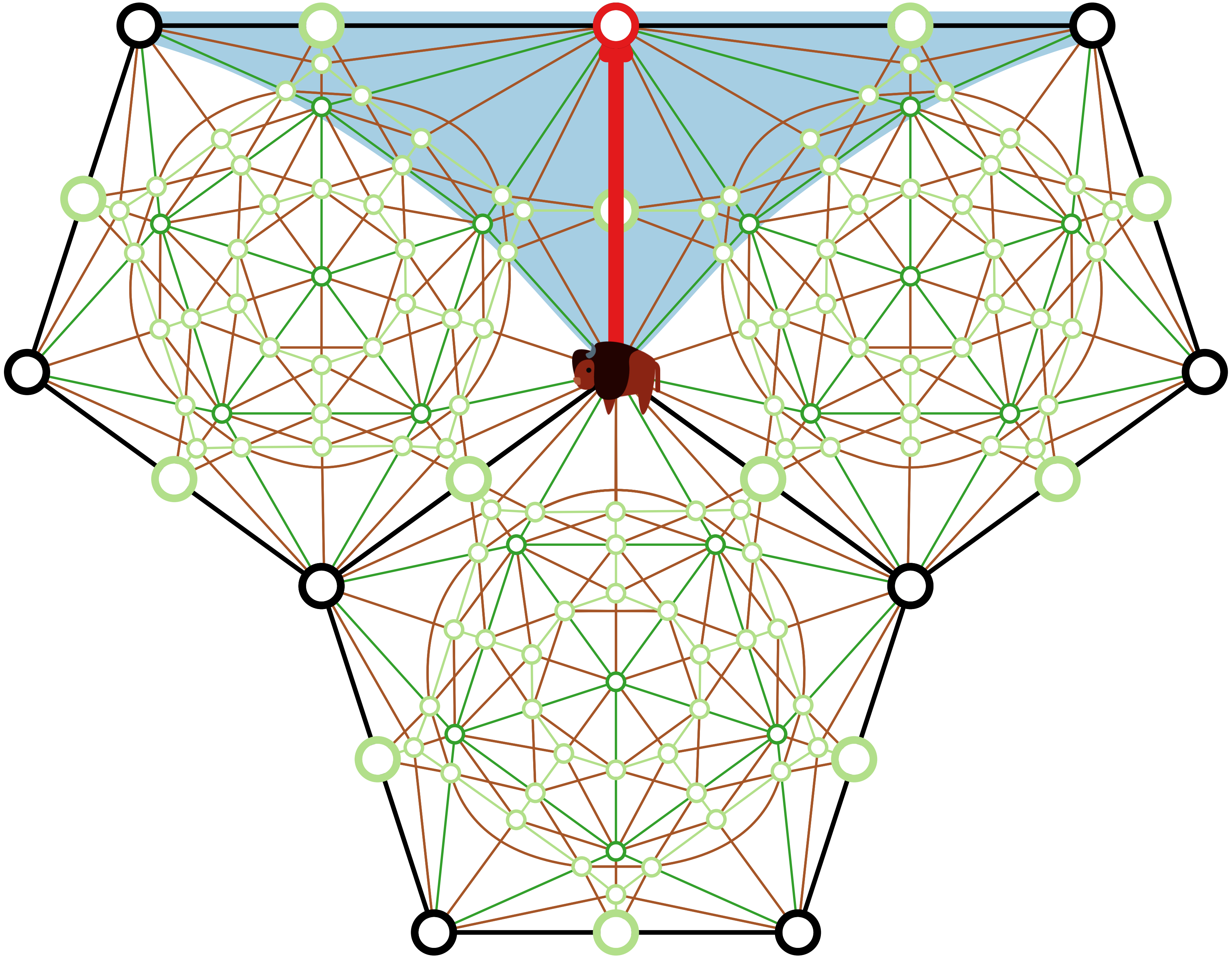

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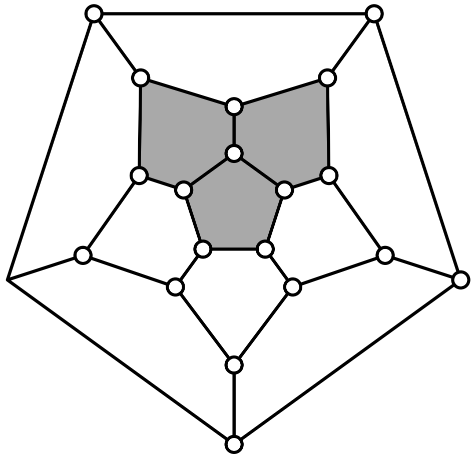

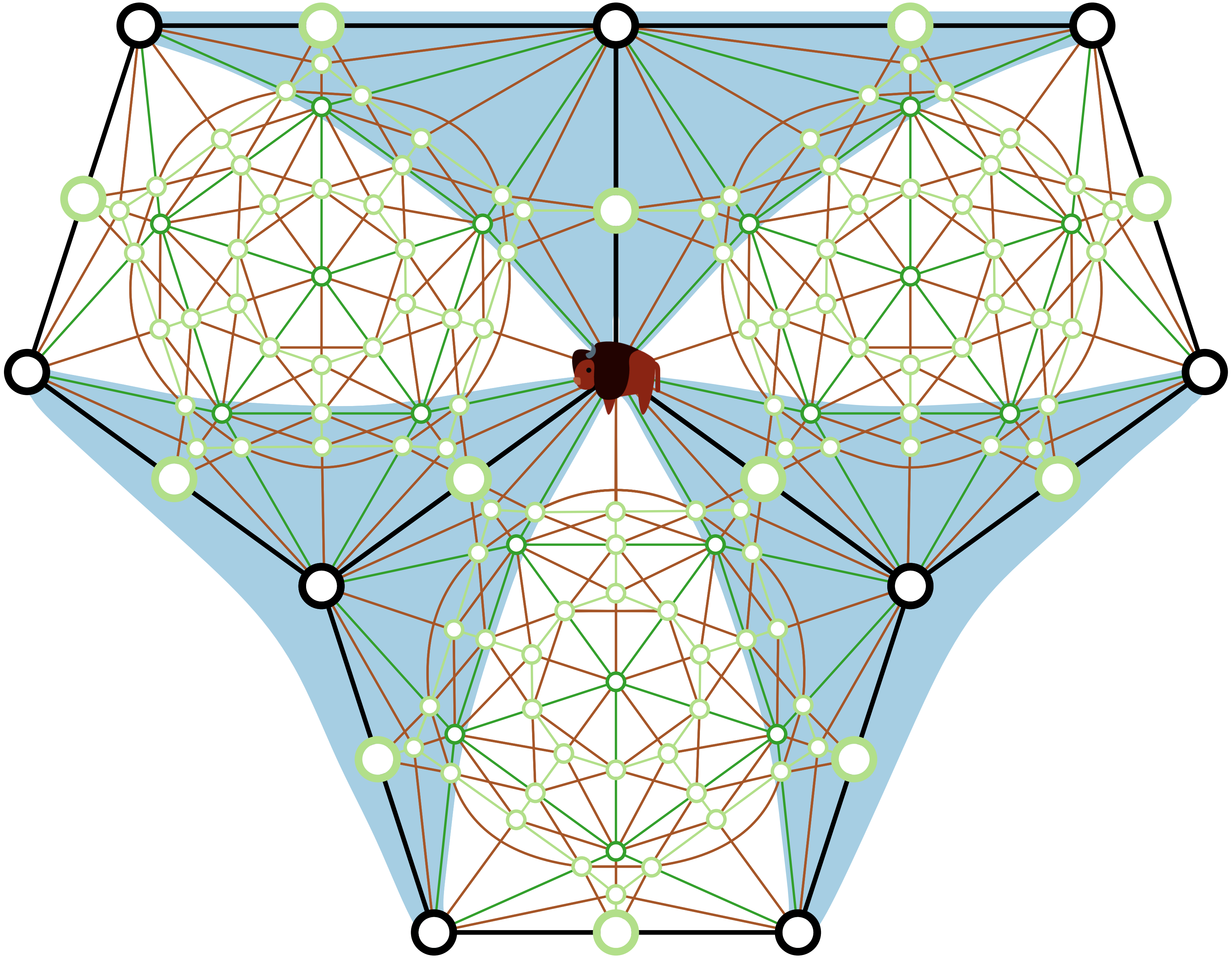

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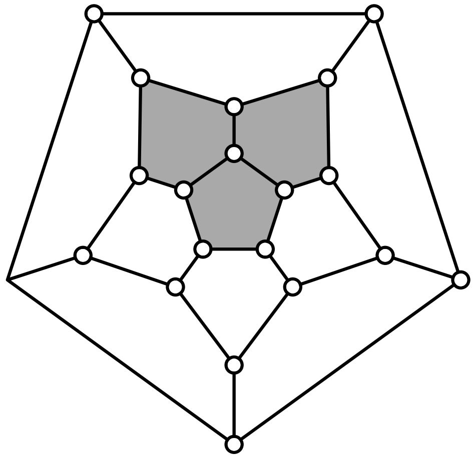

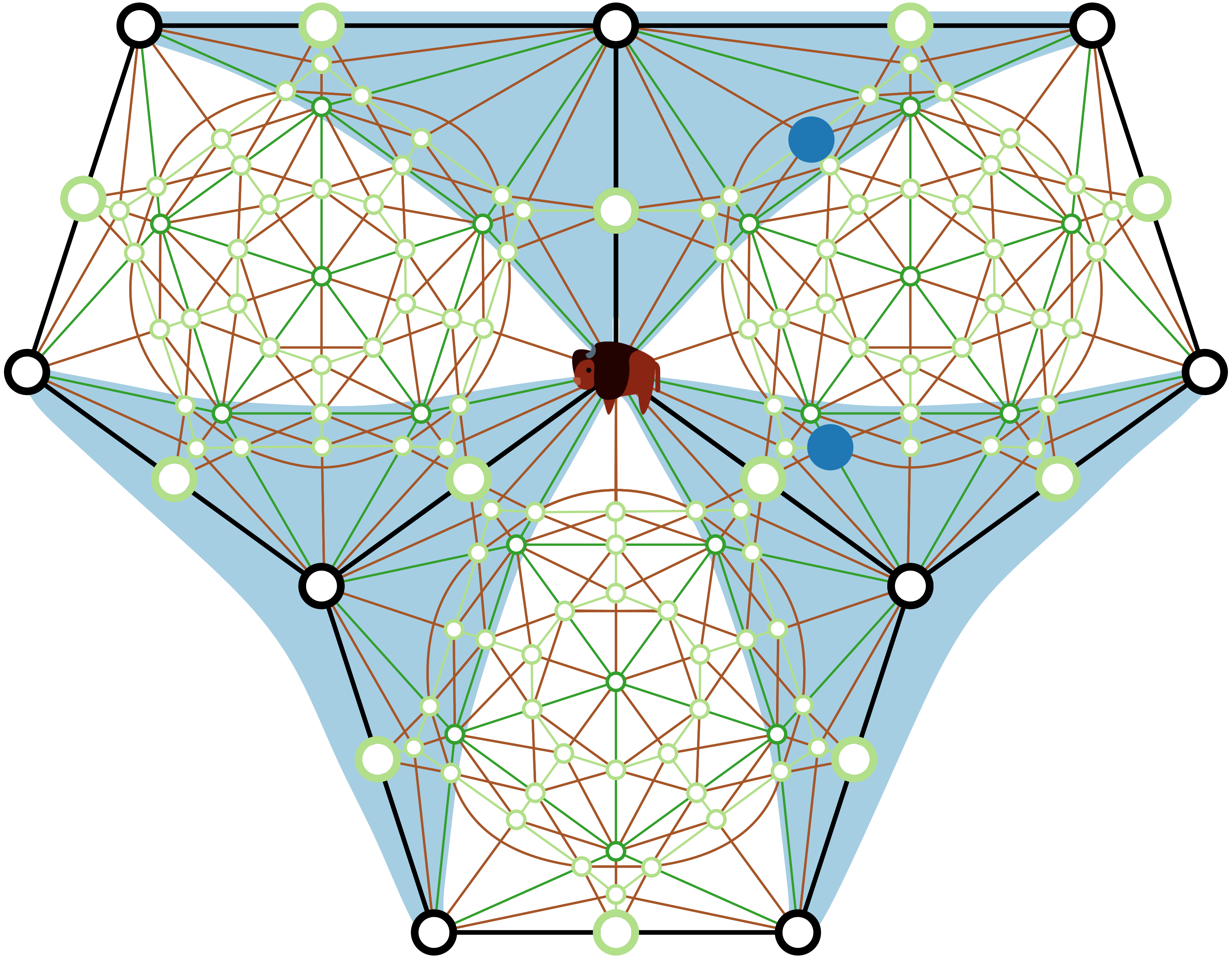

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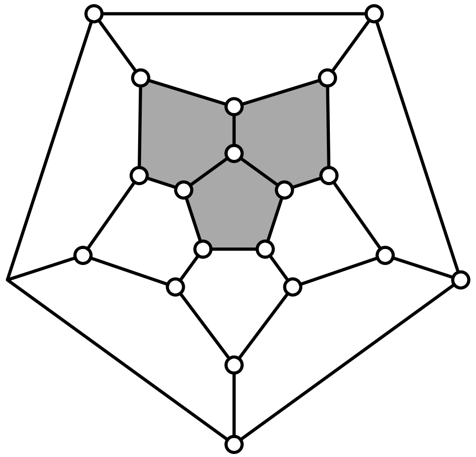

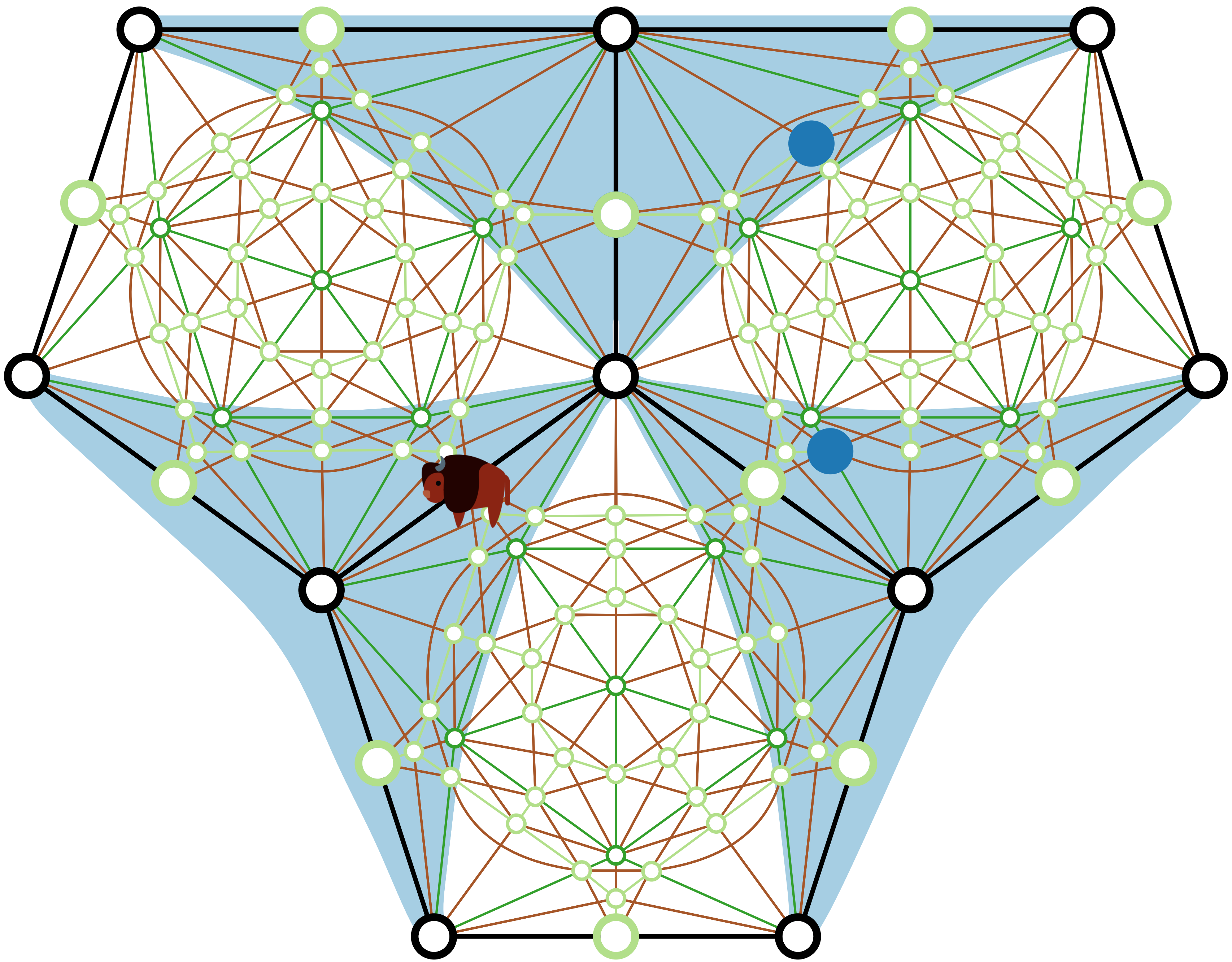

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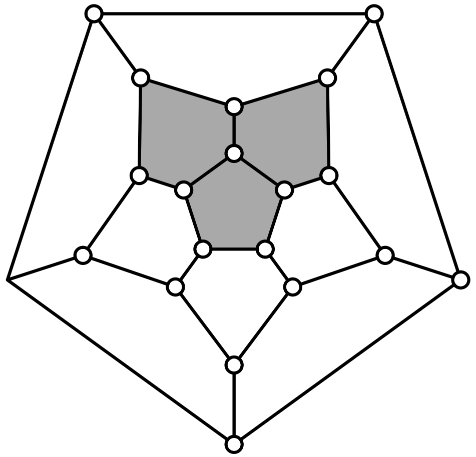

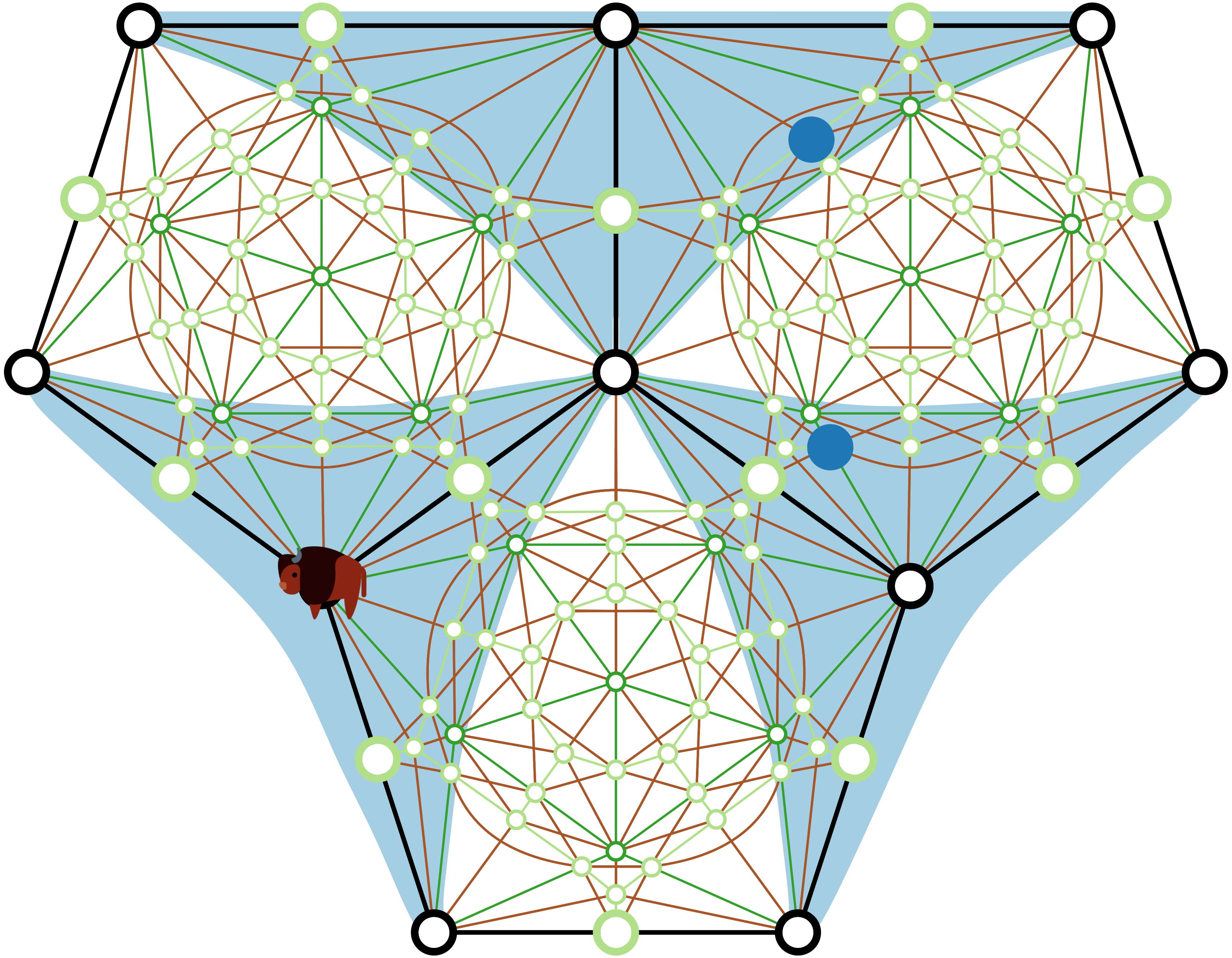

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