Fixed-Parameter Algorithms for Computing RAC Drawings of Graphs

Cornelius Brand · Robert Ganian · **Sebastian Röder** · **Florian Schager**

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Right-angle crossing (RAC) drawings

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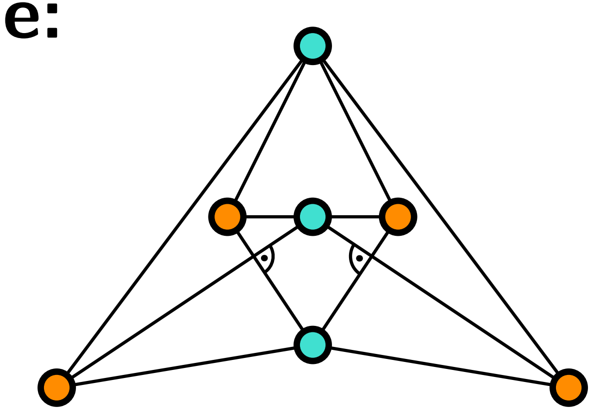

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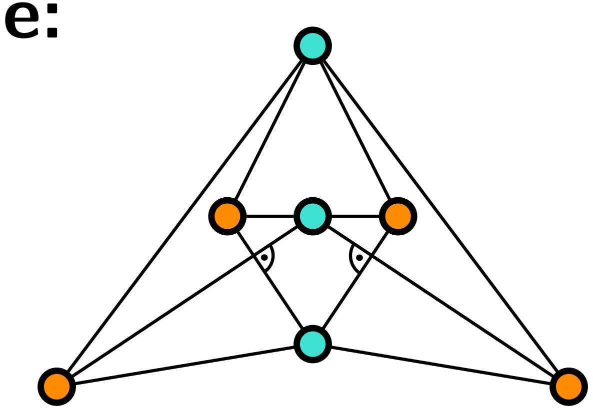

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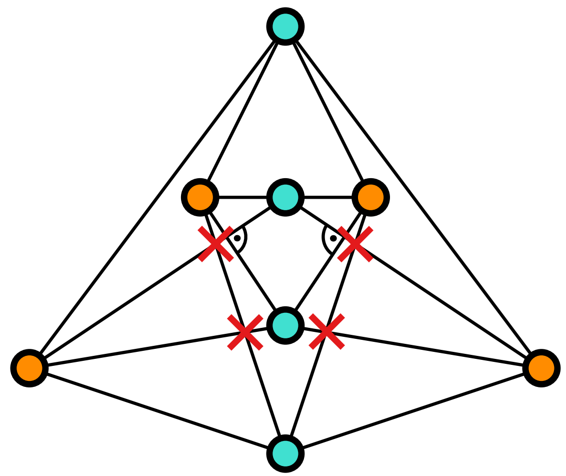

$K_{4,4}$ does not admit a RAC drawing

Generalized RAC drawings

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Question: Does G admit a straight-line RAC drawing?

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Theorem: Deciding whether a graph G admits a RAC drawing is NP-hard

[Argyriou et al. 2010]

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β-Bend RAC DRAWING

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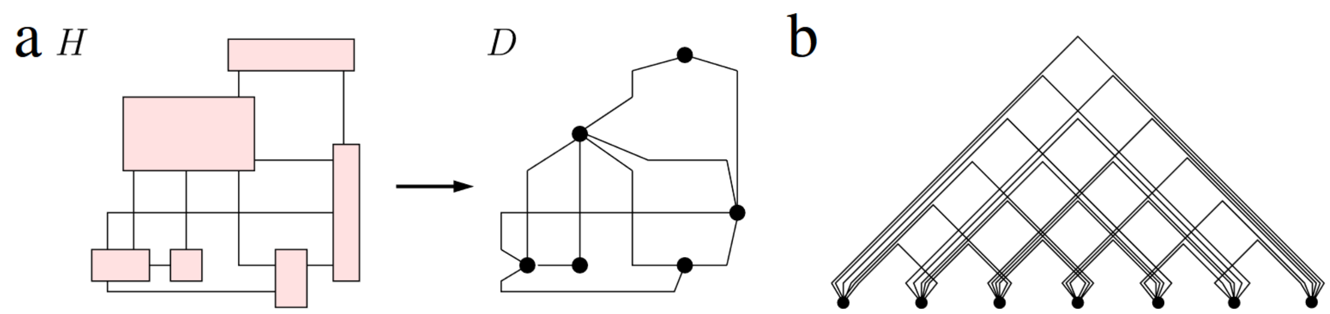

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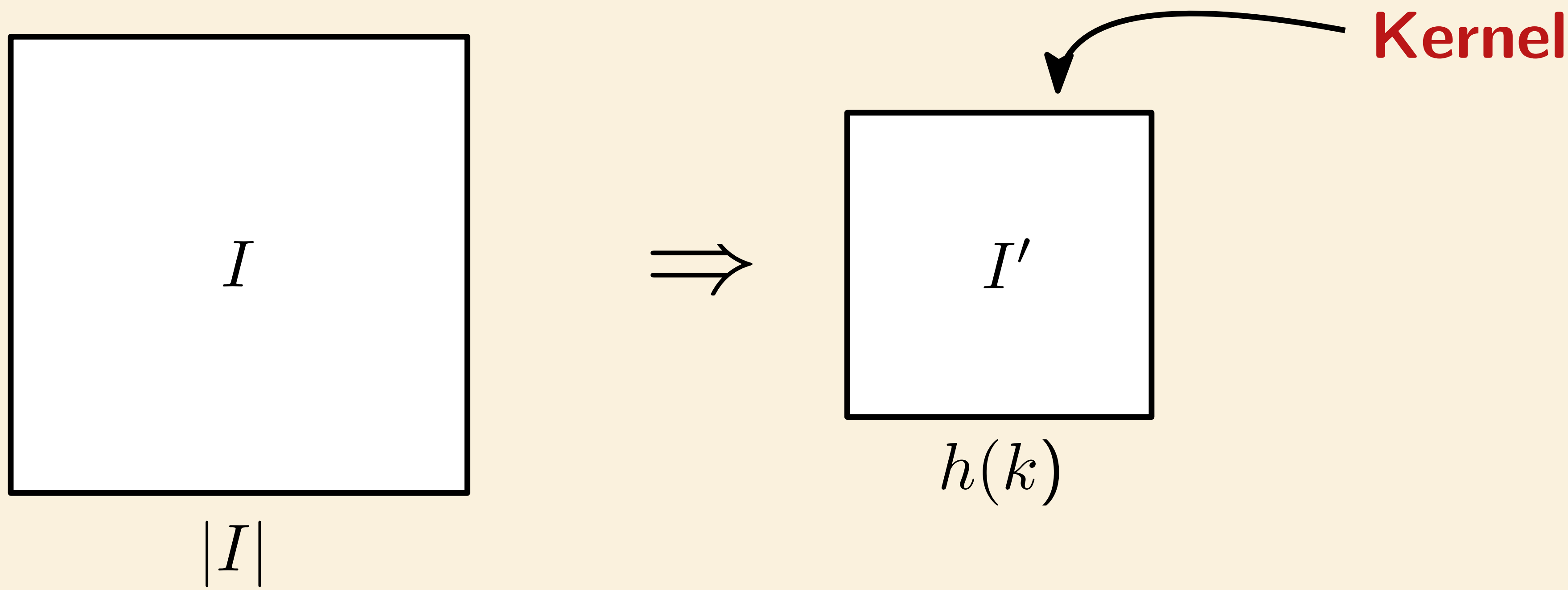
BEND-RESTRICTED RAC DRAWING (BRAC)

Instance: A graph G, an integer $b \geq 0$, edge labelling $\beta : E \mapsto \{0, 1, 2, 3\}$.

Question: Does G admit a RAC drawing, with

- at most b **total bends** and
- at most $\beta(e)$ bends for each edge e?

Side note: Kernelization



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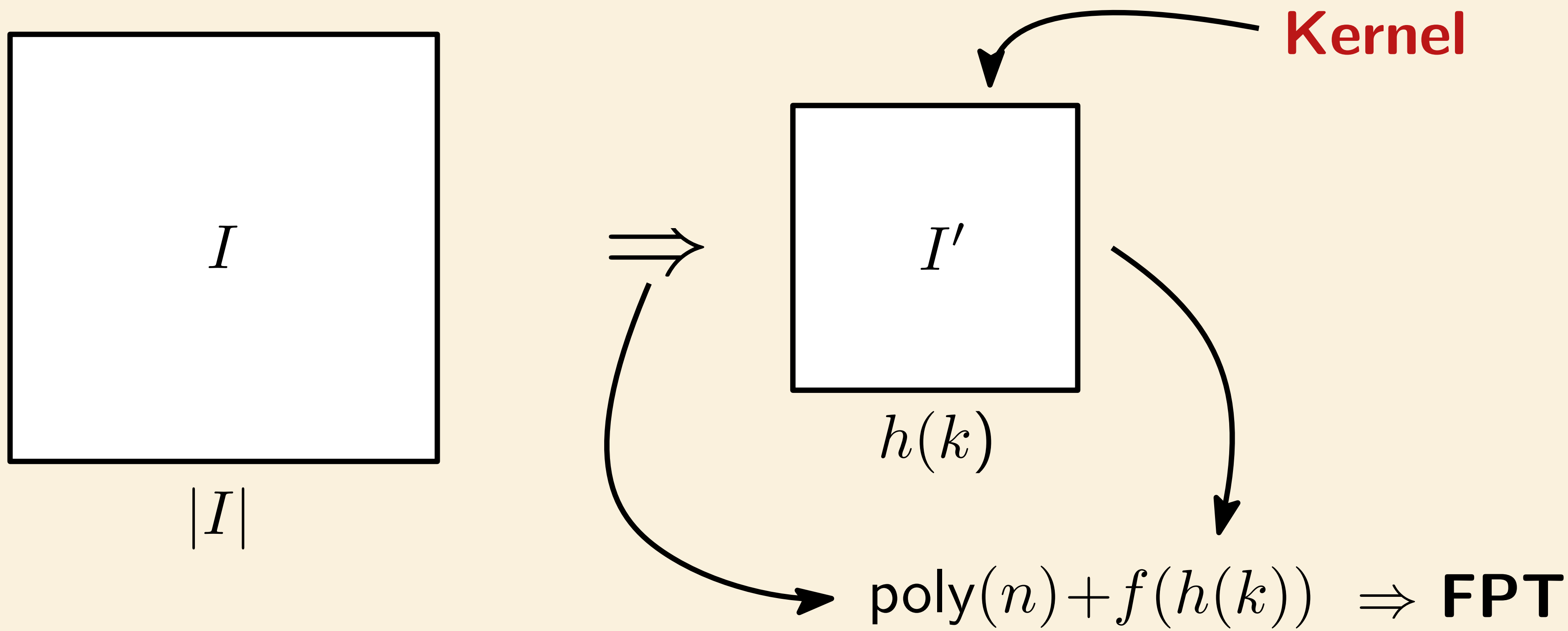

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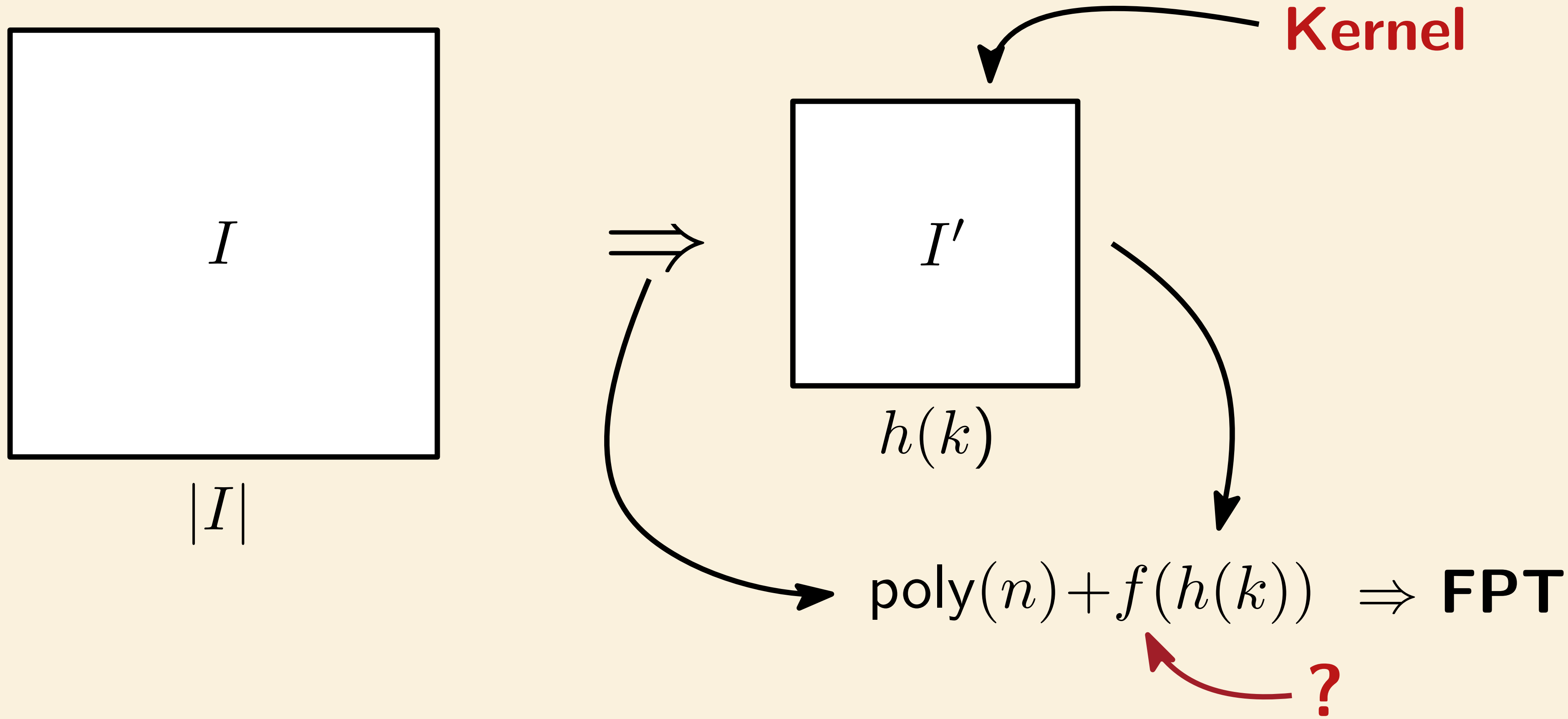

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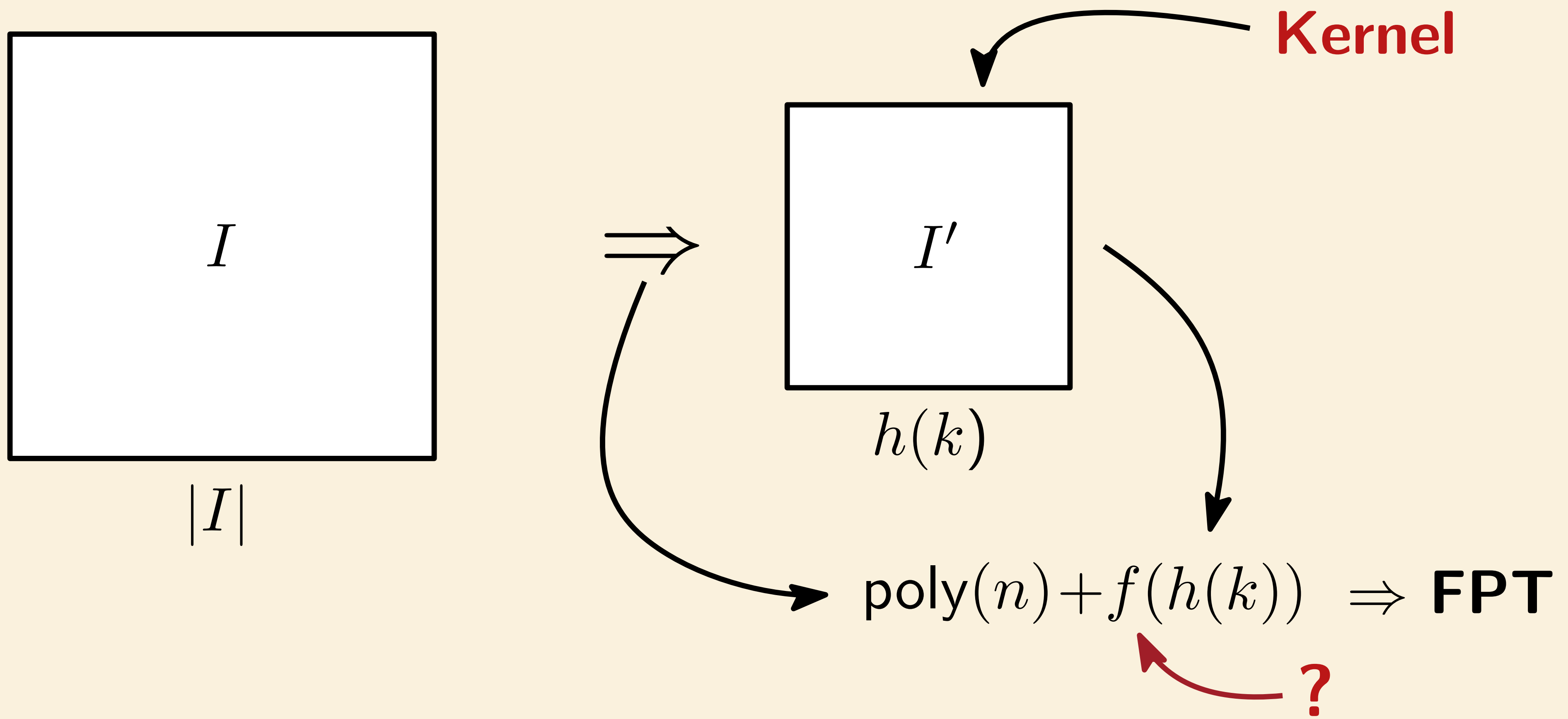
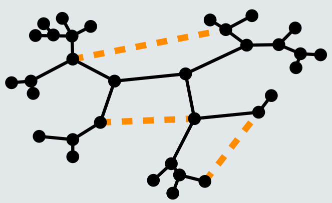
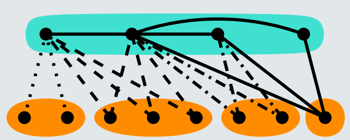
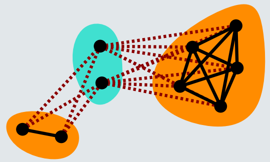


Theorem. An instance (G, b, β) of BRAC can be solved in time $m^{\mathcal{O}(m^2)}$.

Results

- b-BEND β-RESTRICTED RAC DRAWING (BRAC) is *fixed-parameter tractable* when parameterized by

	Feedback edge number $\mathbf{fen}(G)$	$2^{\mathbf{fen}(G)^{\mathcal{O}(\mathbf{fen}(G))}} + \mathcal{O}(\lvert E(G)\rvert)$
	Vertex cover number $\mathbf{vcn}(G) + b$	$2^{2^{\mathcal{O}(\mathbf{vcn}(G)+\log b)}} + \mathcal{O}(\lvert E(G)\rvert)$
	Neighborhood diversity $\mathbf{nd}(G) + b$	$2^{b^{\mathcal{O}(\mathbf{nd}(G))}} + \mathcal{O}(\lvert E(G)\rvert)$

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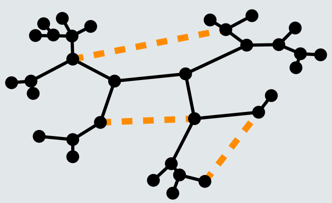
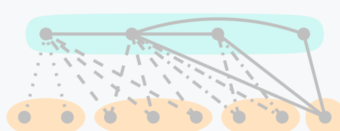
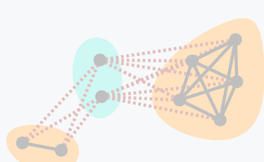
	Feedback edge number $\mathbf{fen}(G)$	$2^{\mathbf{fen}(G)^{\mathcal{O}(\mathbf{fen}(G))}} + \mathcal{O}(\lvert E(G)\rvert)$
	Vertex cover number $\mathbf{vcn}(G) + b$	$2^{2^{\mathcal{O}(\mathbf{vcn}(G)+\log b)}} + \mathcal{O}(\lvert E(G)\rvert)$
	Neighborhood diversity $\mathbf{nd}(G) + b$	$2^{b^{\mathcal{O}(\mathbf{nd}(G))}} + \mathcal{O}(\lvert E(G)\rvert)$

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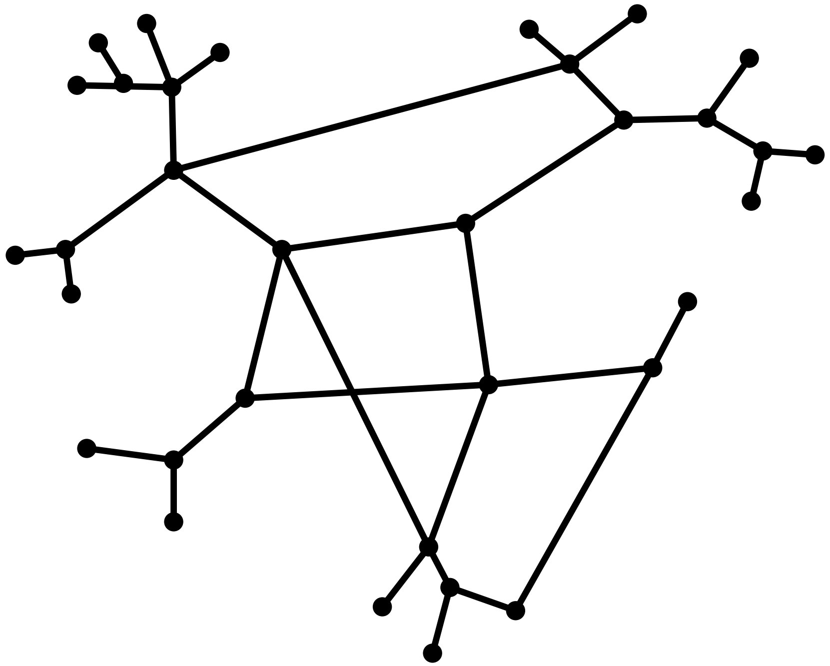

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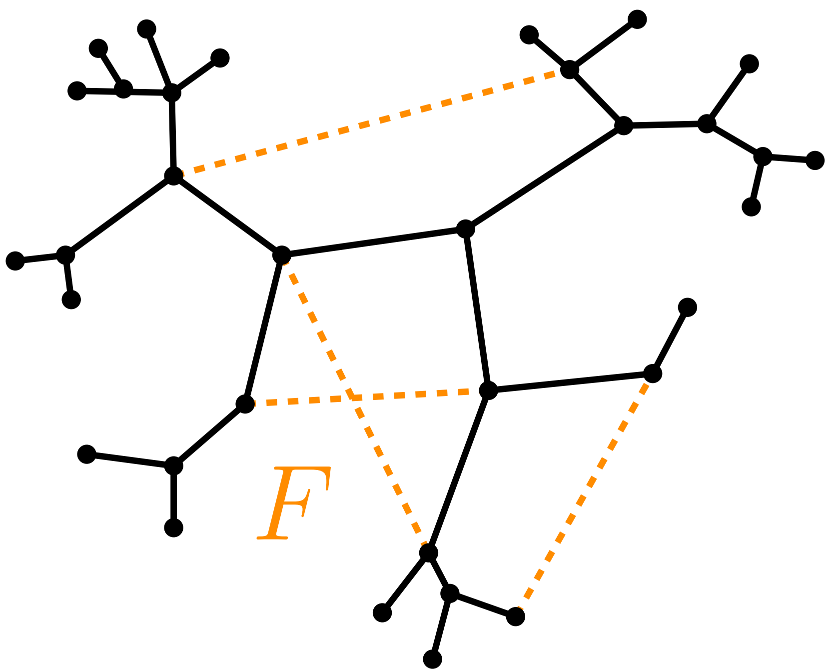

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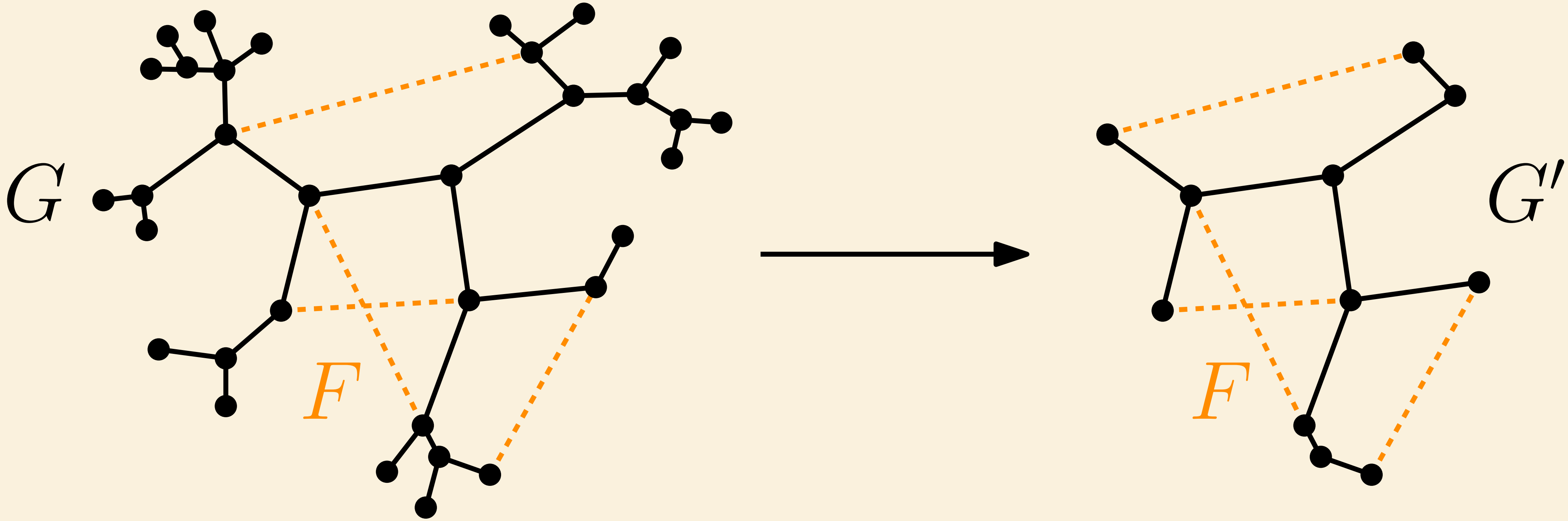

Now $G' - F$ is a tree with at most $2 \cdot \mathbf{fen}(G)$ leaves

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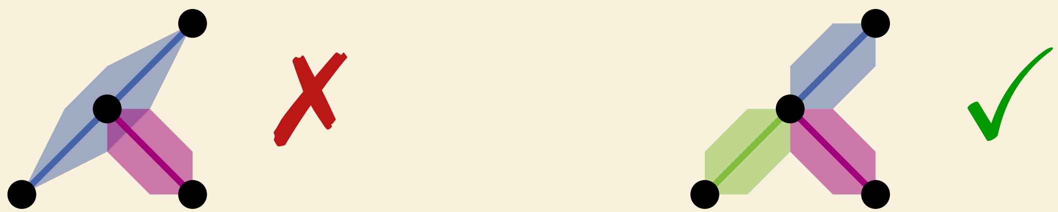
We allow the subpaths only to intersect at their respective endpoints



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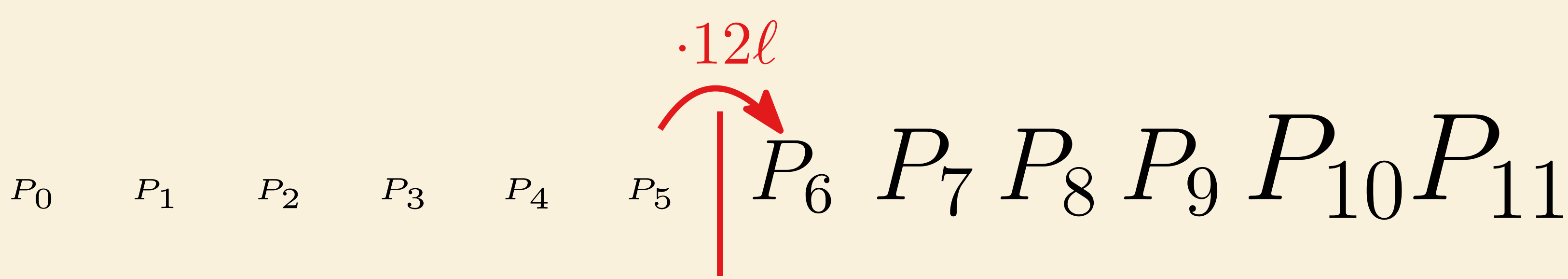

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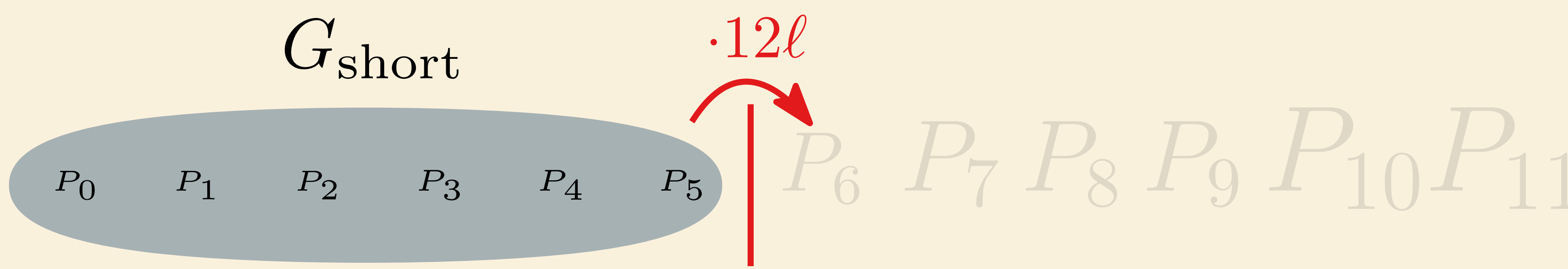

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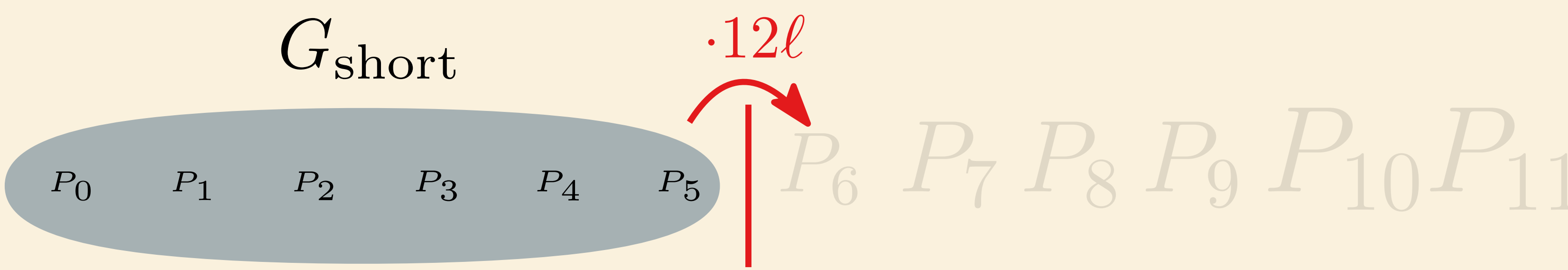

kernel of size $\mathbf{fen}(G)^{\mathcal{O}(\mathbf{fen}(G))}$

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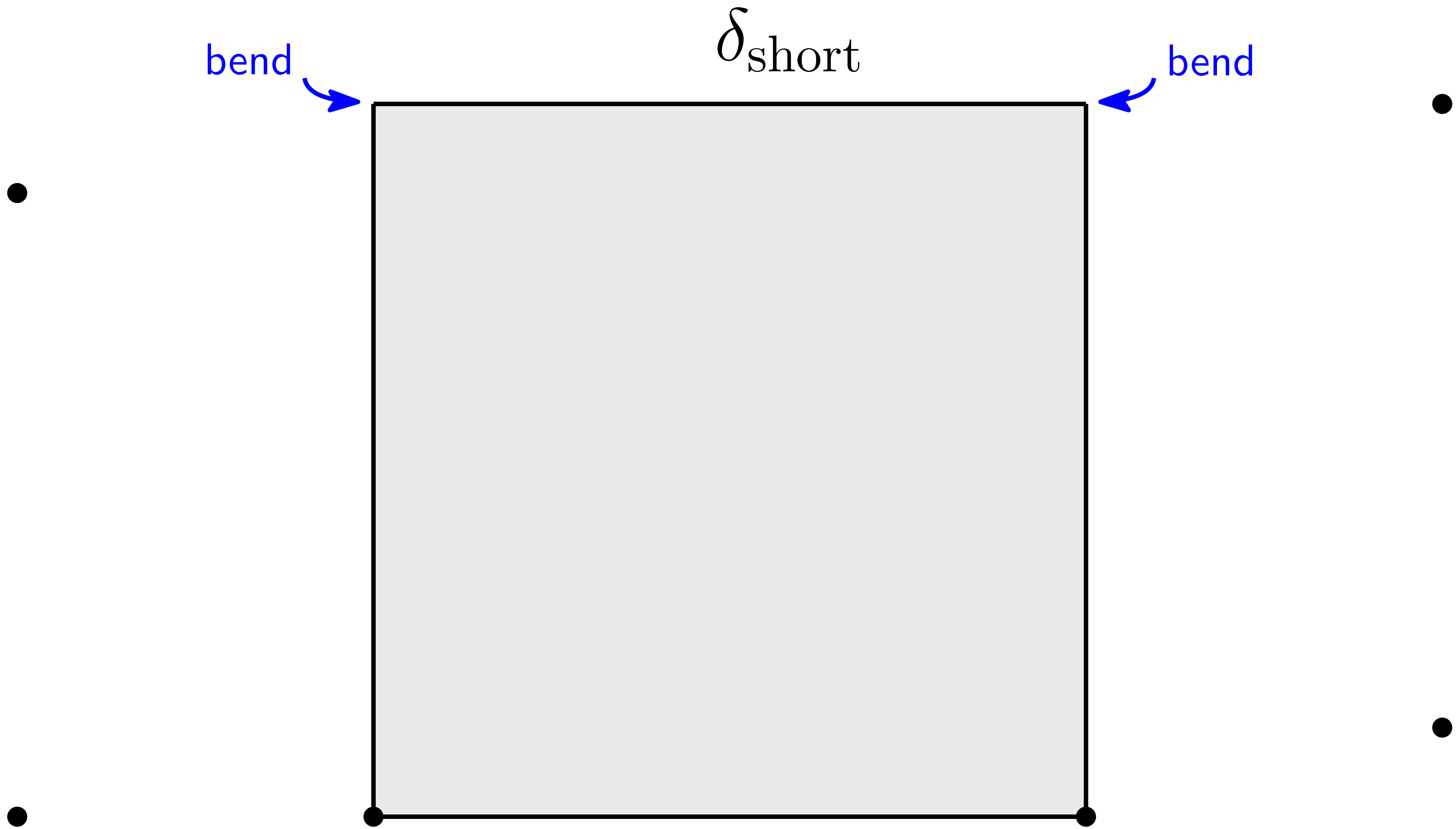

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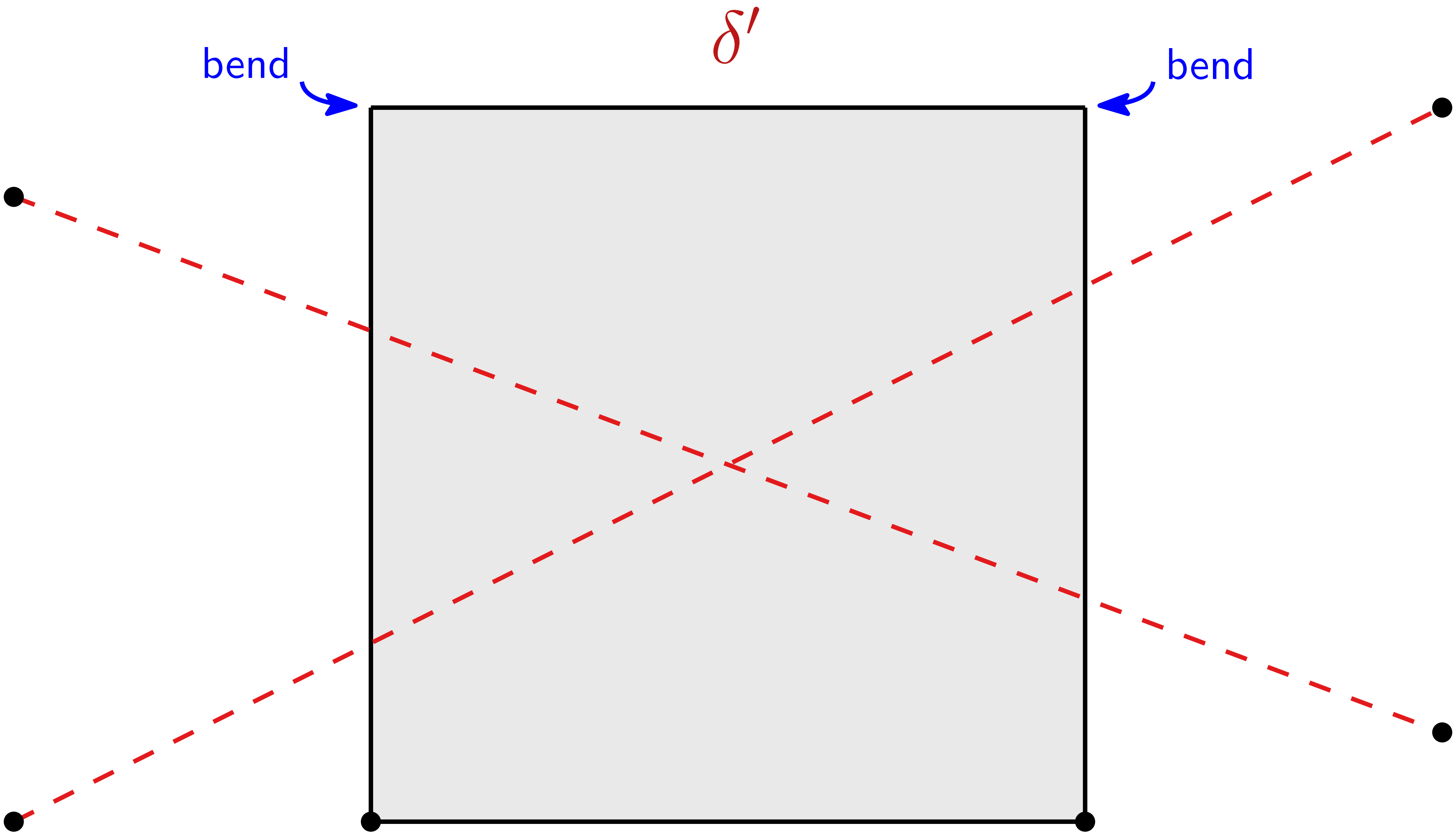

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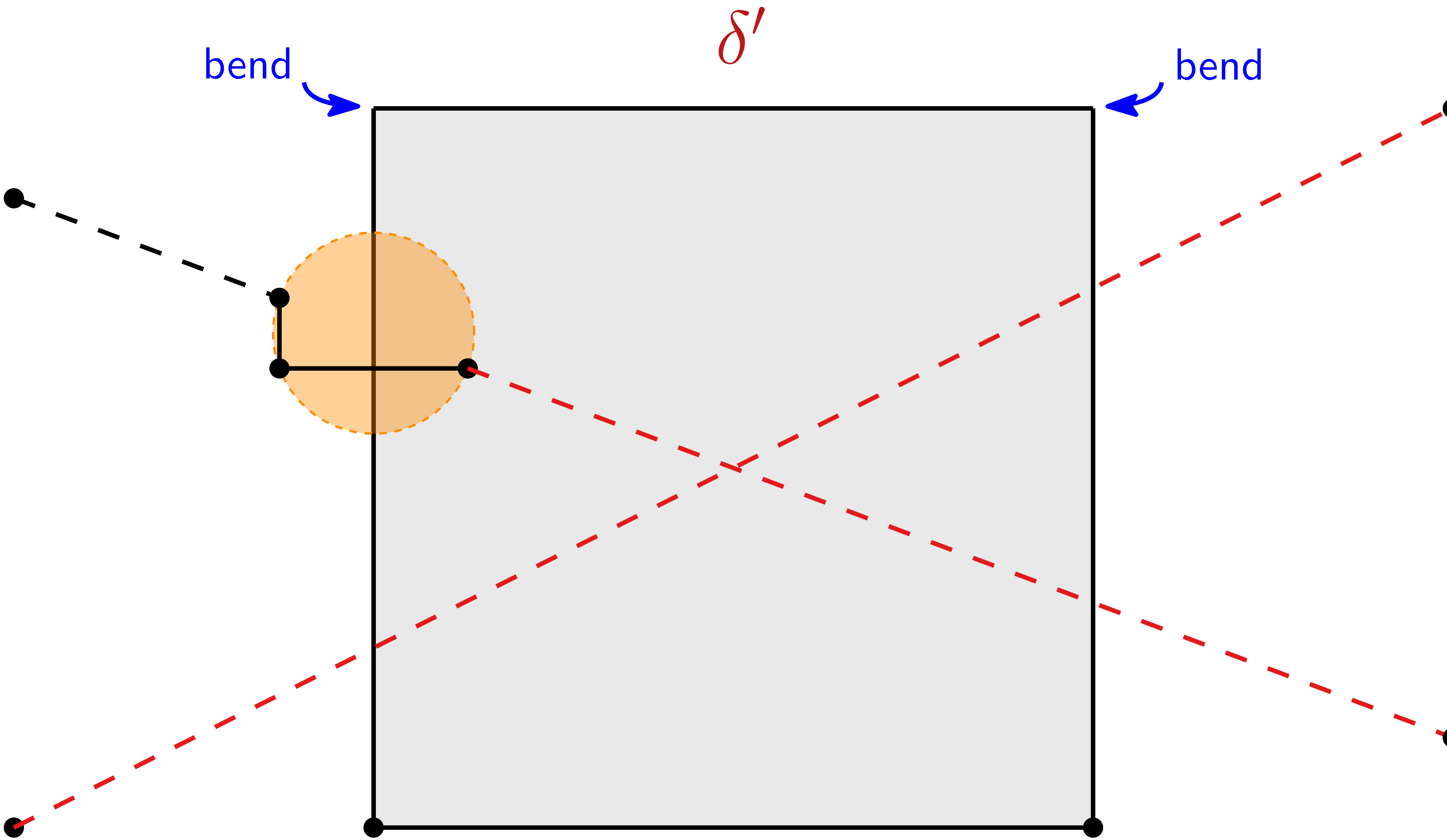

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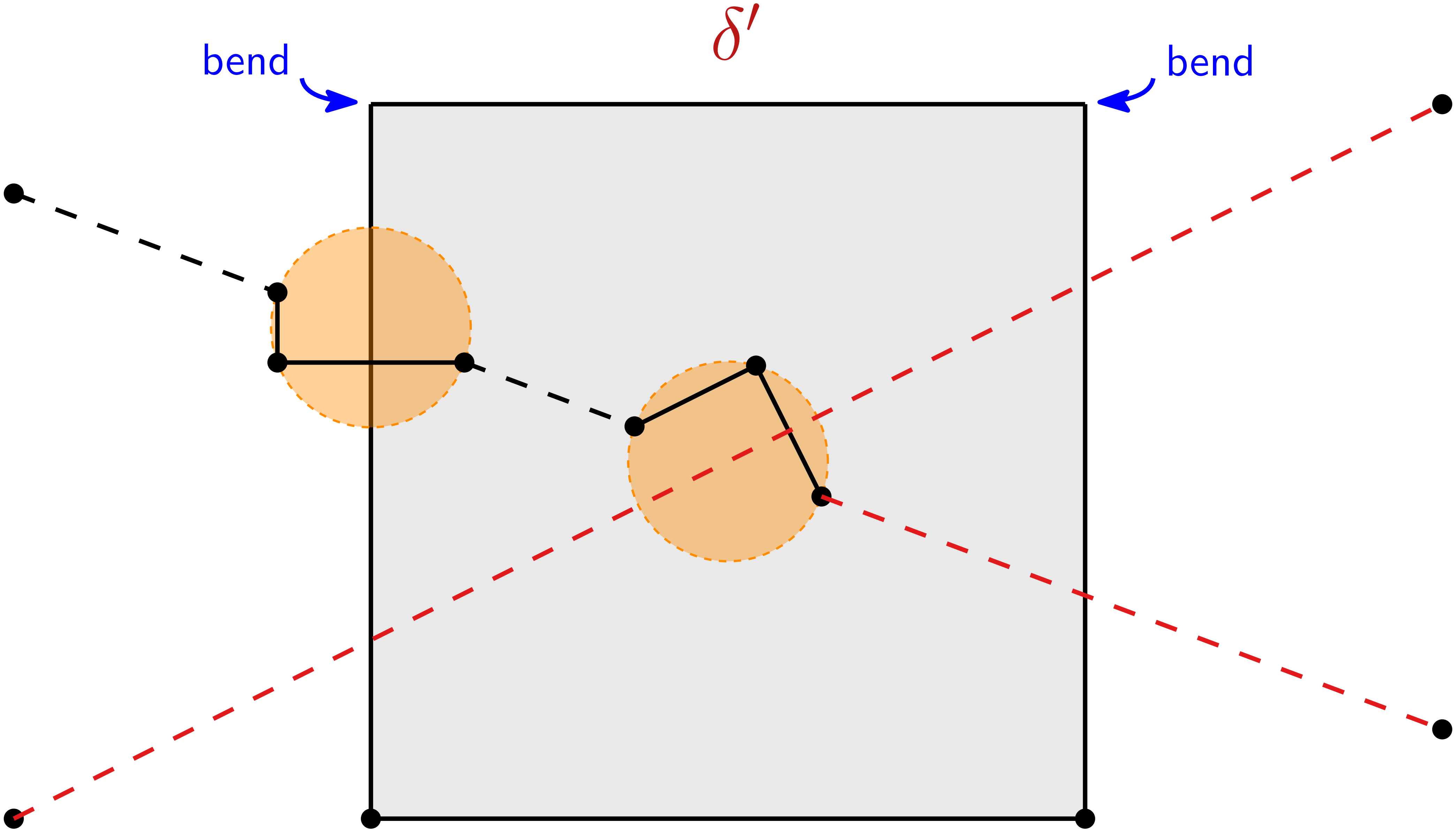

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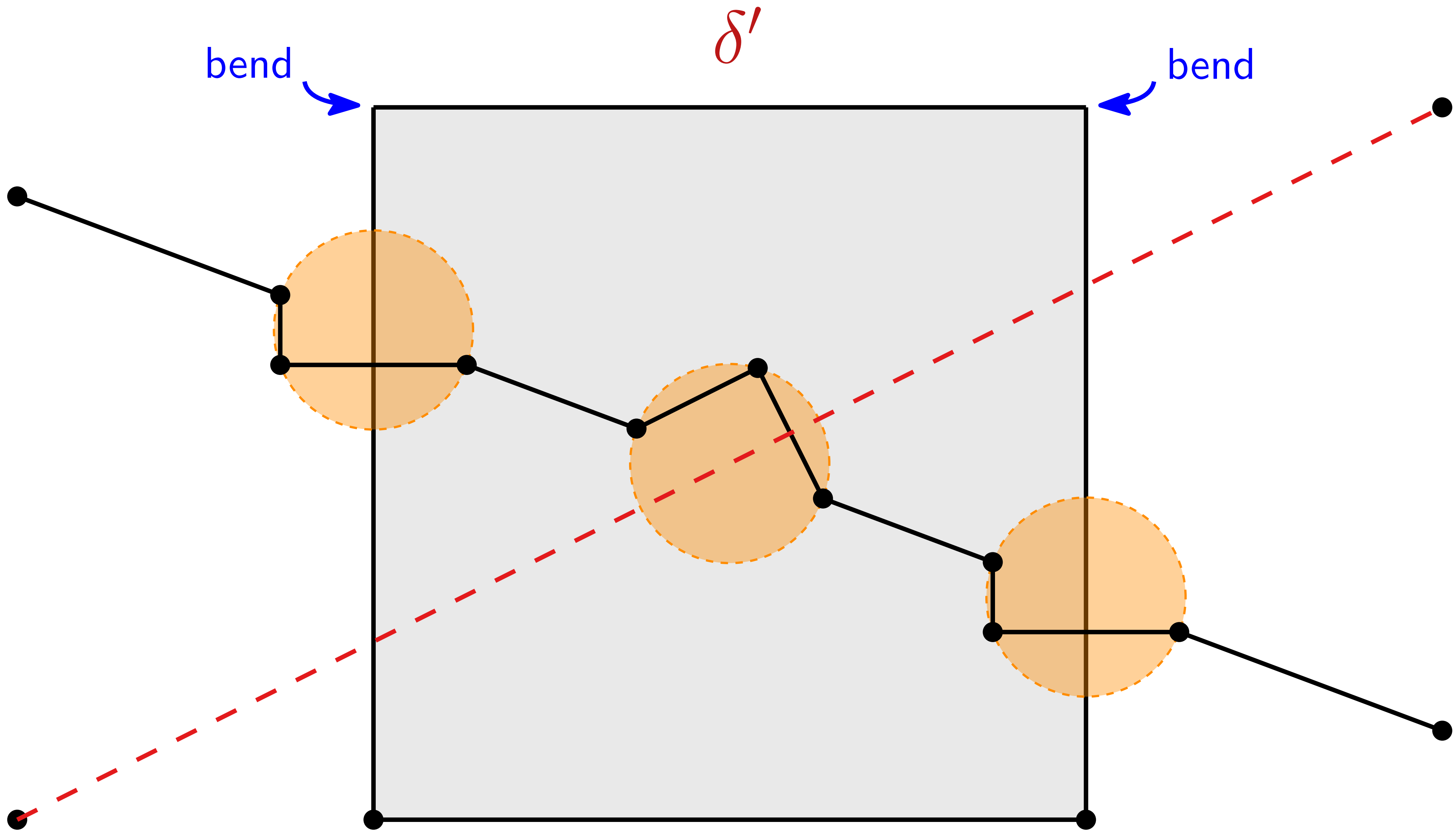

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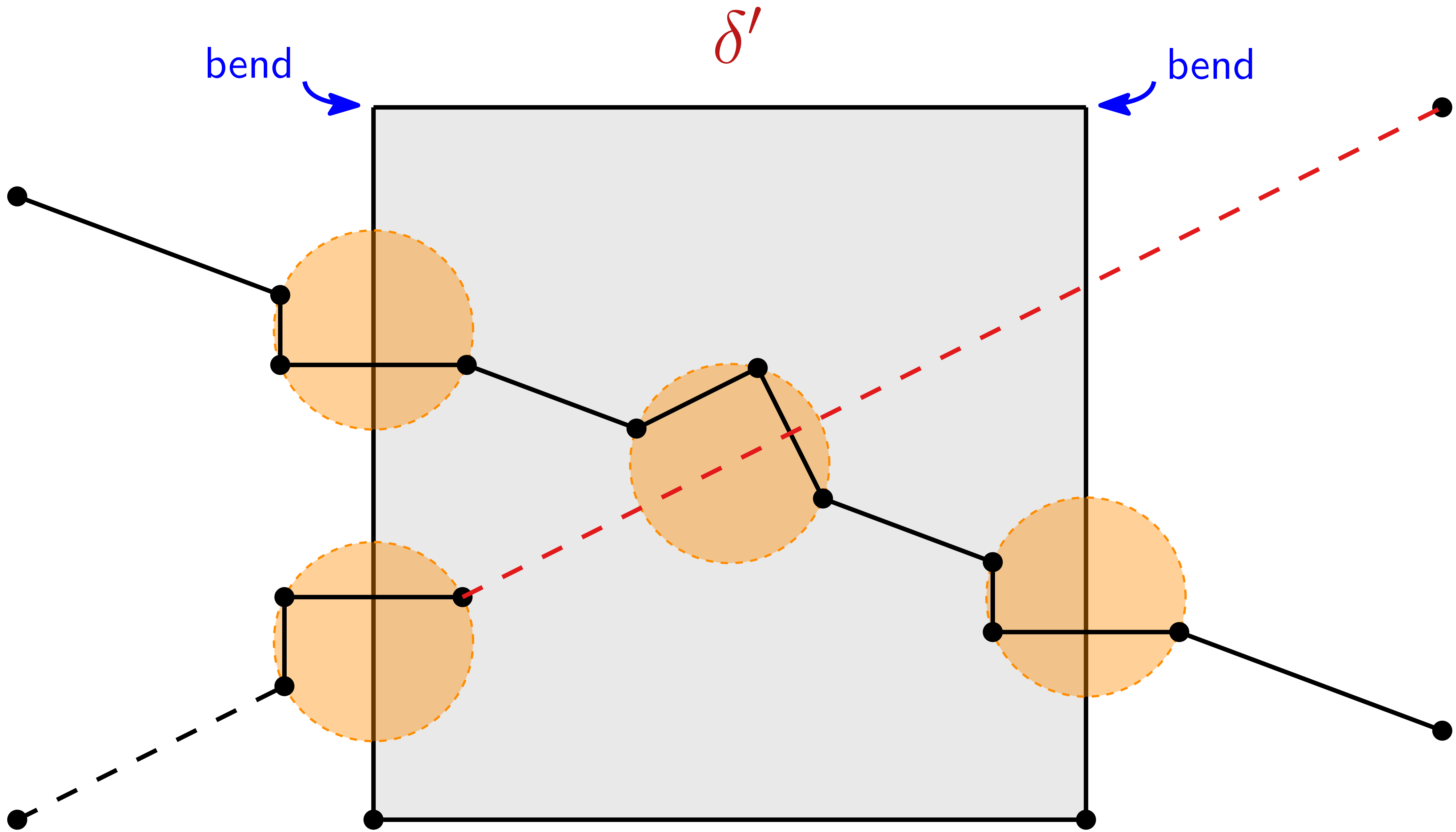

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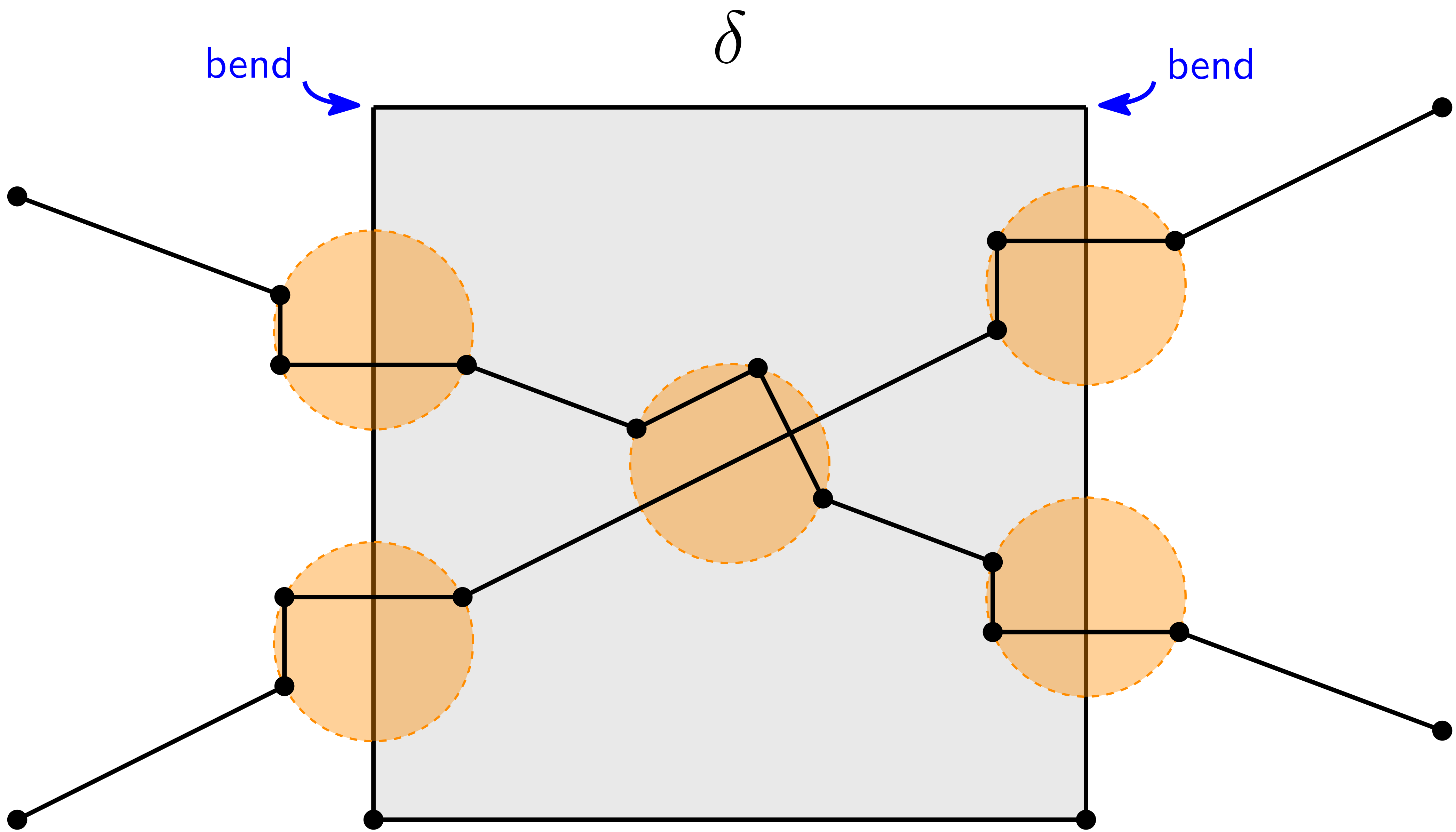

Kernel size

Let i_0 be the smallest i such that $p_{i_0+1} > 12\ell \cdot p_{i_0}$.

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  - Each edge in G_{short} has at most 3 bends
  - At most $4\sum_{i=0}^{i_0} p_i$ crossings involving edges from G_{short}

  Total: $4\sum_{i=0}^{i_0} p_i + (\ell - i_0) \leq 4\ell \cdot p_{i_0}$ crossings

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Solve time $m^{\mathcal{O}(m^2)}$

Kernel size $\mathbf{fen}(G)^{\mathcal{O}(\mathbf{fen}(G))}$

Kernel time $\mathcal{O}(|E(G)|)$

Total $2^{\mathbf{fen}(G)^{\mathcal{O}(\mathbf{fen}(G))}} + \mathcal{O}(|E(G)|)$

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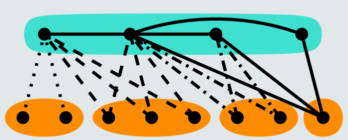

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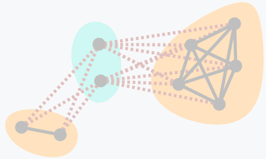

Neighborhood diversity $\mathbf{nd}(G) + b$ — $2^{b^{\mathcal{O}(\mathbf{nd}(G))}} + \mathcal{O}(|E(G)|)$

Vertex Cover

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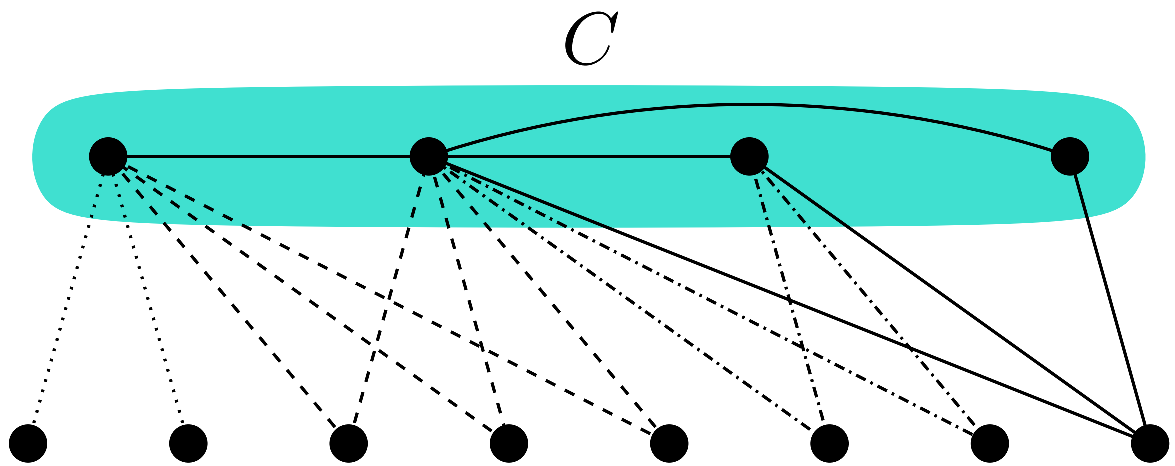

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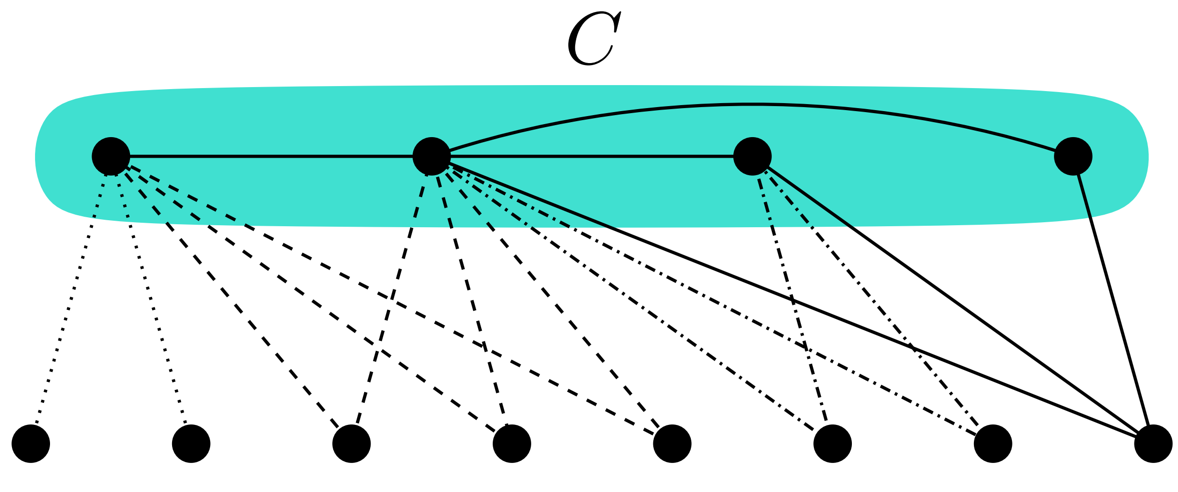

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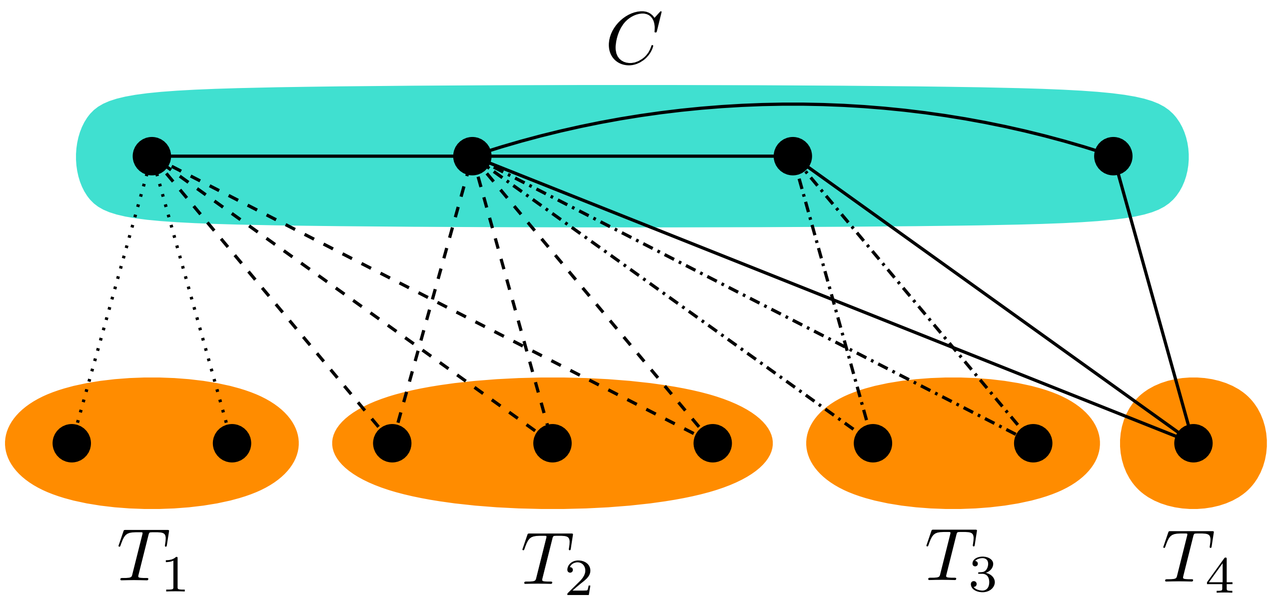

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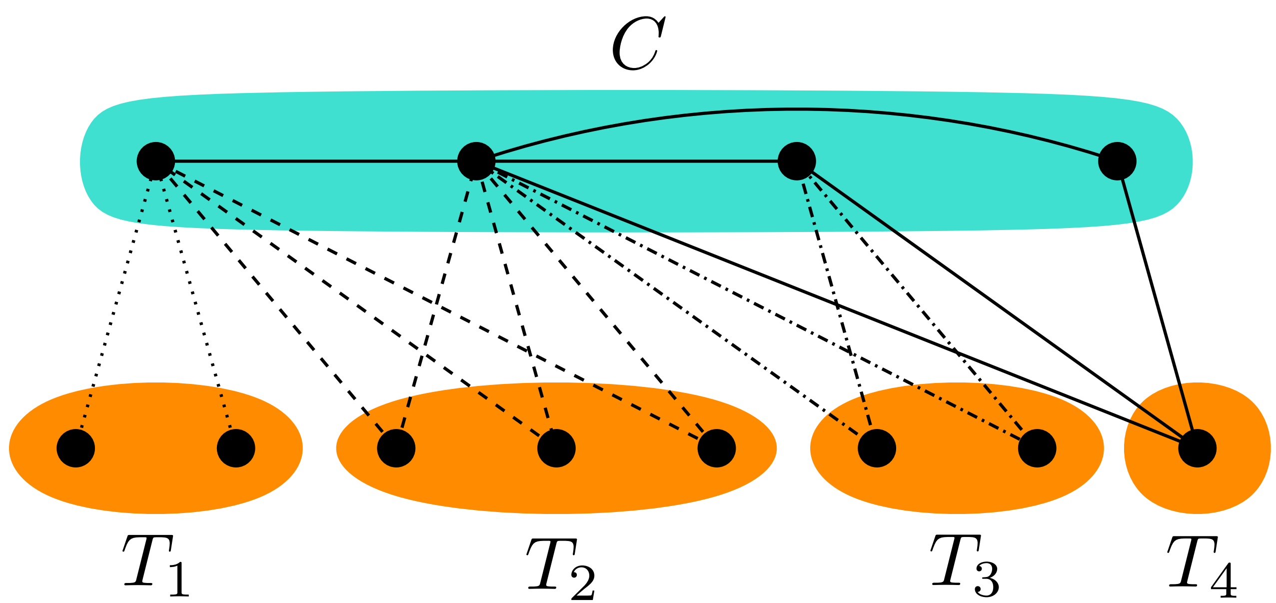

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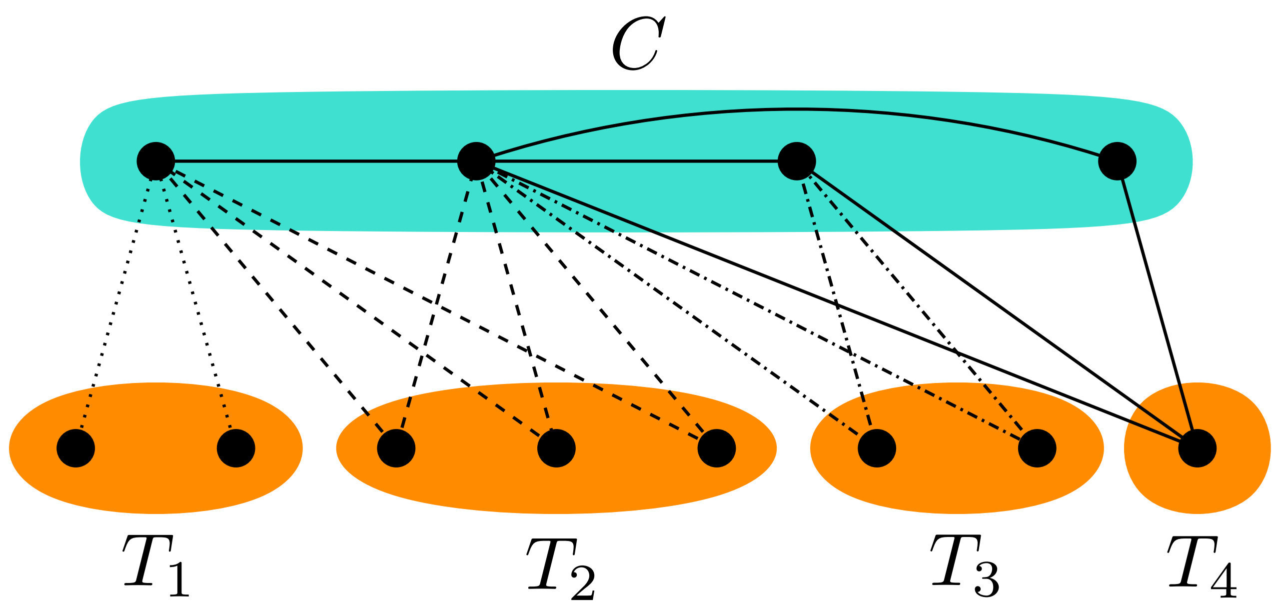

- $k = \mathbf{vcn}(G) = |C|$
- **Type**: Set of vertices in $G - C$ with the same neighbourhood
- At most 2^k types
- **Idea:** limit the number of vertices in each type
  by a function of $\mathbf{vcn}(G) + b$

FTP via $\mathbf{vcn}(G) + b$

- Distinguish types by number of neighbours $|T_i|$

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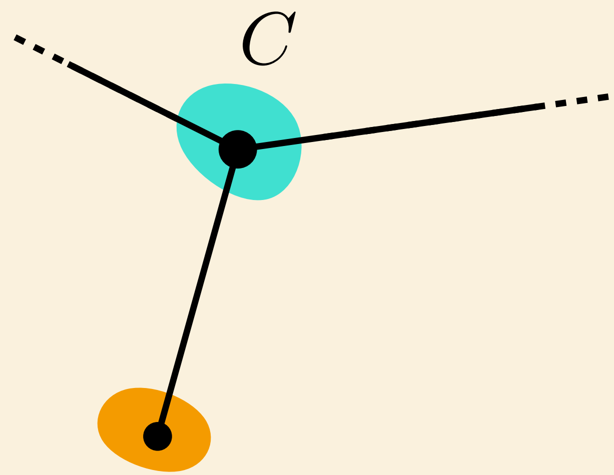

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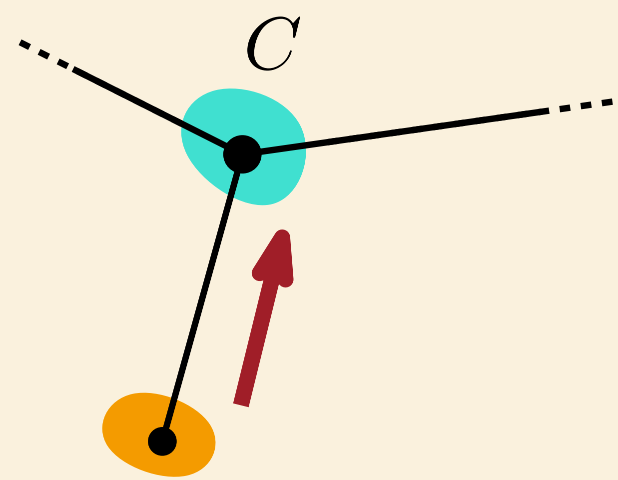

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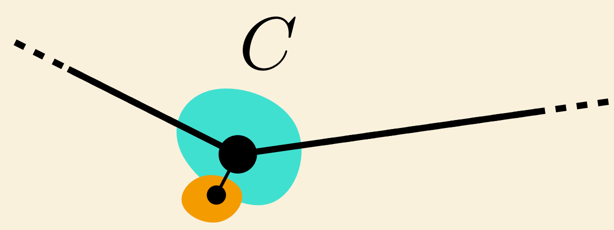

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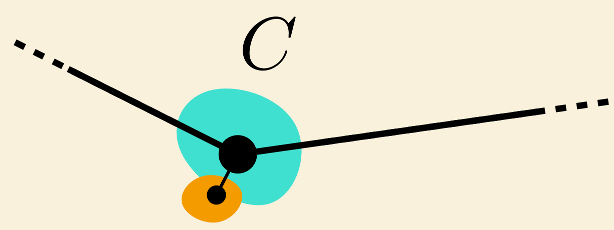

$\Rightarrow$ can remove all types with $|T_i| = 1$

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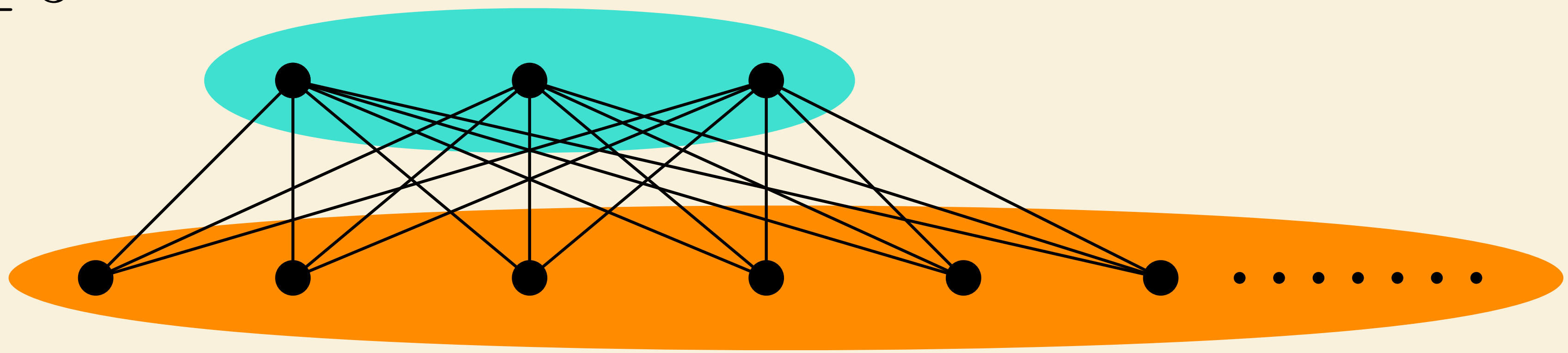

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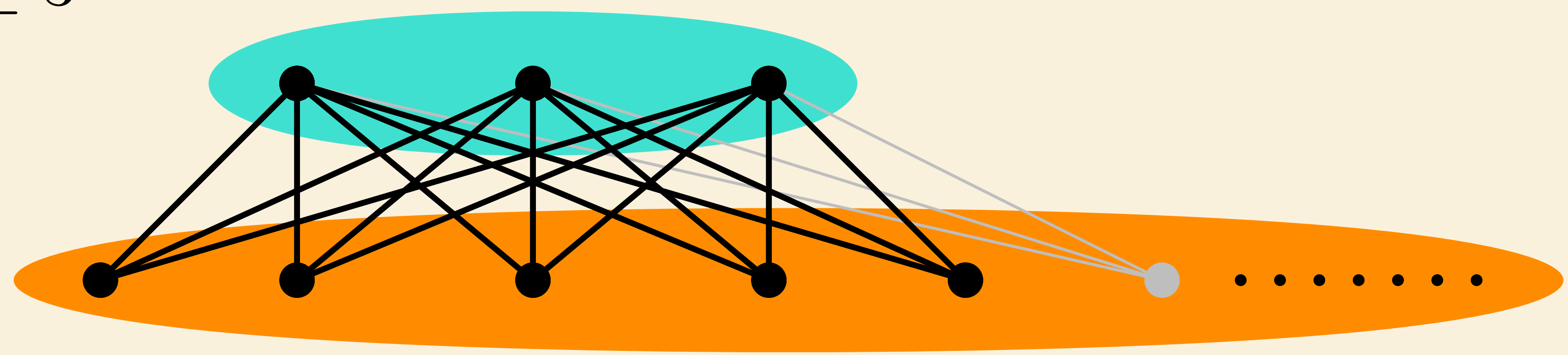

$K_{3,5}$

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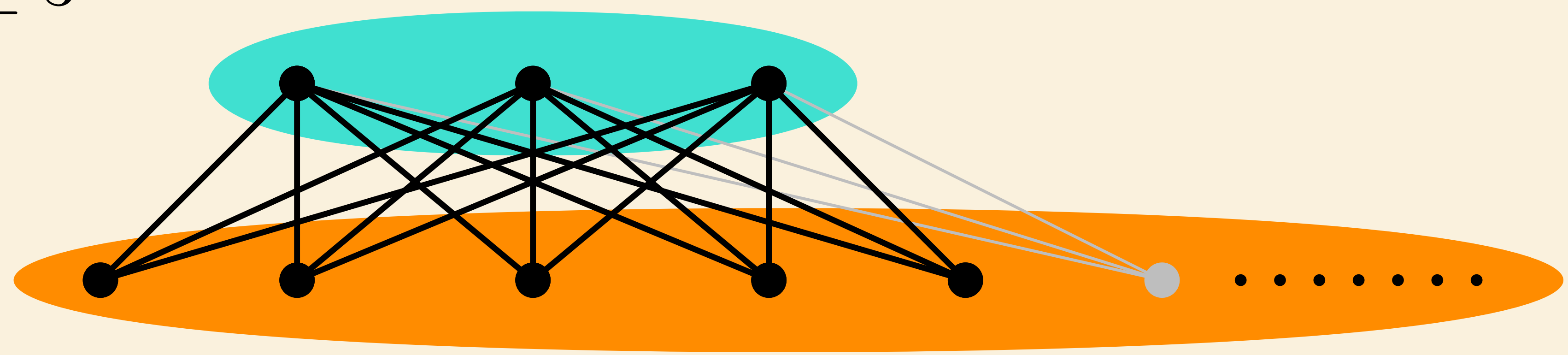

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Lemma. $K_{3,5}$ does **not** admit a RAC drawing [Didimo, Eades, Liotta 2010]

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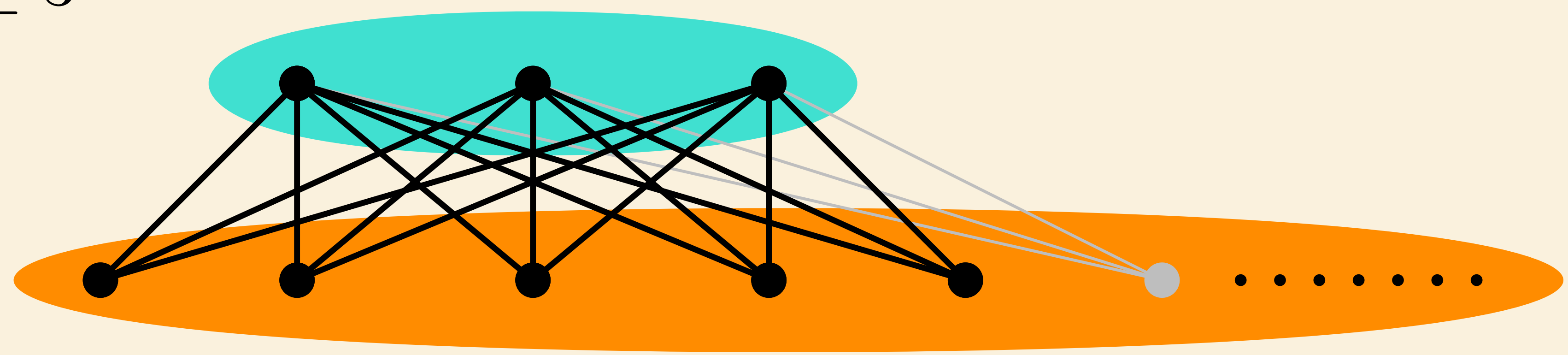

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$\Rightarrow$ always no-instance with $\geq 5 + b$ vertices

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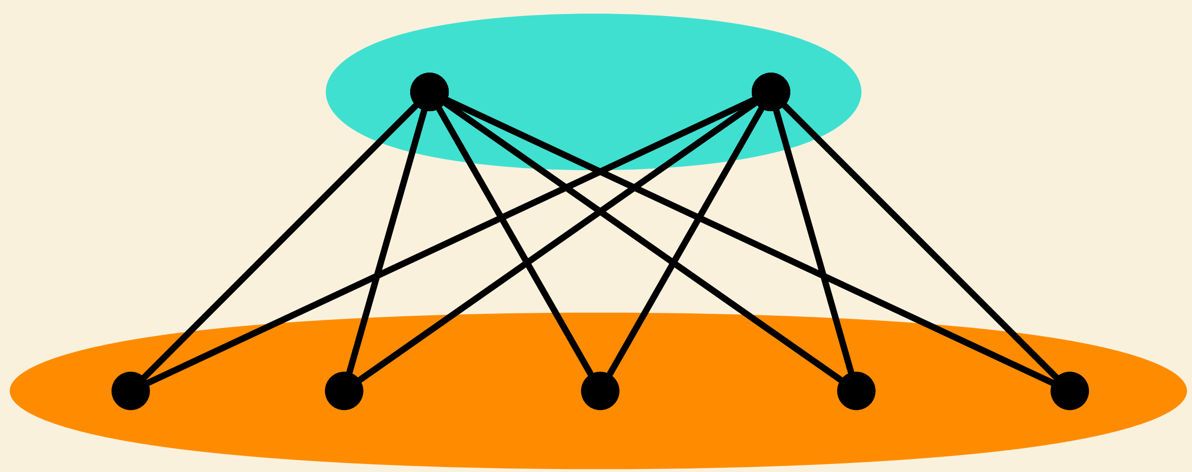

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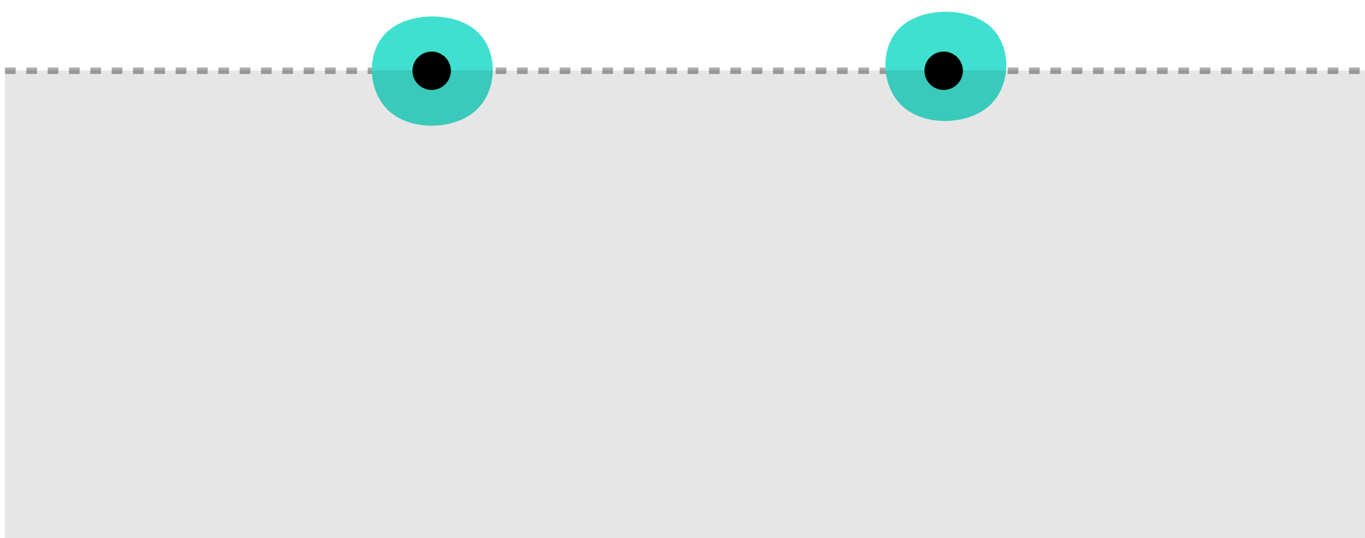

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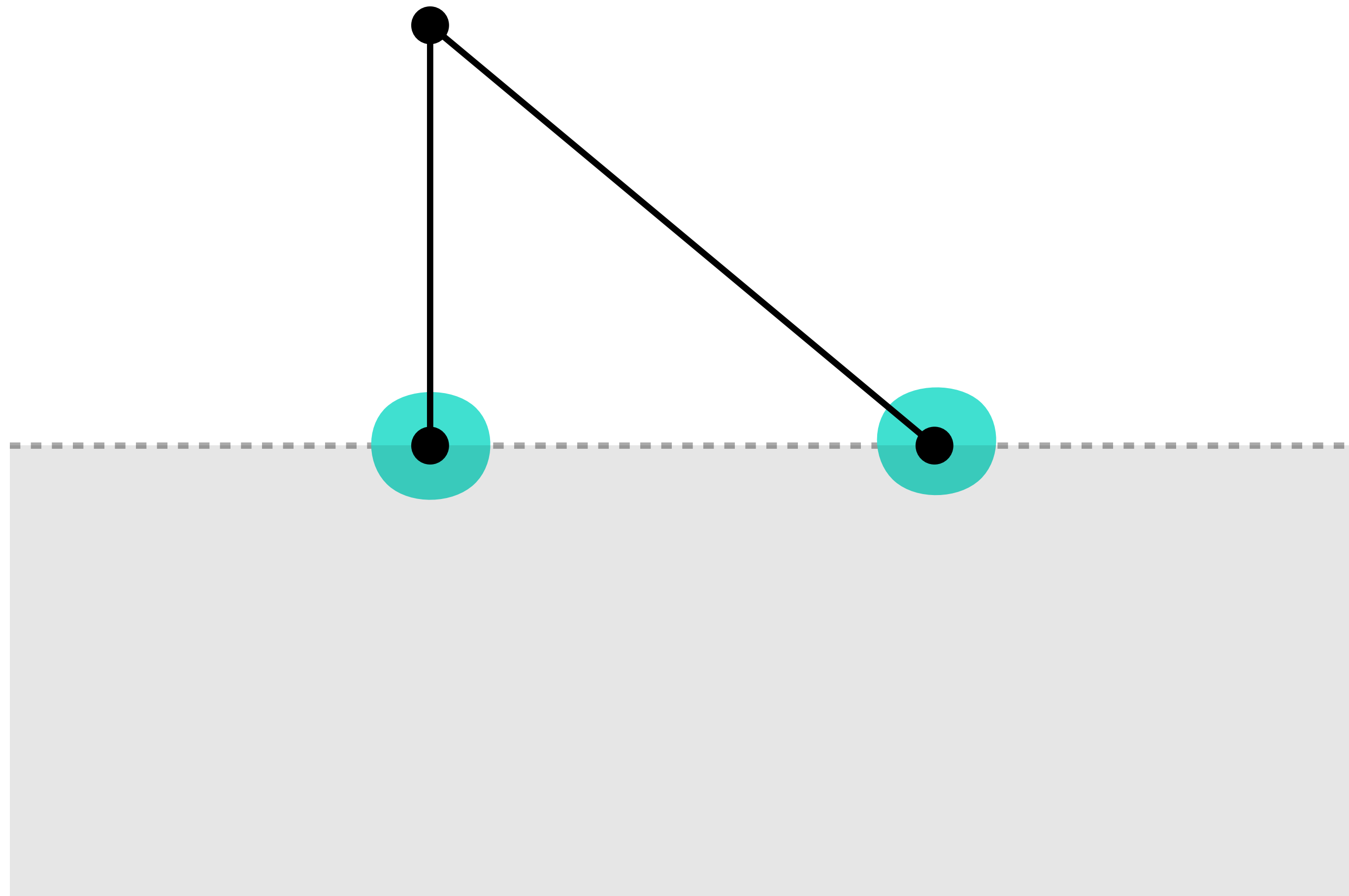

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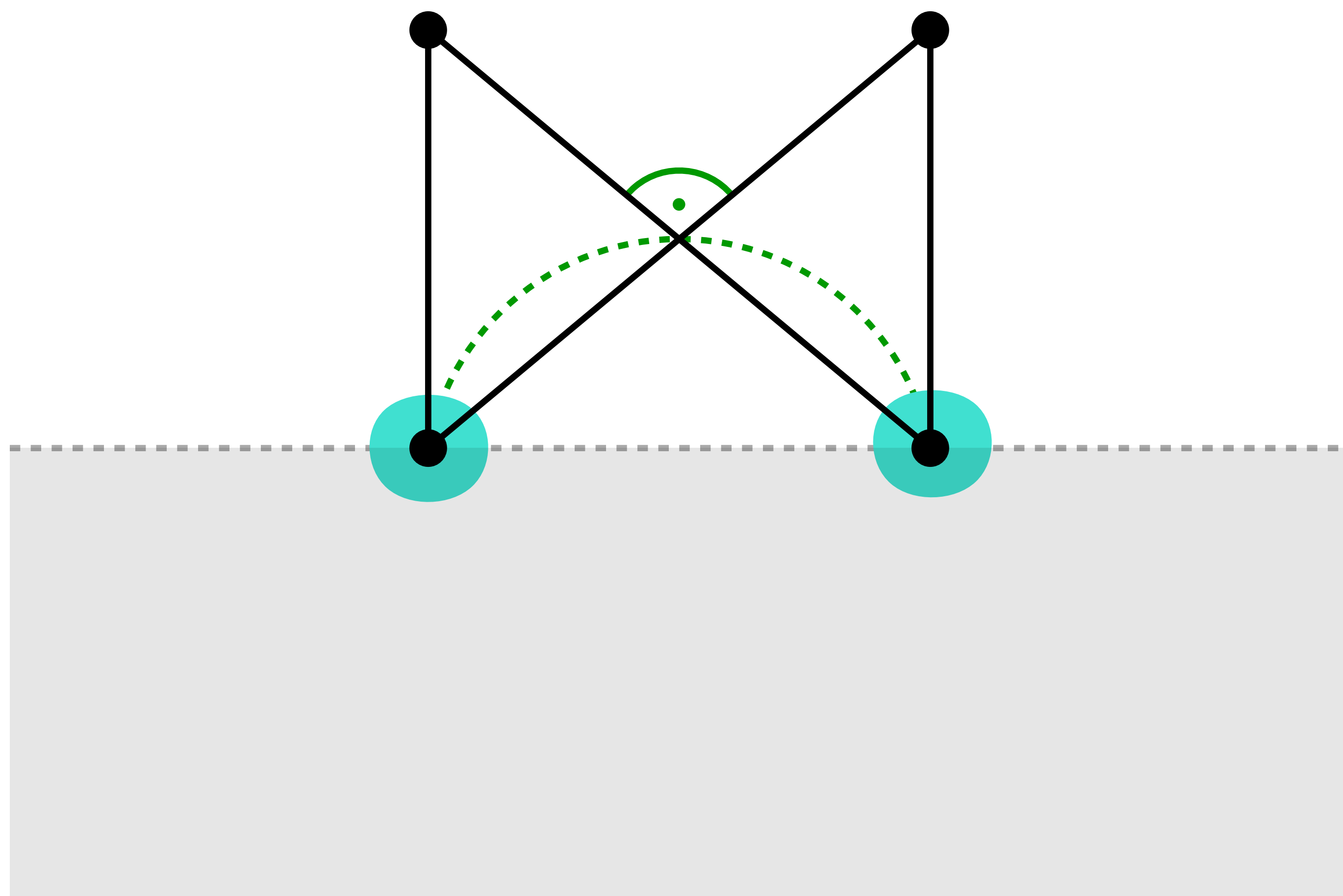

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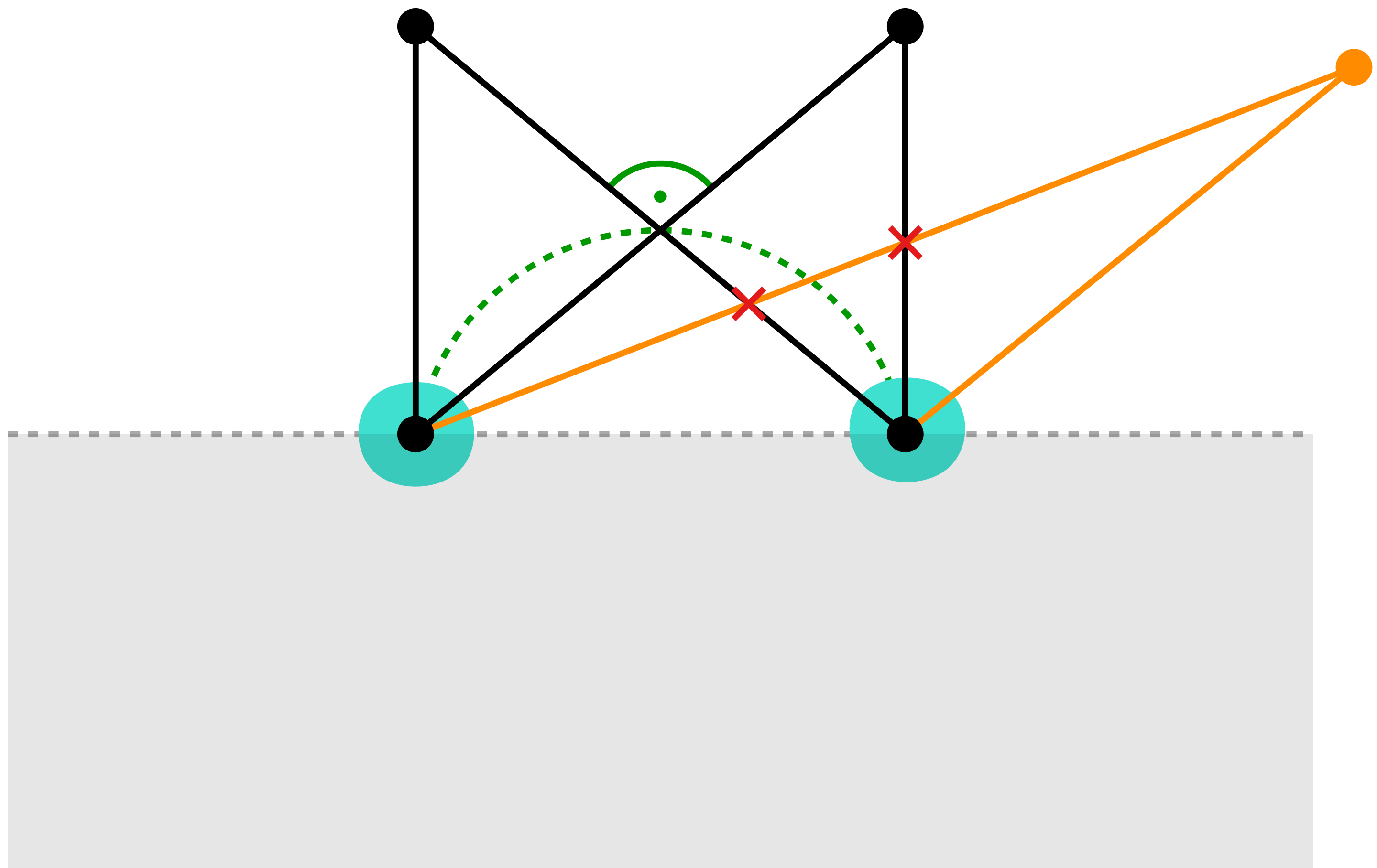

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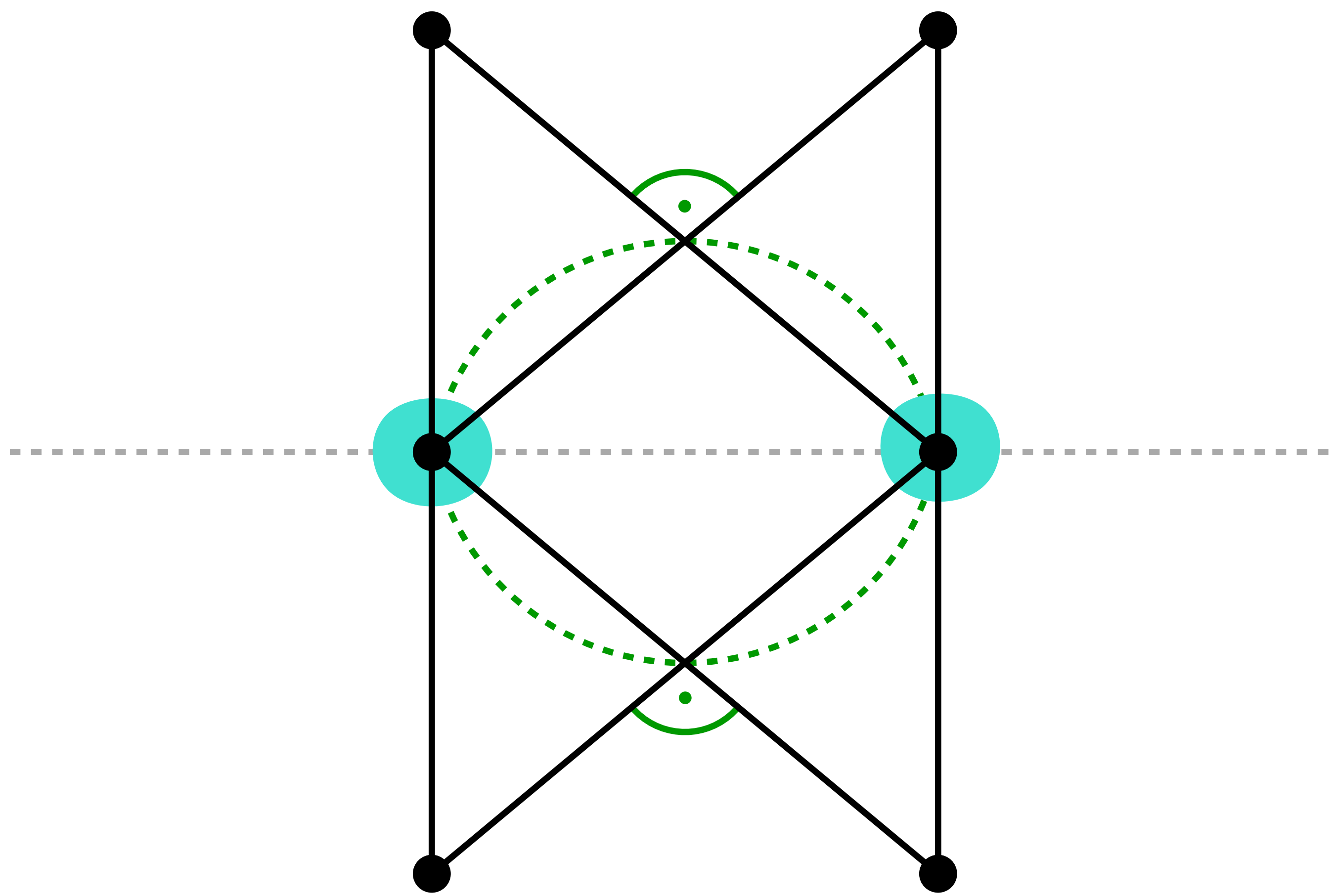

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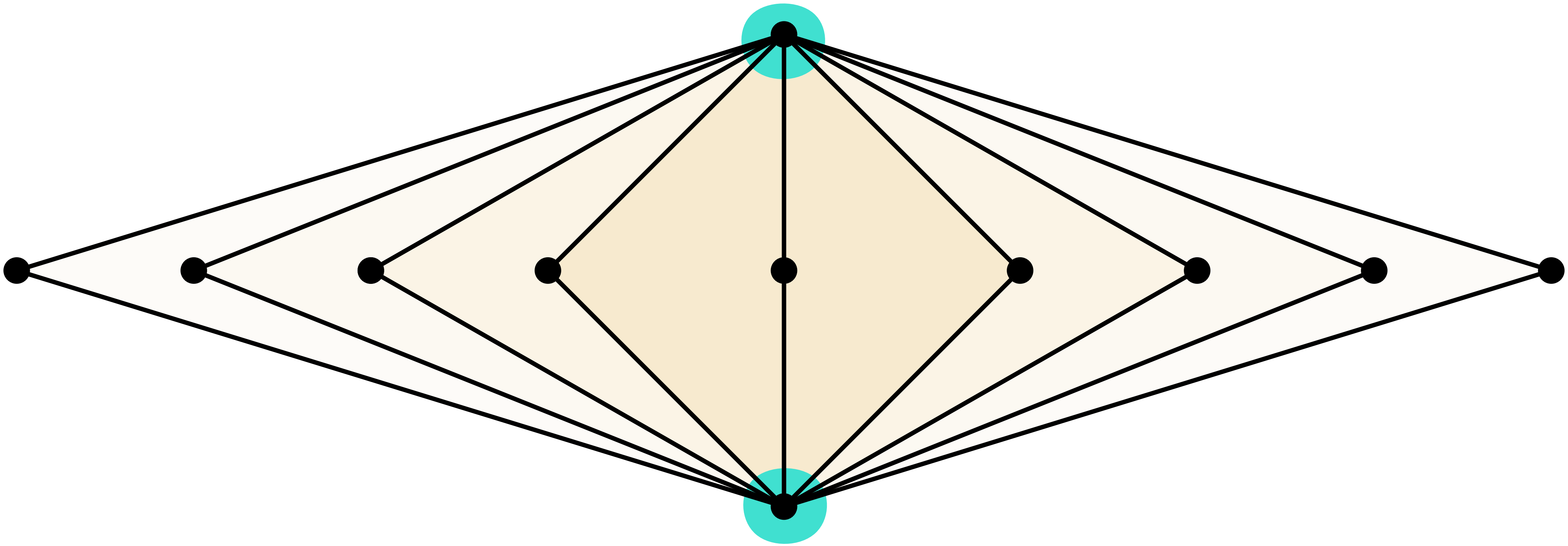
- All other vertices of T_i form a hierarchy



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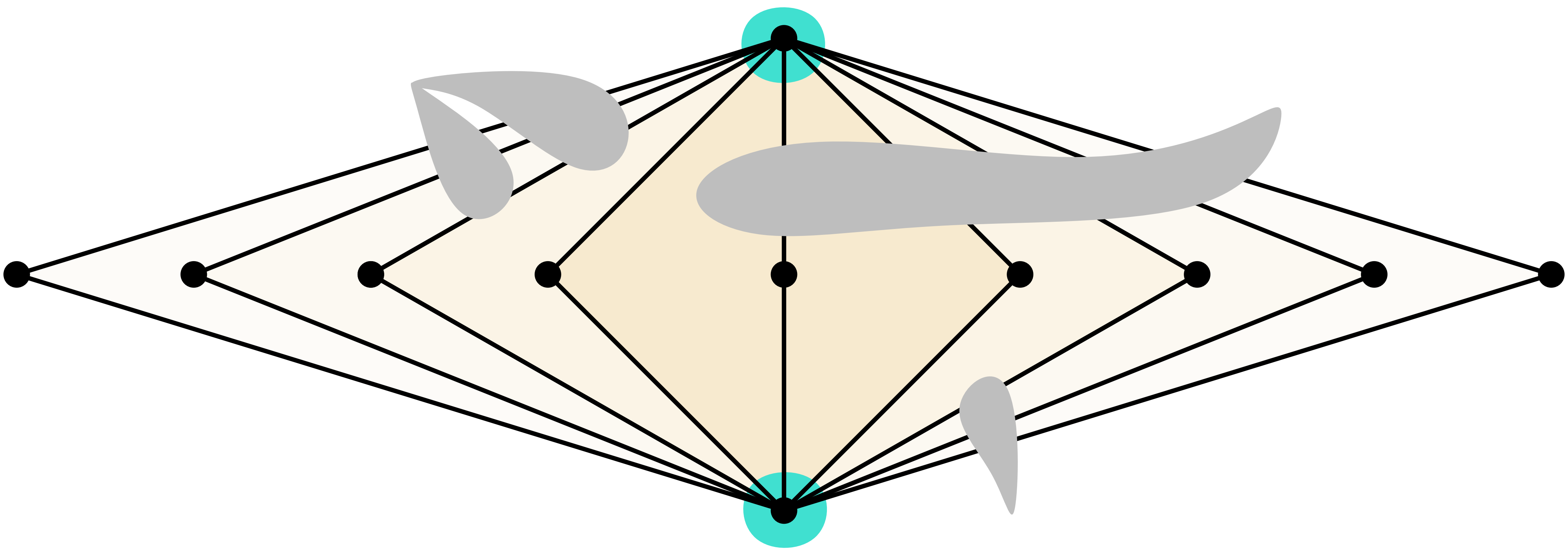

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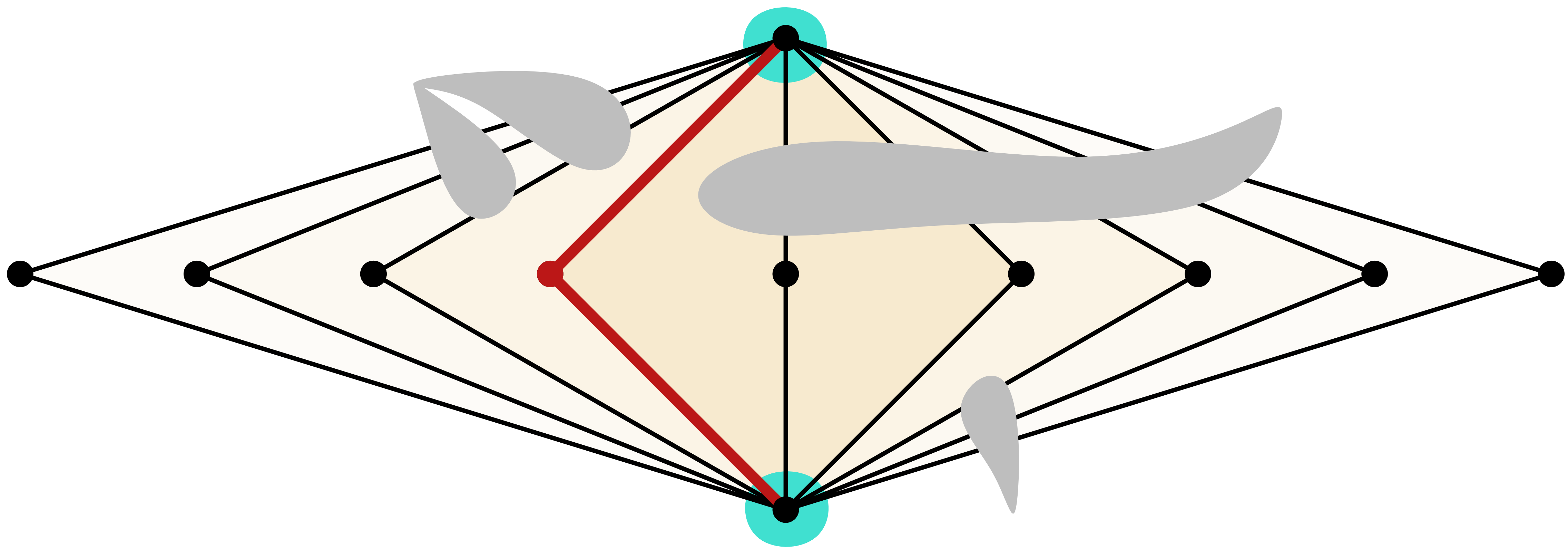

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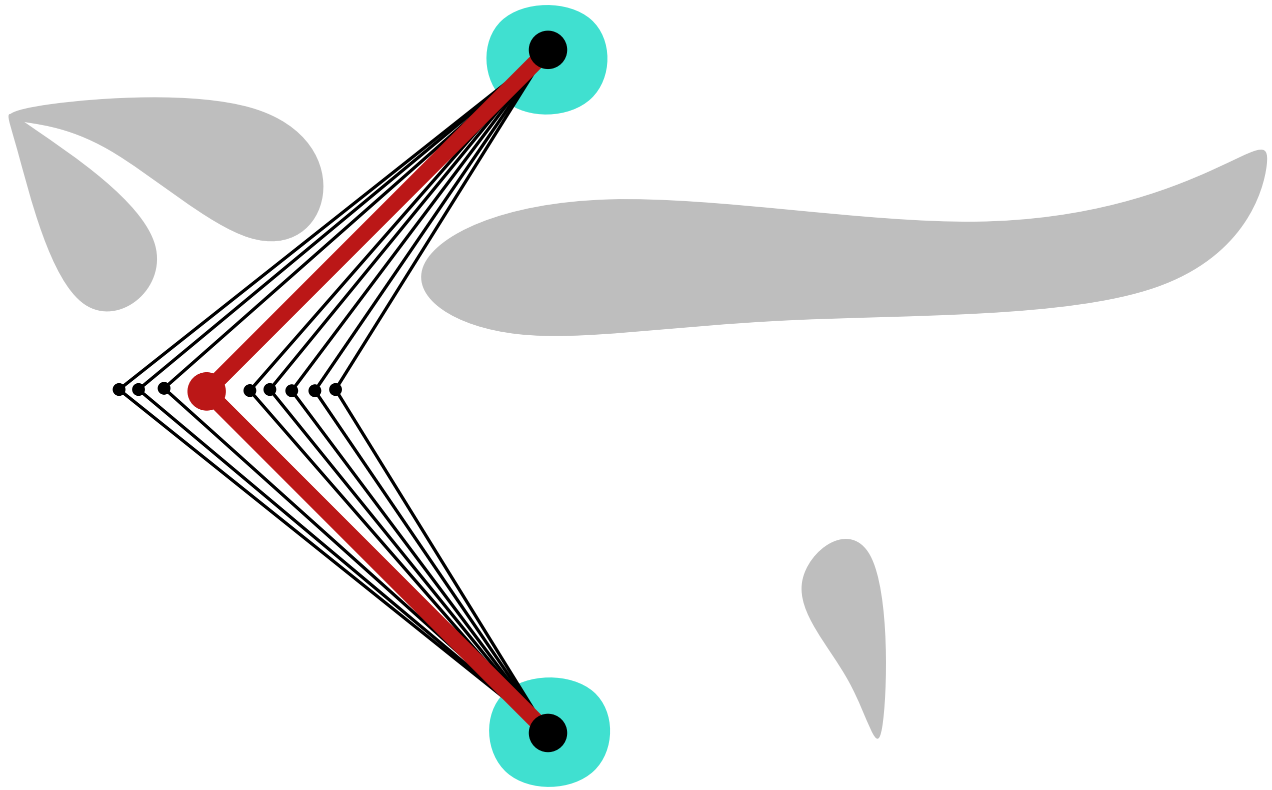

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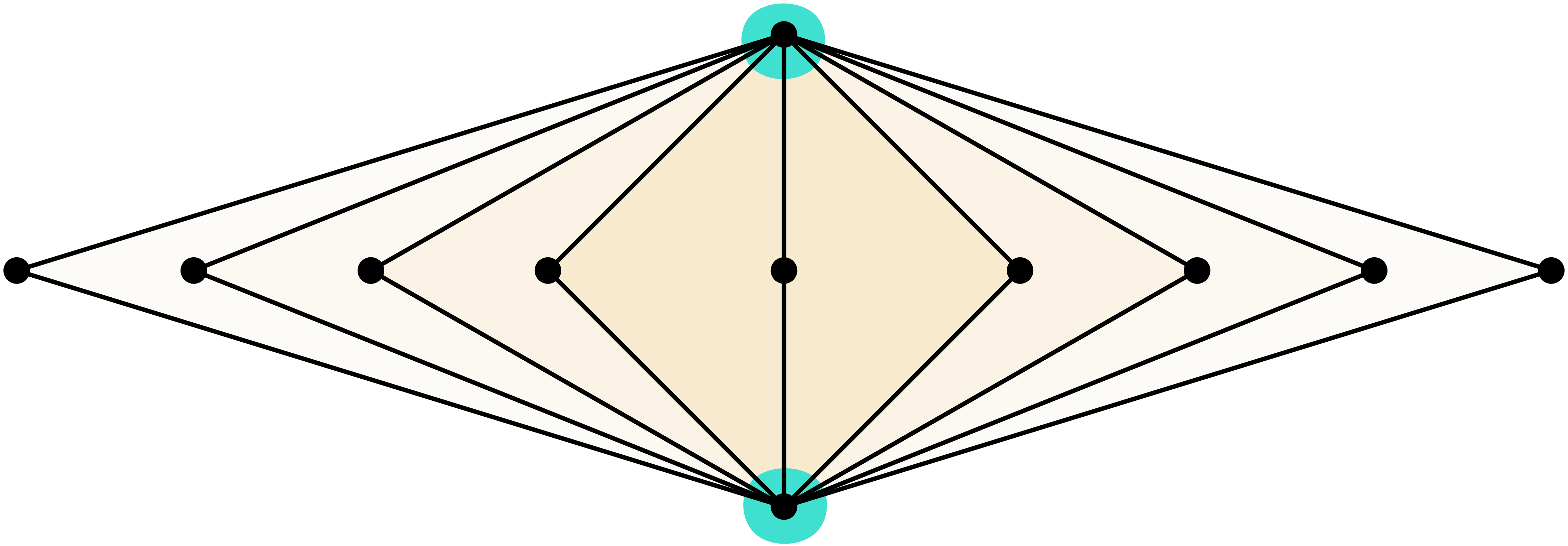

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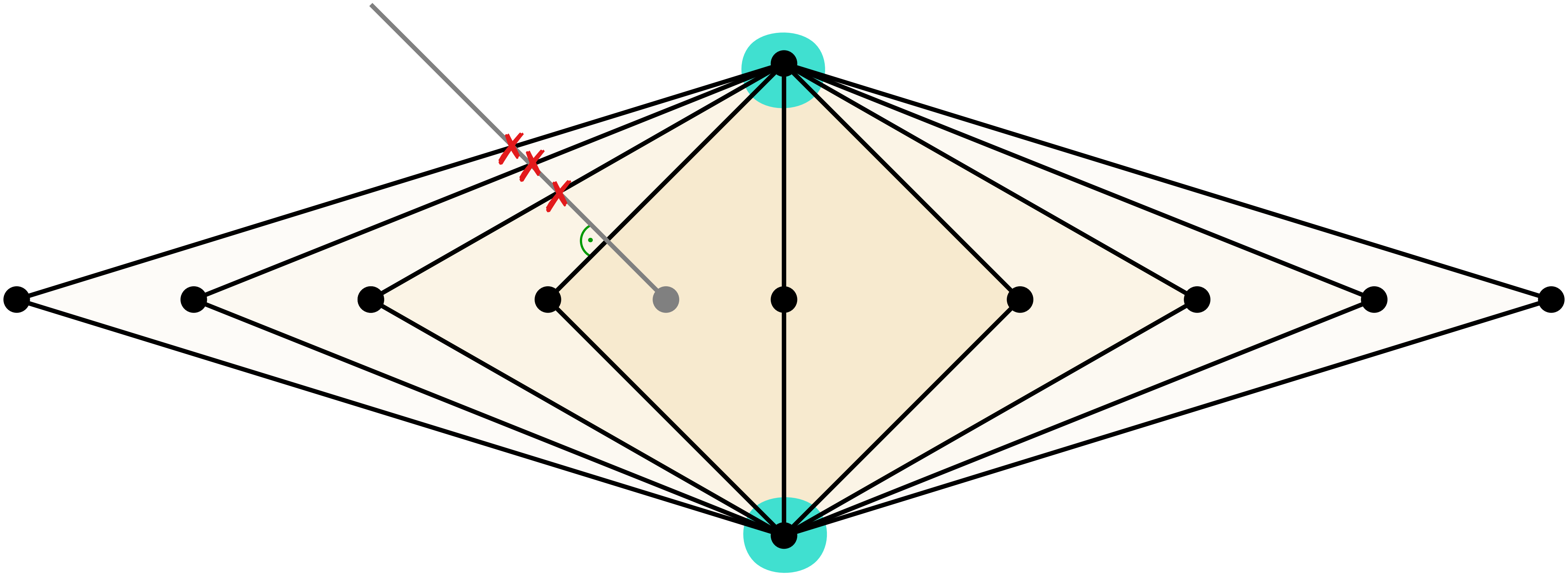

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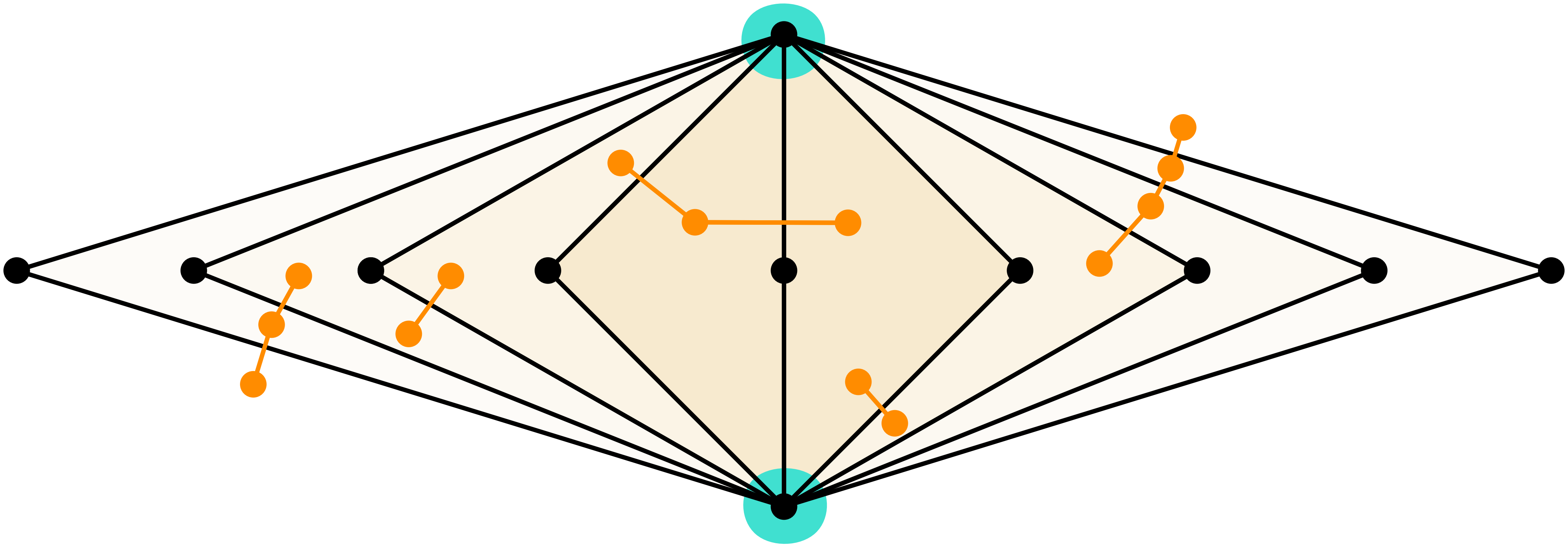

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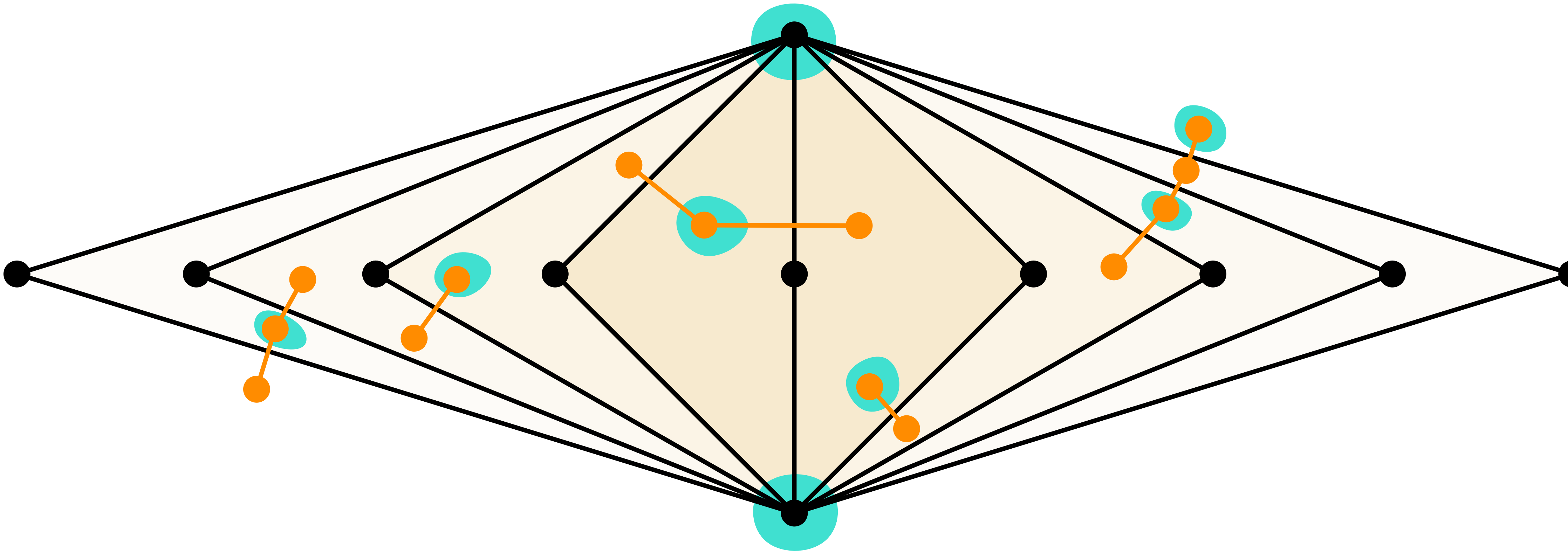

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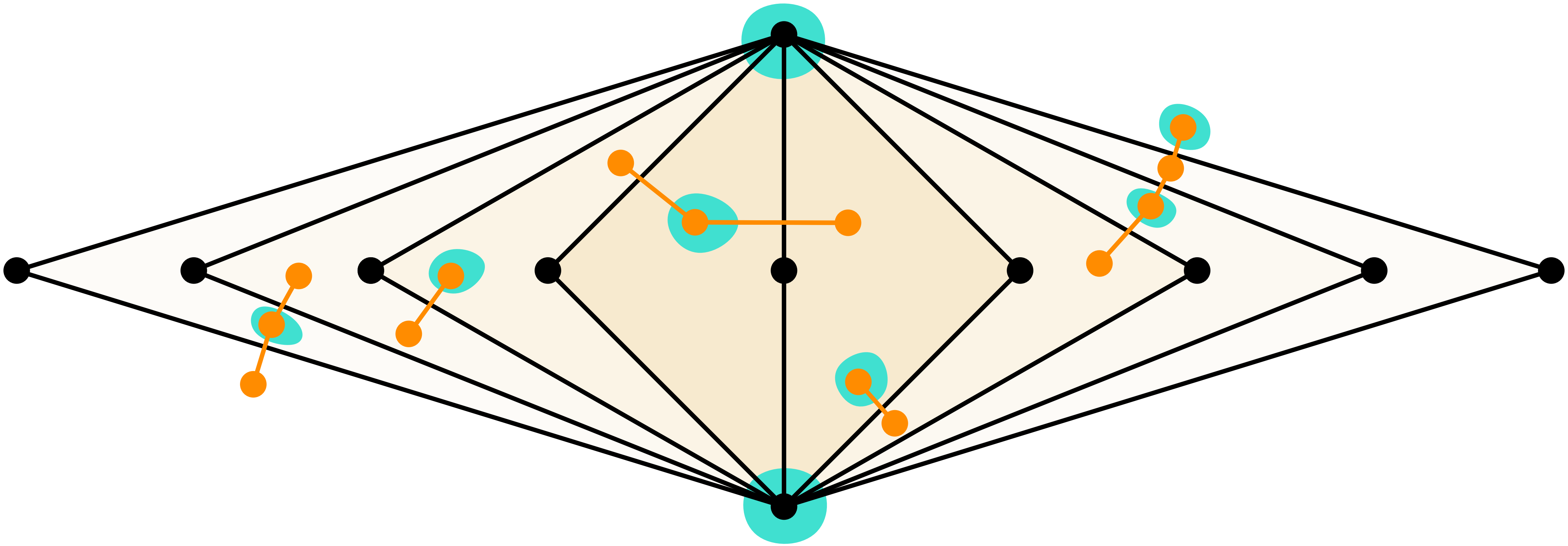

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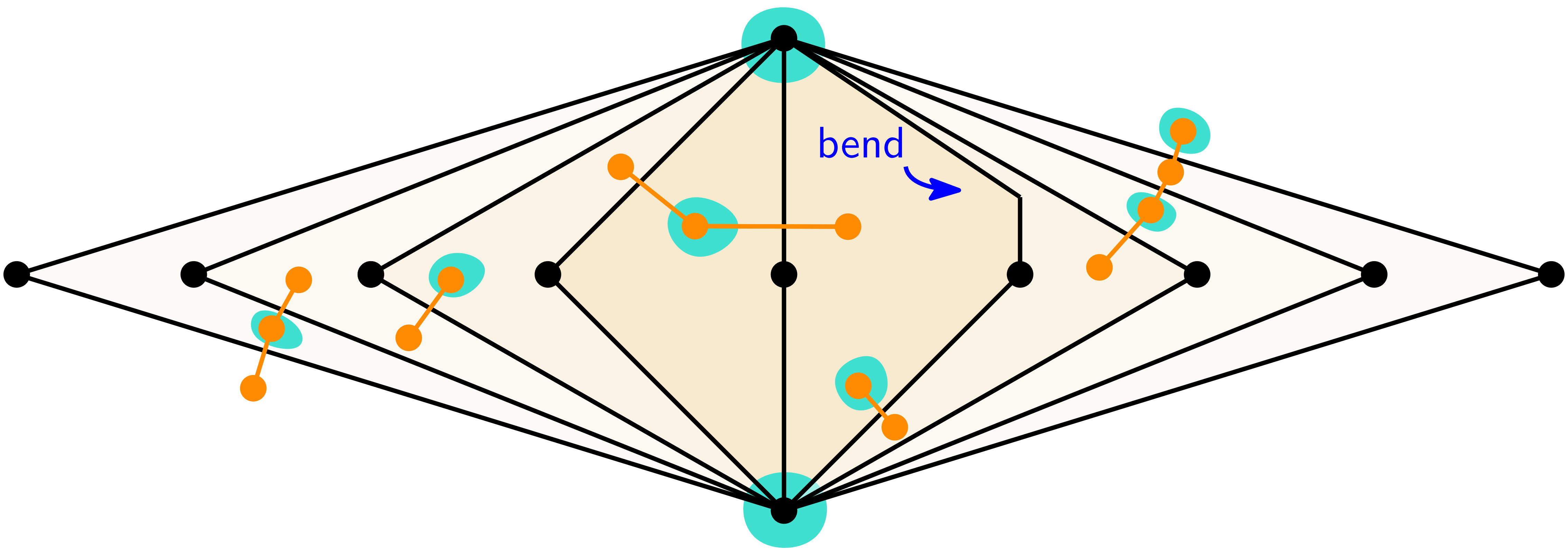

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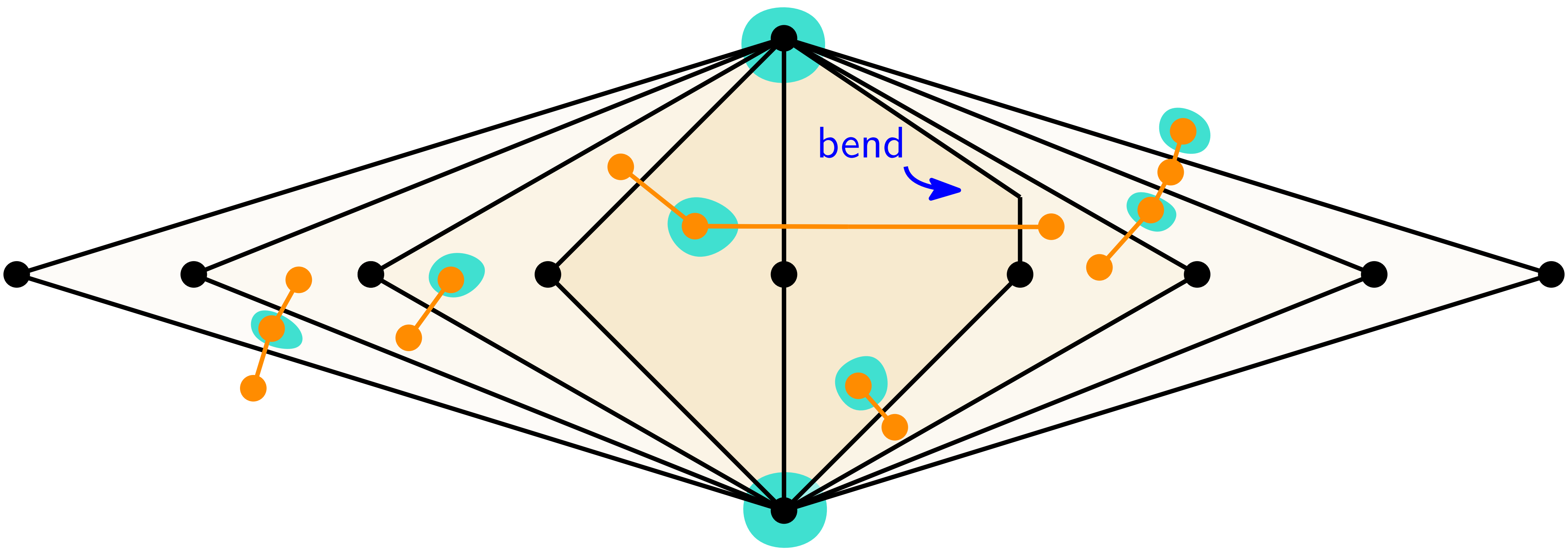

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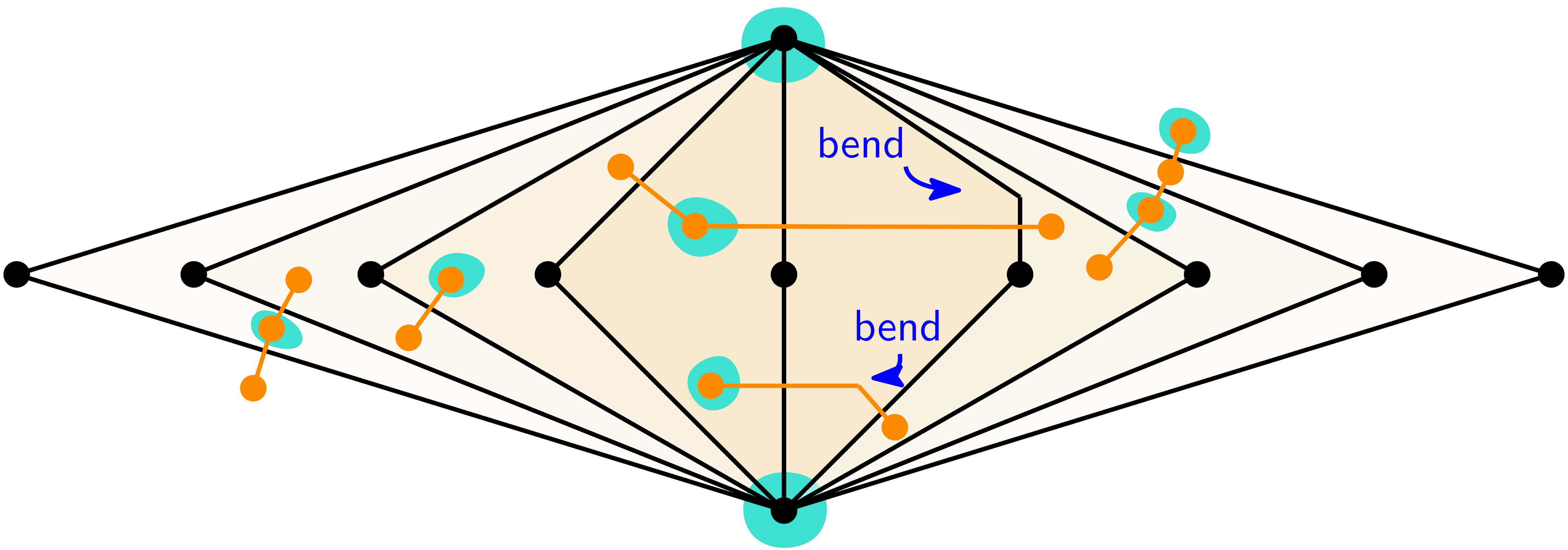

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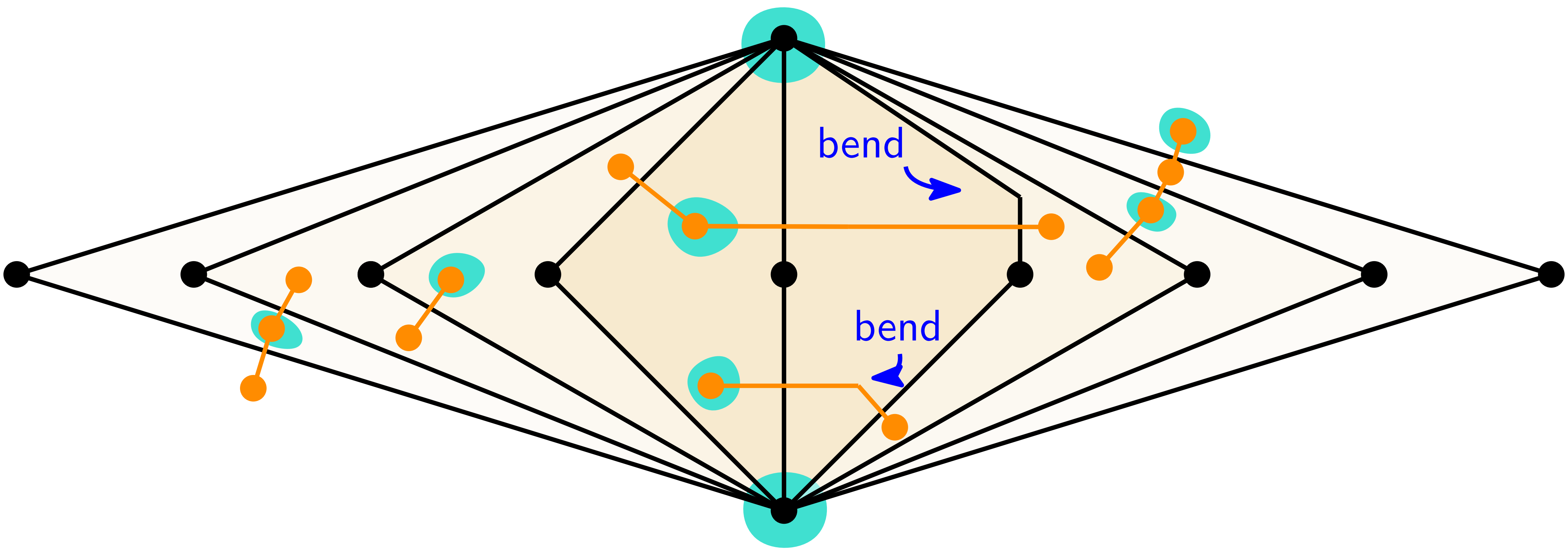

- Crossing-free edge guaranteed with $> 3k$ $+b$ bends members

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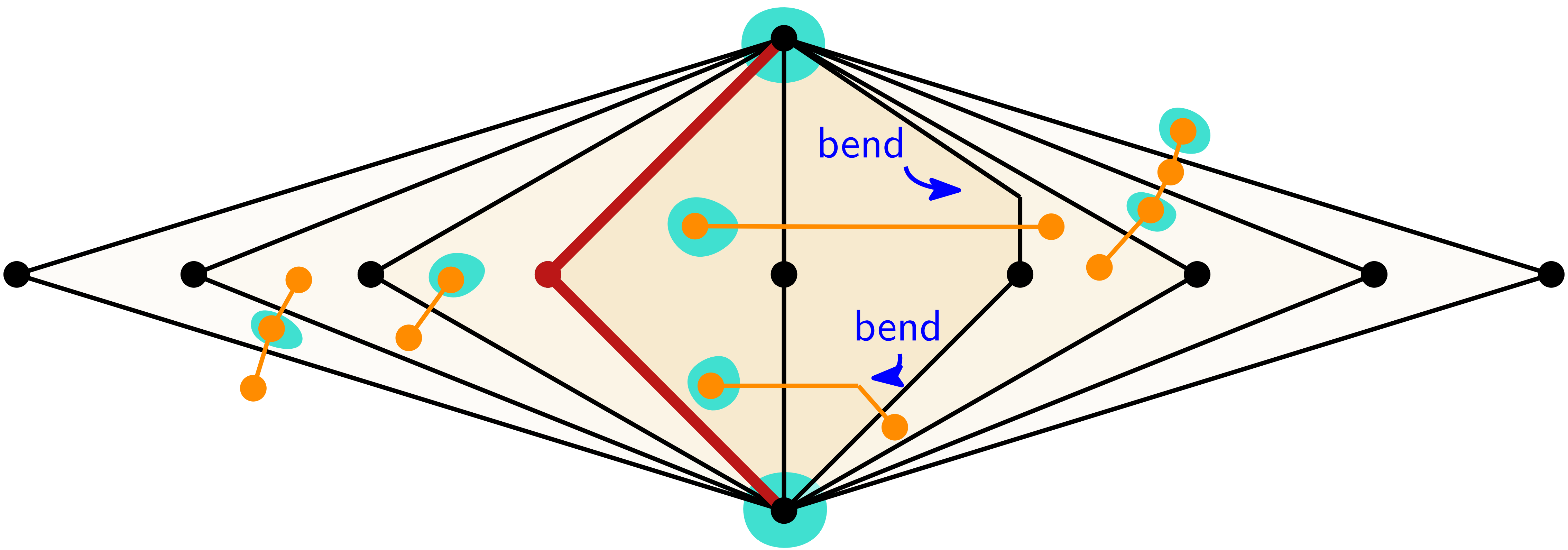

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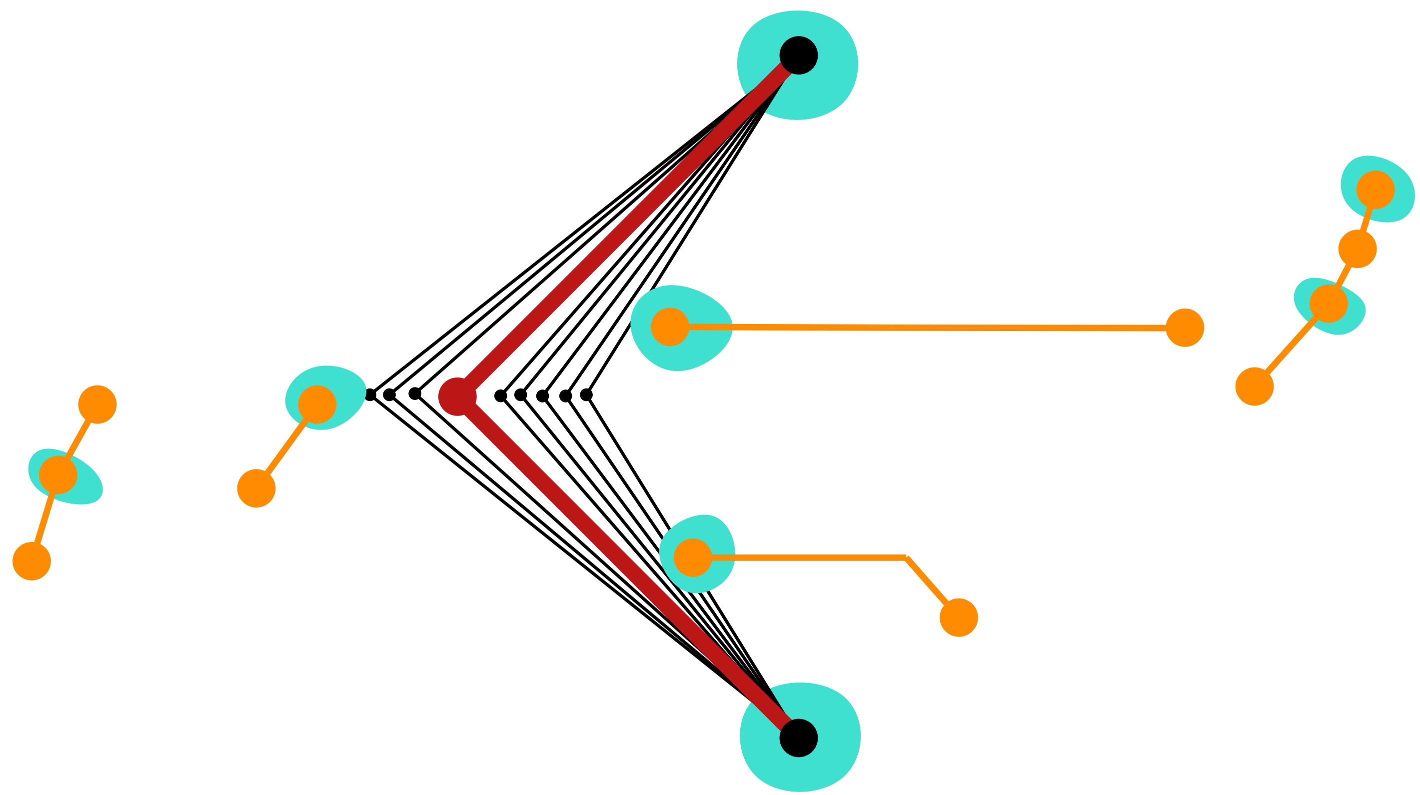

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FPT via $\mathbf{vcn}(G) + b$

- Distinguish types by cardinality $|T_i|$

Case 1: $|T_i| = 1$ $\Rightarrow$ keep 0

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Solve time $m^{\mathcal{O}(m^2)}$

Kernel size $\mathcal{O}(b \cdot 2^k)$

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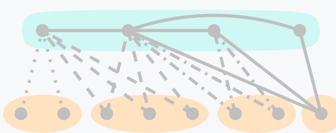
Total $2^{2^{\mathcal{O}(k+\log b)}} + \mathcal{O}(|E(G)|)$

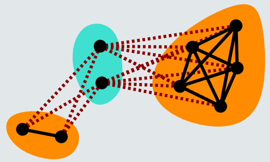
Results

- b-BEND β-RESTRICTED RAC DRAWING (BRAC) is *fixed-parameter tractable* when parameterized by

Feedback edge number $\mathbf{fen}(G)$ | $2^{\mathbf{fen}(G)^{\mathcal{O}(\mathbf{fen}(G))}} + \mathcal{O}(|E(G)|)$



Vertex cover number $\mathbf{vcn}(G) + b$ | $2^{2^{\mathcal{O}(\mathbf{vcn}(G)+\log b)}} + \mathcal{O}(|E(G)|)$



Neighborhood diversity $\mathbf{nd}(G) + b$ | $2^{b^{\mathcal{O}(\mathbf{nd}(G))}} + \mathcal{O}(|E(G)|)$

Neighbourhood Diversity

Definition. **Neighborhood diversity** $\mathbf{nd}(G)$: *min* k *s.t.* $\exists$ k*-partition* neighborhood equivalent for each vertex in same partition

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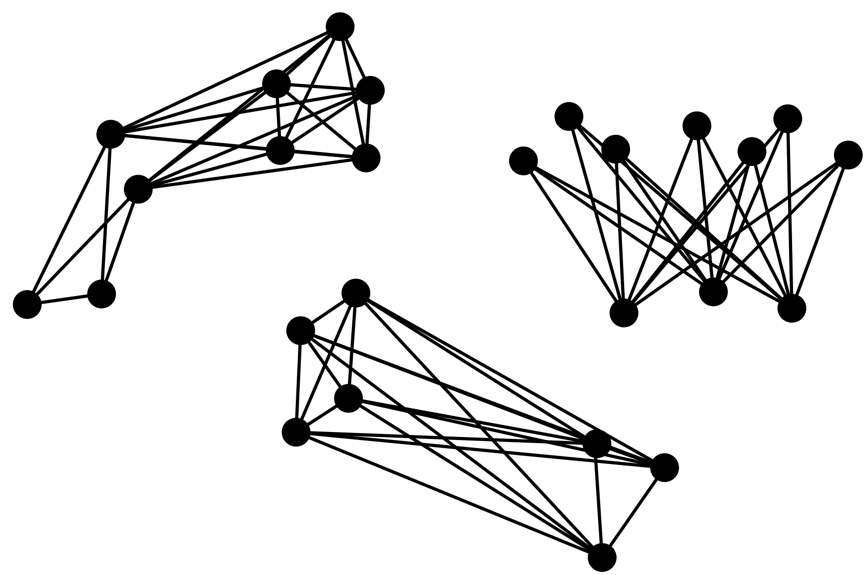

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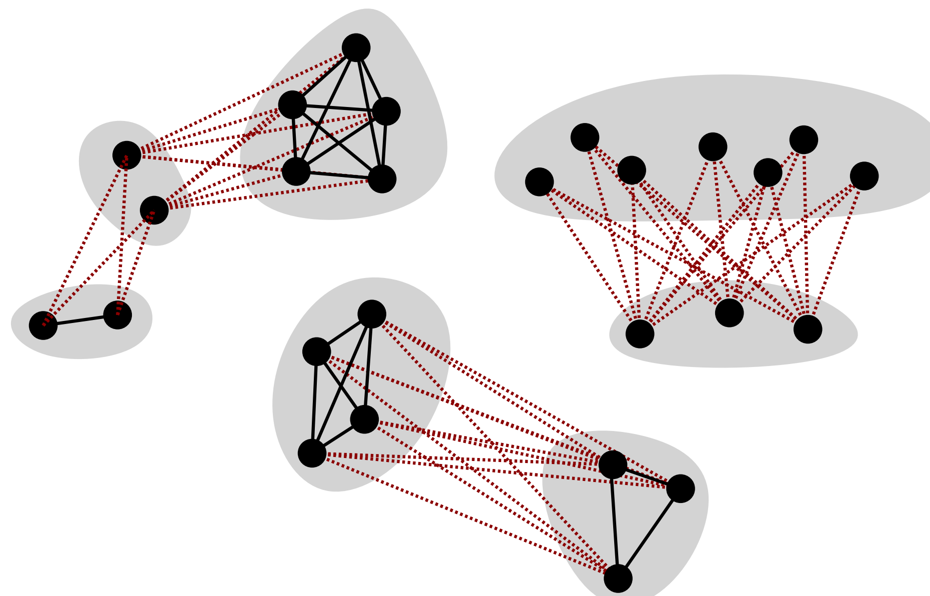

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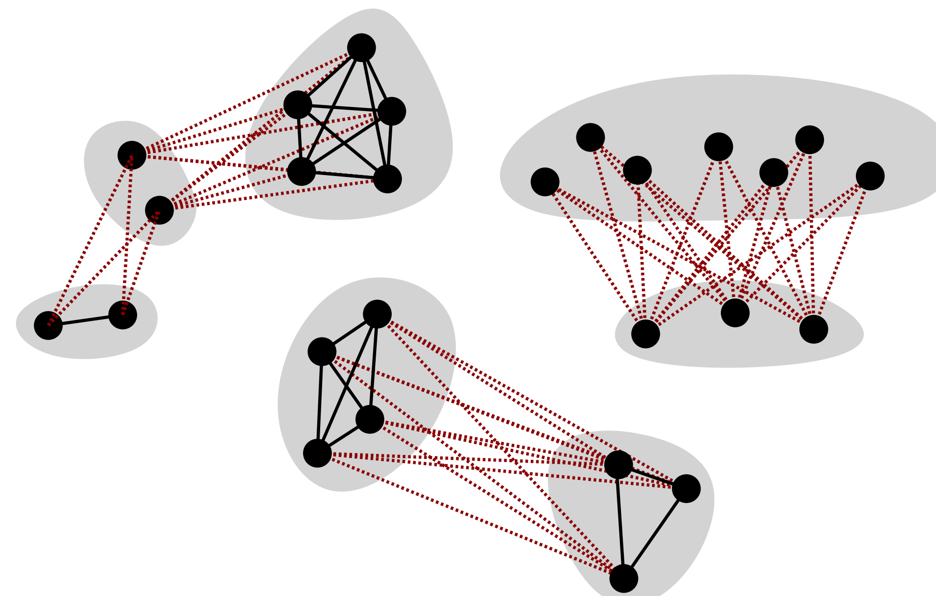

$\mathbf{nd}(G) = 7$

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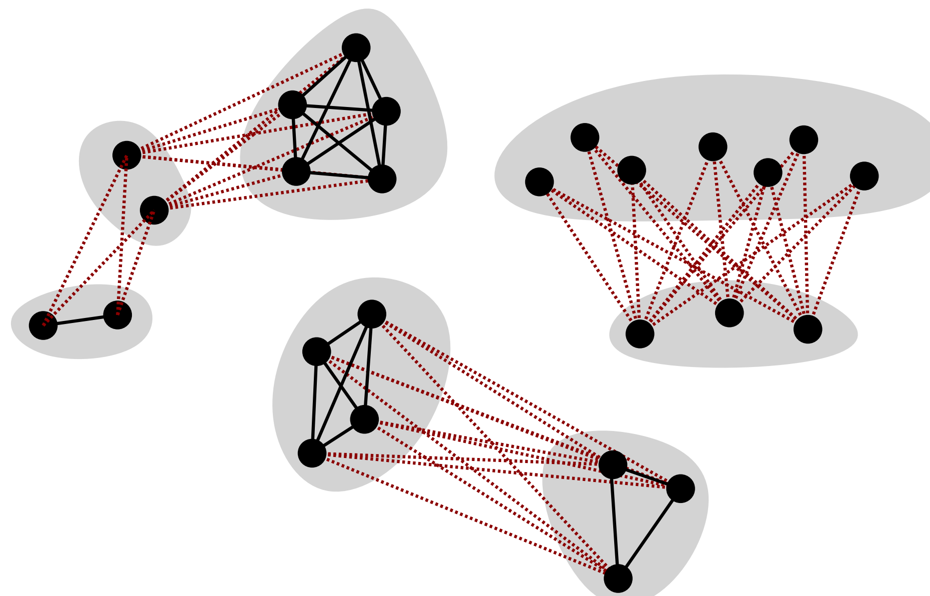

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- $\mathbf{nd}(G) \leq f(\mathbf{vcn}(G))$

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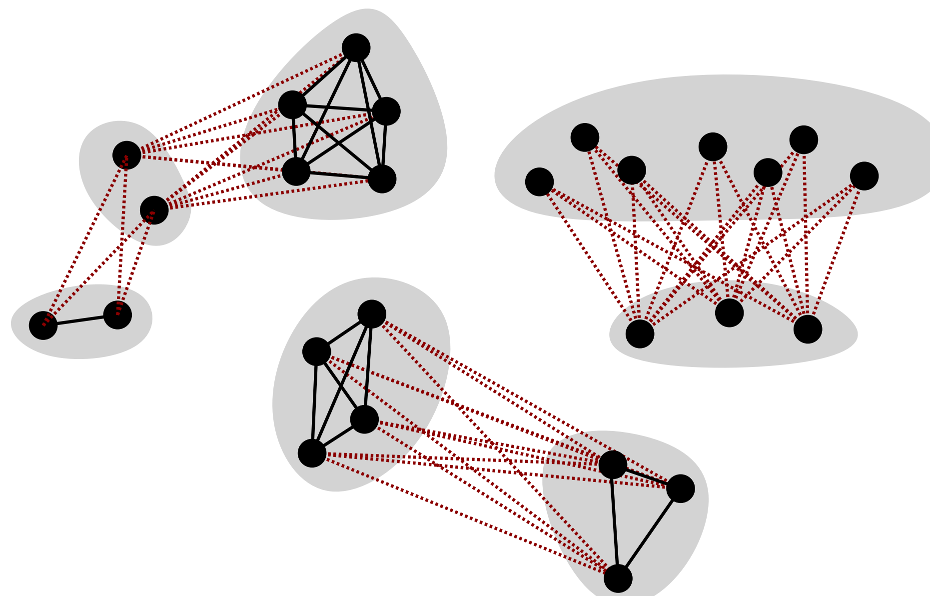

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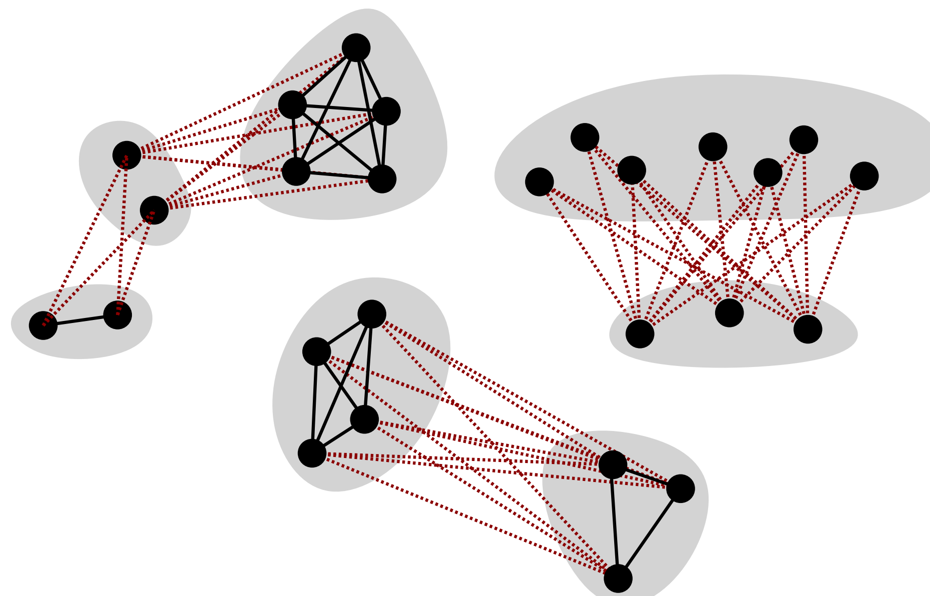

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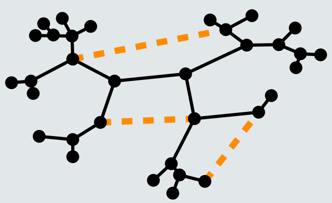
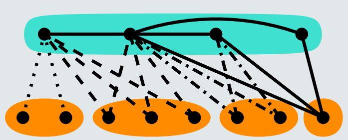
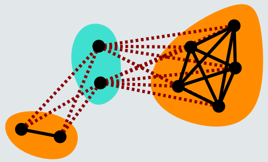
Lemma. G b-bend RAC drawable $\Rightarrow \mathbf{vcn}(G) \leq 5 \cdot \mathbf{nd}(G) + b$.

- Implies BRAC is FPT by $\mathbf{nd}(G)$

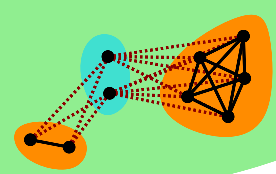
Results

- b-BEND β-RESTRICTED RAC DRAWING (BRAC) is *fixed-parameter tractable* when parameterized by

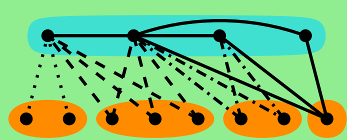
	Feedback edge number $\mathbf{fen}(G)$	$2^{\mathbf{fen}(G)^{\mathcal{O}(\mathbf{fen}(G))}} + \mathcal{O}(\lvert E(G)\rvert)$
	Vertex cover number $\mathbf{vcn}(G) + b$	$2^{2^{\mathcal{O}(\mathbf{vcn}(G)+\log b)}} + \mathcal{O}(\lvert E(G)\rvert)$
	Neighborhood diversity $\mathbf{nd}(G) + b$	$2^{b^{\mathcal{O}(\mathbf{nd}(G))}} + \mathcal{O}(\lvert E(G)\rvert)$

Open questions

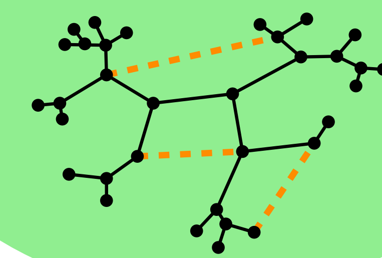
Neighborhood diversity



Vertex cover



Feedback edge set



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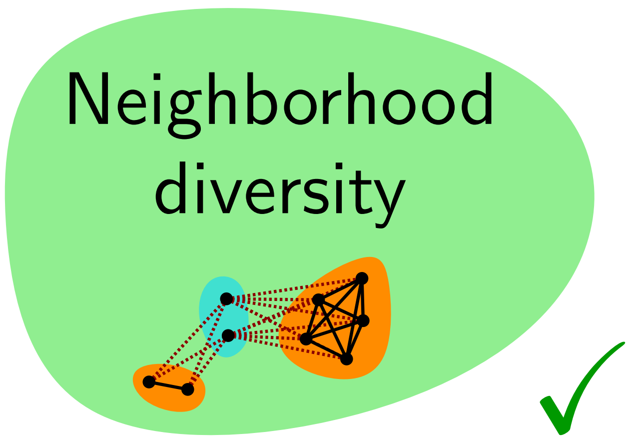
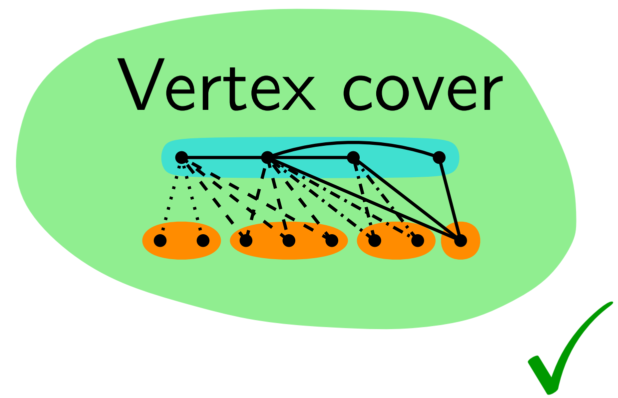
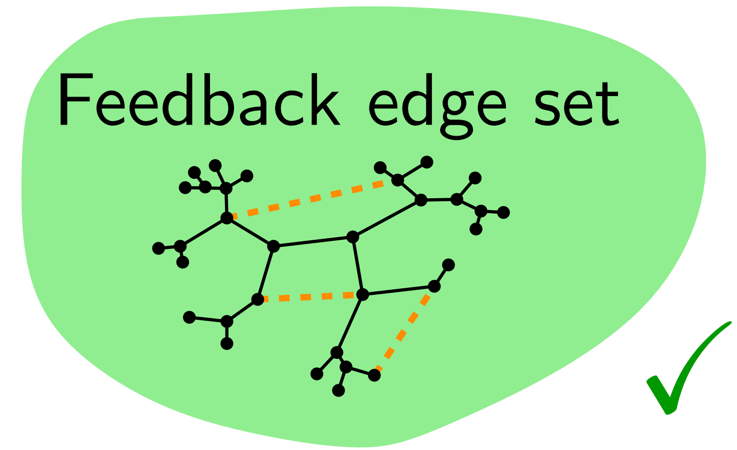
- Also FPT by $\mathbf{vcn}(G)$ alone? (instead of $\mathbf{vcn}(G) + b$)







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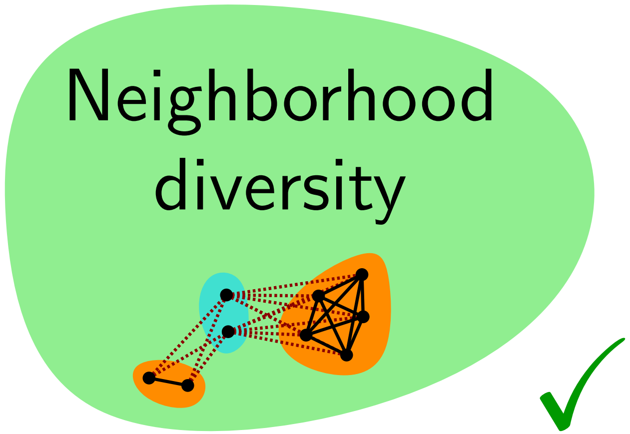
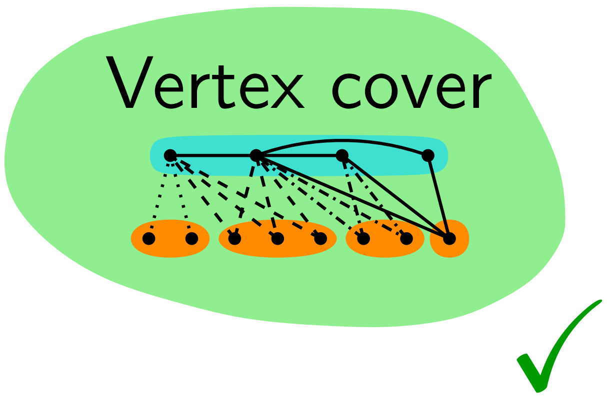
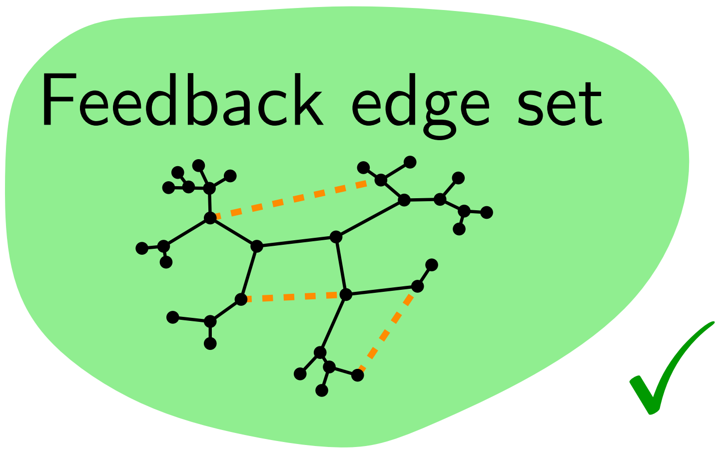
- Also FPT by $\mathbf{vcn}(G)$ alone? (instead of $\mathbf{vcn}(G) + b$)
- Obtain smaller (polynomially sized) kernels







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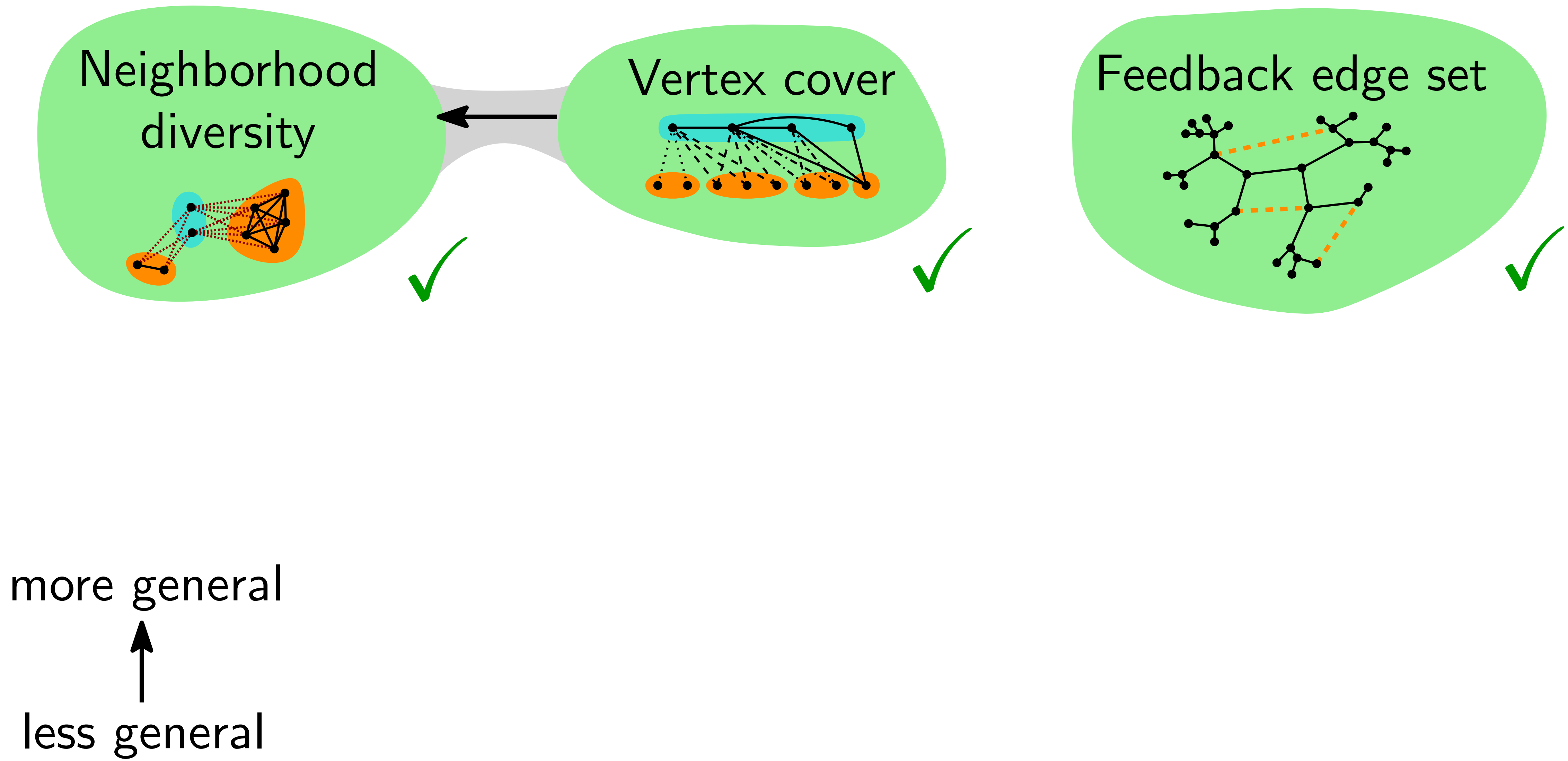

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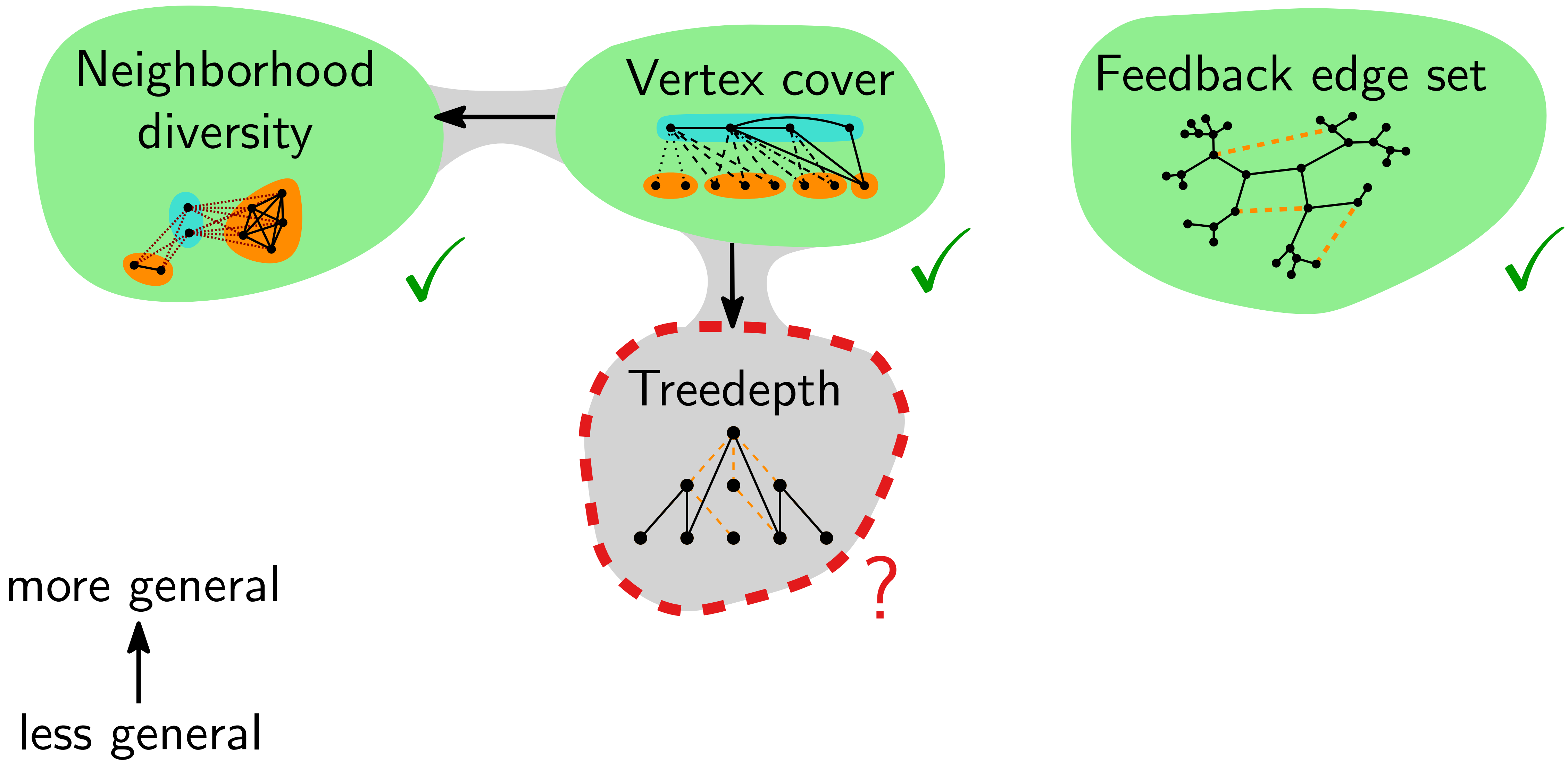

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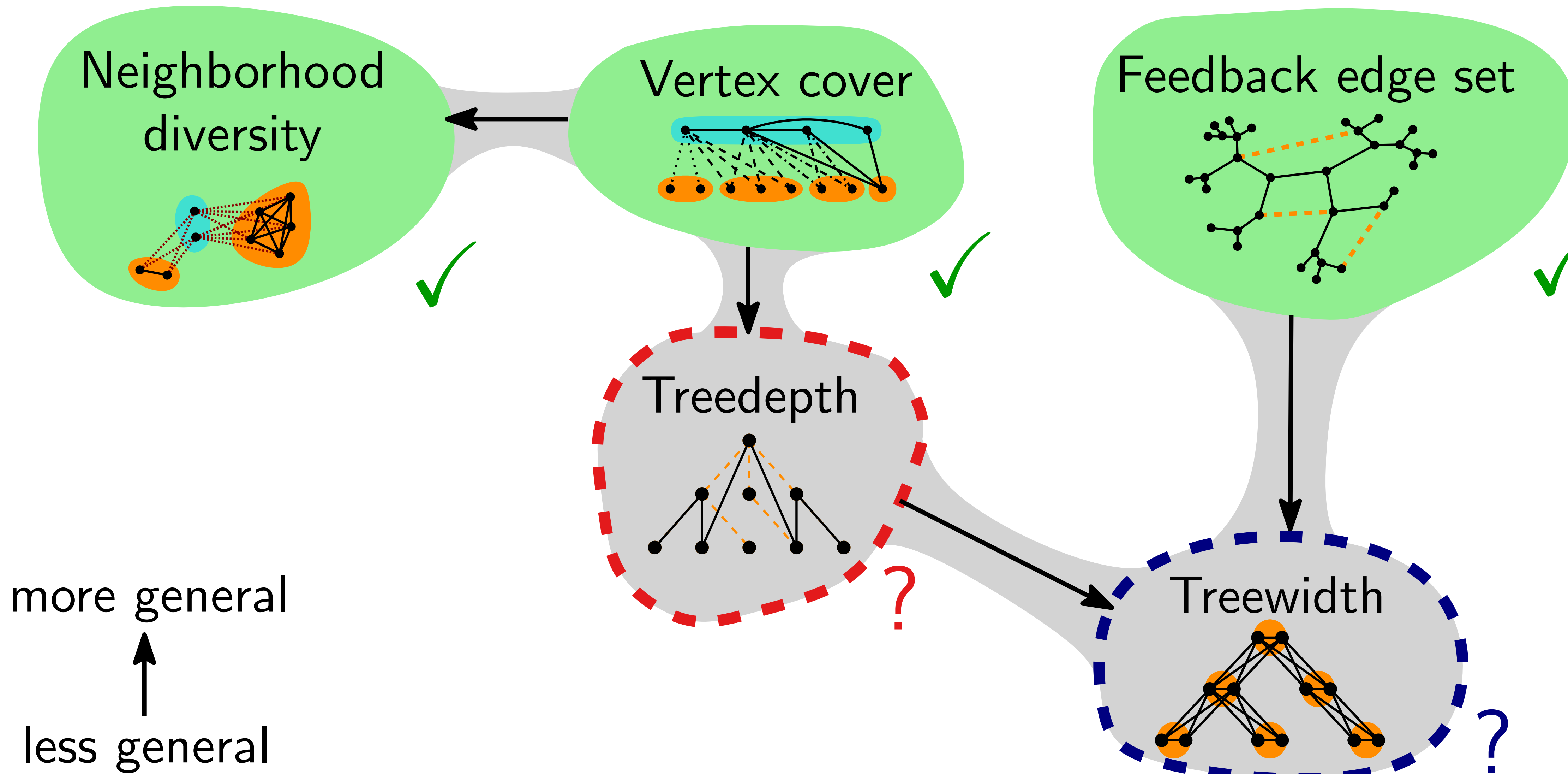

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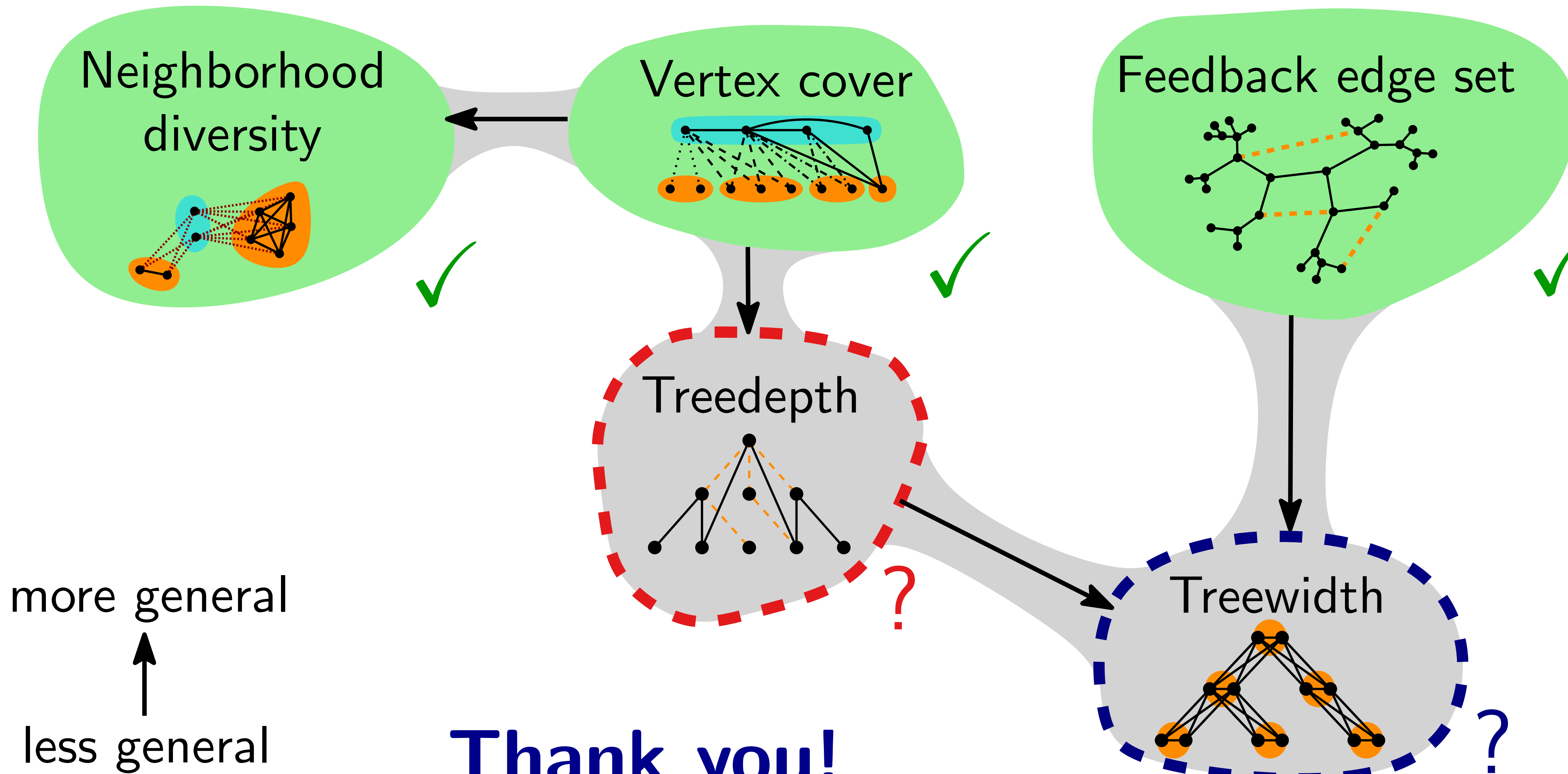

Thank you!